MODELLING OF THE BEHAVIOR OF STEEL-CONCRETE-STEEL COMPOSITE BEAMS WITH A FULL OR A PARTIAL COMPOSITE ACTION

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Abstract: Steel-concrete-steel (SCS) composite structures are modular structures combining two steel plates in the center of which a core of concrete is poured. This material presents the advantages of reinforced concrete while offering higher tightness, durability and strength under some extreme solicitations [1]. The efficiency of this structure is essentially based on the connection system between steel plates and concrete, which is performed by steel ties or dowels. Usually designed to assume a full composite action, in some cases (connector failure, construction choice, etc.), this connection may not be sufficient, which leads to the apparition of a partial composite action between steel plates and concrete core. This study focuses on the modelling of the behavior of SC structures in the case of a full or a partial composite action. This involves assessing the influence of simulation hypothesis on the global (force-displacement) and local (mechanical degradation, failure mode) behaviors and proposing a simulation methodology adapted in both cases. From the simulation of two three point bending beams [2], it is especially demonstrated that the interface conditions strongly influence the simulation results. If a "perfect" ("no-slip") relation between steel plates and concrete is sufficient for a high number of connectors, an additional methodology is necessary when a partial composite action is expected. The methodology is presented and validated by a comparison to the experimental results.

1 INTRODUCTION

Initially designed as an alternative solution for the building of submerged tubular tunnels [3, 4], steel-concrete-steel sandwich structures (SCS) are composite structures composed of a concrete core caught between two steel plates. These two components are linked by a connection system that ensures the overall behavior of the structure (Fig 1).

The main role of the structural concrete is to resist stress, especially compressive ones. The steel-plates fulfil several aims. First of all, prefabricated in factory, steel plates are employed as formwork for the concrete. This part allows a gain in the construction schedule (about 30% on the pouring of the concrete or on the complete schedule of 4% the construction work [5]). The steel plates play also the role of reinforcement in this reinforced concrete composite. Their installation in the out fiber of the structure increases the lever arm. It improves the stiffness, the sustainability and the strength under some extreme solicitations [1, 6-13]. It also allows to reduce the thickness and the overall weight [6]. These qualities make SCS structures a competitive choice for constructions especially for shear walls in high buildings, bridge decks (Fig 2), blast and impact shield walls, liquid and gas containers, submerged tube tunnels or boat hull. In the nuclear field, the SCS provides good feature to meet the need for the construction of the containment internal structures like in the AP1000 [19] (Fig 3) and the US-APWR [20].



Figure 1: Schema of a SCS structure with dowels and ties



Figure 2: Schema of a SCS sandwich deck [14]



Figure 3: AP1000 [19]

The efficiency of this structure is essentially based on the connection system between steel plates and concrete, which is performed by steel ties or dowels. The connection must ensure the bond between the plate and the core while allowing an interfacial slip to limit the stress concentration. It also contributes to the shear resistance and serves as an additional shield against plate buckling. Different types of connection system can be found (cohesive [14] or mechanical connections [3-4, 6-7, 15-16]) (Fig 4-5). For mechanical connection, the number of connectors defines the nature of the adhesion. Usually designed to assume a full composite action, in some cases (connector failure, construction choices, etc.), it may not be sufficient, leading to the appearance of a partial composite action between the steel plates and the concrete core [17-18].



The increasing use of SCS structures leads to the development of standards ([21-24]) and of experimental and numerical studies on the behavior of these structures. In this latter case, one of the main points is the modelling of the connection system and its influence on the bond between the steel plates and the concrete core. In the state-of-the-art, different modelling techniques exist for the connection: explicit modelling with 1D beam elements [25-26] or 3D massive elements [27-28], implicit modelling or homogenized materials. For example, Nguyen et al [26] propose a numerical model of a SCS wall with dowel and ties under a cyclic in-plane stress. The concrete core is modeled using 3D solid elements while the steel plates are modeled with 2D shell elements and the connectors with 1D beam elements. A perfect bond hypothesis is assumed between each component of the structures except between the core and the plates for which friction is considered. These choices lead to good results in particular for the global strength and stiffness of the wall. Another study, carried out by Yan et al [8], presents the modelling of a SCS beam with J-Hook connectors subjected to flexural strain. Each component of the structure is modelled with 3D elements. The bond between two connectors facing each other is simulated by an axial spring element whose stiffness is proportional to the stiffness of the connection system obtained through a push-out test. The core to plates contact is also modelled by friction conditions and unilateral contact to prevent interpenetration between concrete and steel. This modelling matches well the behavior law (stress-displacement curve) but some differences are noticed with the experimental results on the final strain.

The objectives of this contribution are a better understanding of the behavior of SCS

structures and the evaluation of modelling choices by comparison between numerical and experimental responses. Especially, the influence of simulation hypothesis on the global (force-displacement) and local (mechanical degradation, failure mode) behaviors will be investigated and an adapted simulation methodology will be proposed in the case of full and partial composite actions.

The selected test specimens and the modelling set up are first described. Then the simulations of two SCS composite three point bending beams with different numbers of dowels are proposed and a comparison with experimental results is done.

2 TEST SPECIMENS AND MODELLING SET UP

The behavior of two SCS beams with a high or a low number of connectors is investigated. For the comparison, the experimental study from Varma et al [2] is considered. Two beams (named SP1-1 and SP1-2) are selected from the



Figure 6: Schema of SP1 beams



Figure 8: Mesh of SP1-1 beam

2.2 Materials and models

The material properties are given in Varma et al [2].

8 SCS experimentally tested beams. The associated simulation is performed using the finite element code Cast3M [29].

2.3 Geometry and mesh

SP1 beams are SCS composite beams loaded in three point bending with a connection system between steel and concrete performed by welded dowels. The geometry is given in Figures 6 and 7.

SP1-1 and SP1-2 beams have both the same geometry except for the spacing of the connectors. SP1-1 beam presents twice more dowels than SP1-2 beam.

For the simulation, a 3D model of each beam is performed. Only one fourth of the structure is modelled, due to symmetry. The concrete core, the steel plates and the dowels are meshed with 3D solid elements. A coincident mesh is considered between connectors and the other components. The size of the finite elements is between 1.6 mm and 30 mm (Fig 8 and 9).



Figure 7: Schema of the transversal section of SP1 beams



Figure 9: Mesh of SP1-2 beam

• Concrete core

The concrete core is a self-consolidating concrete. Its characteristics are given in Table 1.

 Table 1: Concrete material characteristics

Compressive strength f_c (MPa)	42.06
Tensile strength f_{ct} (MPa)	3.15
Young modulus E _c (GPa)	33.85
Poisson's ratio ν_c (-)	0.2

It is modelled by an isotropic elastic damage model. Damage is considered through a scalar internal variable: the isotropic damage D ([30]). The behavior of the concrete can be described by the law:

$$\sigma = (1-D) E \varepsilon \tag{1}$$

where σ is the stress tensor, ε is the strain tensor, **E** the Hook tensor and $D = D(\varepsilon)$ is the damage variable:

$$D = \alpha_T^{\beta} D_T + \alpha_C^{\beta} D_C \tag{2}$$

where α_T and α_C are functions of compressive and tensile states. β is a coefficient used to reduce the damage in case of shear in concrete and D_T and D_C are the tensile and compressive damages:

$$D_{c} = 1 - \frac{(1 - A_{c})\varepsilon_{d0}}{\varepsilon_{eq}}$$
$$-A_{c} \exp\left(-B_{c}(\varepsilon_{eq} - \varepsilon_{d0})\right)$$
(3)

where $\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} \langle \varepsilon_i \rangle_{+}^{2}}$ is the equivalent strain, ε_{d0} is the damage threshold and A_c and B_c are model parameters to be identified in compression.

In tension, Hillerborg method is chosen for the energy regularization [31] by introducing the fracture energy in the formulation of the tensile damage. D_T follows the Feenstra evolution law:

$$D_T = 1 - \frac{\varepsilon_{d0}}{\varepsilon_{eq}} exp\left(-B_T(\varepsilon_{eq} - \varepsilon_{d0})\right)$$
(4)

in which D_T only depends on $B_T = h f_{ct}/G_f$ and ε_{d0} . *h* is the mesh characteristic length and G_f is the fracture energy.

The parameters of the model are identified from the uniaxial response:

- $A_c = 1.9$
- $B_{C} = 1900$ $G_{f} = 150 J/m^{2}$
- $\varepsilon_{d0} = f_{ct}/E_b$



The simulated uniaxial behaviors under tension and compression are given in Figure 10.



Figure 10: Simulated concrete behavior a) in tension, b) in compression

• Steel plates

Steel plates have an elasto-plastic behavior with an isotropic hardening. The material parameters are given in Table 2 (from [2]).

Table 2: Steel plates material characteristics

Elastic limit f_y (MPa)	448.2
Young modulus E _s (GPa)	201
Poisson's ratio ν_s (-)	0.3
Tangential modulus E_T (GPa)	0.42

• Steel dowels

Steel dowels also follow an elasto-plastic behavior with an isotropic hardening. Their characteristics are given in Table 3:

Table 3: Steel dowels material features

Elastic limit f_g (MPa)	488.8
Elastic modulus E_g (GPa)	201
Poisson's ratio $\boldsymbol{\nu}_g$ (-)	0.3
Tangential modulus E_T (GPa)	0.42

2.3 Loading and boundary conditions

SP1 beams are simply supported. The load is applied at mid-span. Sensors are used to monitor the applied load and the vertical displacements at mid-span, under the load points and at the location of the supports. Concrete cracking is also measured with gauges. The resulting experimental shear force – displacement curves are illustrated in Figure 11.



Figure 11: Shear forces – displacement curves [2]

<u>Note:</u> to take into account the setting up of the system during the loading, the initial phase of loading of the experimental shear forces - displacements curve is cut for the comparison with the simulation.

Regarding the local behavior for SP1-1 beam, the failure mode is expressed as an outof-plane shear with cracks inclined at 45° between the load point and the lower fiber of the beam. This diagonal tension cracking generally appears after the propagation of flexural cracks. Then the cracks spread to the load point parallel to the lower fiber (Fig 12).



Figure 12: Crack pattern of the concrete of the SP1-1 beam [2]

The failure mode of SP1-2 beam is different because of the lower number of connectors which allows a slip at the steel plates – concrete core interface. The failure of the beam is an interfacial shear failure due to the shear fracture of some dowels. The global stiffness of SP1-2 beam is lower than SP1-1 beam because the composite action is not fully achieved. The beam finally separates into two pieces due to a middle flexural crack (Fig 13).



Figure 13: Failure of the SP1-2 beam [2]

The loading and the boundary conditions are illustrated in Figure 14.

To take into account the symmetrical condition of the simulation, the x = L and y = 0 surfaces have respectively their displacements blocked in X direction and Y direction. The central line of the support has its displacement in the direction Z blocked. The loading is finally imposed through a controlled displacement condition in the Z direction applied at the right end line of the beam, from 0 mm to 35 mm in 100 loading steps.

A perfect bond hypothesis is imposed between the steel plates and the upper part of the support as well at the interface between the steel plates and the connectors. Different bond conditions are considered to model the interfacial conditions between the concrete core and the steel plates and between the concrete core and the connectors

These conditions will be developed in the following section dedicated to the parametric study. The numerical simulations are performed through an implicit method implemented in Cast3M [29].



Figure 14: Boundary conditions on the beams

3 SIMULATION OF STEEL – CONCRETE – STEEL COMPOSITE THREE POINTS BENDING BEAMS

A parametrical study is carry out to identify the influence of the interfacial conditions on the behavior of simulated beams and its representativeness compared to experiment. The different interfacial conditions are presented in the Figure 15.



Figure 15: Interfacial bonds

Six cases are studied (Table 4):

 Table 4: Studied case

Ca	Cases Hypothesis	
SP1-1	SP1-2	nypotnesis
(1)		Perfect bond hypothesis between steel plate
		and concrete without connector system
		(no explicit meshing of the dowel)
 Perfect bond hypothesis between steel plate and concrete with the connector system (2) (explicit meshing of the dowe considering a perfect bond with 		Perfect bond hypothesis between steel plate
		and concrete with the connector system
		considering a perfect bond with
		concrete)
(3)		Partial bond between concrete core and
	(5)	steel plates and explicit modelling of the
		connectors with a perfect bond with
		concrete and with steel plates
(4) (Partial bond between concrete core and
	(6)	steel plates and explicit modelling of the
	(0)	connectors with a partial bond with
		concrete and steel plates.

A Perfect bond supposes the same displacements between the two components while a partial bond is a "simple" contact condition.

3.1 SP1-1, a full composite action beam

SP1-1 beam includes 80 dowels (left/right and top/bottom lines of 20 dowels). It is firstly modelled with a perfect bond hypothesis between the concrete core and the steel plates (cases (1) and (2)). The simulated results are shown in Figure 16.



Figure 16: Shear force - displacement curves of the SP1-1 beams for the case (1) and (2)

Results in both simulations are close and the simulated behavior seems to be equivalent. However, in case (1) higher values of force and displacement are reached and the simulation goes further. This can be explained by a difficult convergence when the stress concentrates around the dowels due to the presence of the heterogeneity (the connectors). This analysis is confirmed by the observation of the final damage distributions for each case (Fig 19 and Fig 20). A 45° inclined shear crack is observed in both cases and spreads in the horizontal direction, close to the lower fiber (especially in case (1)). However, the damage is better distributed in the beam without connectors while it is more located around the connectors in case (2). Globally, the same results are obtained with or without an explicit meshing of the dowels with an easier convergence in the case of no-modelling of the dowels. Cases (3) and (4) are then studied. They especially introduce a partial bond between the steel-plates and concrete. The new results are compared with case (1) (Fig 17).



Figure 17: Shear force - displacement curves for cases (1), (3) and (4)

The linear phase is similar for each case but the loss of stiffness appears earlier when a contact condition is introduced. Furthermore, failure in cases (3) and (4) appears for a higher displacement but a lower strength, compared to case (1). The structure is more ductile. The comparison of cases (3) and (4) also shows that the behavior is globally equivalent except at failure. The loosening of some degrees of freedom between the concrete and the connectors leads to an increase in the strain capacity. The final damage distributions are illustrated in Figures 21 and 22.

The results confirm the previous observations: the shear fracture with the 45° inclined crack is still obtained. Damage is more located around the bond points at the location of the dowels.

The results of the most complex, and probably representative simulation (case (4)) is compared to the experimental ones (Fig 18). A good agreement is obtained. The limit strength and the displacement at failure are close to the experimental values, despite the relative complexity of the structure. However, differences in the evolution of the stiffness after the damage initiation are noticed. These differences may be explained by a possible effect of concrete heterogeneity or by the choice of concrete model parameters.



Figure 19: Damage distributions of the concrete in the SP1-1 beam, case (1)



Figure 21: Damage distributions of the concrete in the SP1-1 beam, case (3)



Figure 18: Comparison of the shear force displacement curves of SP1-1 beam for case (4) and experiment

Finally, the simulated damage pattern (Fig. 22) is rather close to the experimental damage shown in Figure 12.

This first study indicates that an appropriate simulation of a bending SCS beam with a significant number of dowels can be obtained considering the modelling of the connectors with 3D solid elements. However, a simplification of the model is possible with a perfect bond assumption between the concrete core and the steel plates and even in the case where the dowels are not represented.

3.2 SP1-2, a partial composite action beam

SP1-2 beam has 40 dowels (left/right and to/bottom lines of 10 dowels). Because of its



Figure 20: Damage distributions of the concrete in the SP1-1 beam, case (2)



Figure 22: Damage distributions of the concrete in the SP1-1 beam, case (4)

similar geometry compared to SP1-1 beam (except the number of dowels), the results with a perfect bond between the concrete core and the steel plates are the same (even if the connectors are explicitly modeled). We compare the results obtained in the case (1) with cases (5) and (6). The following curves are obtained (Fig 23).



Figure 23: Shear force - displacement curves of the SP1-1 beams for the case (1), (5) and (6)

In this configuration, the perfect bond case (1) and the partial bond cases ((5) and (6)) are significantly different. In case (1), a higher limit strength is obtained and a displacement at failure almost twice lower compared to cases (5) and (6). In this situation (lower number of connectors), the perfect relation hypothesis seems no longer valid. The differences between cases (5) and (6) are less significant but the perfect bond condition at the connectors-concrete interface results in a higher force for a given displacement. The damage distributions of cases (5) and (6) are provided in Figure 25 and 26.



Figure 25: Damage distributions of the concrete in the SP1-2 beam, case (5)

These distributions confirm that SP1-2 beam has a different behavior than the SP1-1 beam. The failure occurs with a single-vertical crack at the mid-span of the beam instead of a 45° inclined crack in SP1-1 case. This out-of-plane shear failure corresponds to the experimental failure (Fig 13). This behavior is not well simulated with the perfect bond assumption (Fig 19).

The numerical and experimental results are finally compared (Fig 24):



Figure 24: Comparison of the shear force displacement curves of SP1-2 beams for case (6) and experimental results

The experimental behavior is globally reproduced. However, the degradation phase is slightly different, probably due to the calibration process.

As a conclusion, in the case of a lower number connectors, a perfect bond hypothesis is not adapted. The partial composite action need to be taken into account through additional interface models.



Figure 26: Damage distributions of the concrete in the SP1-2 beam, case (6)

4 SUMMARY AND CONCLUSION

Steel-concrete-steel composite structures are sandwich composite structures combining steel plates and a concrete core thanks to a connection system which ensures the overall behavior of the structure. The structure combines the advantage of reinforced concrete and provides greater resistances to impact and blast forces, sustainability and durability.

All these qualities tend to make the SCS construction a competitive choice. Experimental and numerical studies are nevertheless needed to investigate their local and structural behaviors.

In this study, the simulation of the connection system was particularly studied. Two SCS composite beams, including different number of connectors were modeled. The results show that the proposed methodology was able to reproduce the main features and the global behavior of SCS beam as well as the damage mode of the concrete core. A perfect bond hypothesis between the concrete core and steel plates can be justified if there is enough connectors and a refined model including all the components and the appropriate interface conditions is not so necessary. In case of a lower number of connectors, this refined methodology becomes necessary to correctly capture the partial composite action.

These study leads to a better comprehension of the behavior of SCS beam and is a first step to the proposition of a complete simulation methodology. Future works will focus on the shear behavior of the connection system and the associated best modelling choices in case of a partial composite action. Simplifications in the 3D methodology will be particularly investigated as the proposed 3D modelling is very resource-consuming.

5 ACKNOWLEDGEMENTS

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