THE UNIVERSALITY OF B-VALUE AND SIZE EFFECT IN ACOUSTIC EMISSION: EXPERIMENTAL OBSERVATIONS IN QUASI-BRITTLE FRACTURE

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Abstract. Convergence of the b-value, the exponent of Gutenberg-Richter law, to unity near failure is considered as a universal phenomenon observed in many natural and artificial complex systems. The existence of universality implies that the underlying mechanism in various systems at the critical stage is universal too, irrespective of size, shape, and ingredient (heterogeneity) of the medium. The present work intends to discuss the size effect phenomenon on the b-value and apparent universality through experimental evidences. To study the size effect on the acoustic emission in concrete, geometrically similar, single edge notch beams of three different sizes are tested under three-point bending. The trend exhibited by critical b-value with respect to specimen size confirms its size-dependence which indeed originates from the size dependence of the fracture process zone.

1 INTRODUCTION

The systems with many interactive components are often characterized as complex systems. For such systems, the mathematical representation of the underlying dynamics is not always possible. However, despite of inherent complexity, these systems converge to a unique state after a sufficient time of evolution or at a specific critical point. This distinctive phenomenon of state convergence is widely known as *universality*. The existence of universality implies that the dynamics of the various complex systems is independent of their constituents, level of complexity and size. Power-law like longtailed distribution is the hallmark of complex system intimating self-similarity and scale free behavior of the system. Consequently, for the exploration of complexity, simple statistical approach is adequate than complicated mathematical equations. This is the beauty of Gutenberg-Richter law. The Gutenberg-Richter (GR) [1] law is an excellent archetype of simplistic approach devised for representation of the seismic complexity. The GR law can be expressed as follows,

$$log_{10}(N > m) = a - bm \tag{1}$$

where *N* is the number of events greater than magnitude *m*, *a* is referred as productivity and *b* is the negative slope of cumulative frequency distribution (CFD) on a log-linear scale. GR can also be expressed as an exponential distribution of magnitudes as follows,

$$f(m) = \beta e^{\beta(m-m_{th})} \qquad m > m_{th} \tag{2}$$

where m_{th} is threshold magnitude and β is the rate parameter. The maximum likelihood estimate of b-value is given by Aki [2] as,

$$b = \frac{log_{10}(e)}{\mu - m_{th}} \quad (3)$$

where μ is arithmetic mean of the observed magnitudes. Equations 1 and 3 are related by $\beta = b/log_{10}(e)$.

The application of the GR law is not only constrained to seismicity but it is also applicable in many other areas too. Due to its broad generality, the GR law has received criticism in recent years. The self-similarity and universality implied by GR law has become questionable. Although the existence of universality and scale-invariance has been justified on the basis of self-organized criticality in literature, self-organization itself is not yet well understood [3, 4].

The acoustic emission (AE) phenomenon of material cracking resembles to seismogenesis and the GR law has been used extensively for *b-value* analysis of AE events. Acoustic emission occurs due to sudden release of energy in the form of stress waves which travel through material and can be acquired using piezoelectric sensors. Fracture process in concrete like heterogeneous material is a result of multi-scale cracking. Various sized cracks interact to make fracture process correlated and long-ranged consequently resulting in power-law like long-tailed magnitude distribution. Hence, it is appropriate to consider it as a complex system.

The GR law, as expressed in Equation 1, is a linear relationship between the number of events and magnitudes on the semi-log scale. However, such a linear relationship is only valid for a finite range of magnitudes [5, 6]. The linearity of the GR law facilitates the extrapolation of large size earthquake in seismology as a consequence of intrinsic self-similarity of power-law. While the b-value analysis in structural health monitoring (SHM) is performed to predict the critical state of a structural element as the b-value converges to unity. This basic difference in the application of b-value in both seismology and SHM is important and needs to be enunciated.

In spite of simplicity, self-similarity and universality of the GR law, the recent studies in seismology have raised alarm against the validity of GR law [7]. Such concern has not been addressed in AE studies and GR law remains persuasive. We demonstrate shortcomings of GR law using AE in concrete through experimental evidence.

The present work is organized as follows. The factors affecting b-value analysis is briefed in Section 2. The experimental details are given in Section 3. Observational insights of fracture process zone are summarized in Section 4.1. Size-effect on GR b-value is discussed in Section 4.2. Finally we conclude the arguments for refutation of the GR law and the other notions attached to it like scale-invariance, universality and criticality in Section 5.

2 Variability of b-value

The factors affecting b-value can be classified as material, numerical and geometry dependent [7]. Heterogeneity, bond strength between particles, existing flaws, and porosity are the few material dependent factors which may cause variability in b-value. In the literature, majorly two methods are used for b-value determination namely by least square fitting (Equation 1) and Aki's maximum likelihood method (Equation 3). It is known that both of these methods are biased towards either large or small magnitude events. Therefore, use of such biased method also causes errors in b-value. From the numerical point of view, improper sample size and magnitude binning also cause variation in b-value. Above all these, the major cause of variation in b-value is due to geometry and boundary conditions of the specimen. Although all these factors are important, the present work mainly focuses on the effect of size on b-value. Size effect is not a new concept for fracture community, and accounting it, is the main concern of the many fracture studies. The energy dissipation by fracture process zone (FPZ) in concrete is considered to be the reason for the size effect in concrete [8]. Development of FPZ is dependent on the material but also on the size of the specimen too. The length and width of FPZ changes during crack propagation according to prevailing stress conditions and hence micro to macrocrack proportion changes with specimen size in spite of geometrical similarity. Usually, larger specimens fail with relatively larger FPZ volume than smaller size specimens. To demonstrate the size-effect on b-value, we experimented on geometrically similar concrete beams. The experimental details are given in following section.

3 EXPERIMENTAL SETUP

An experimental program is designed to study the b-value behavior of geometrically similar plain concrete beams of three different sizes. The beams are cast from the same concrete mix. The mix design of concrete is done using the ACI method and the mix proportion of the cement, fine aggregate and coarse aggregate obtained is 1:1.86:2.61 by weight. A water to cement ratio of 0.5 is used for preparing the concrete mix. Table 1 gives the geometrical details of the beams. A computer controlled servo-hydraulic machine is employed for testing the beams in flexure under three-point loading using crack mouth opening displacement (CMOD) control. Monotonic loading rate is set to 1μm/sec for all specimens. Midpoint deflection of beams is measured using a linear variable differential transformer (LVDT), while the load is recorded using a load cell of 35 kN capacity.

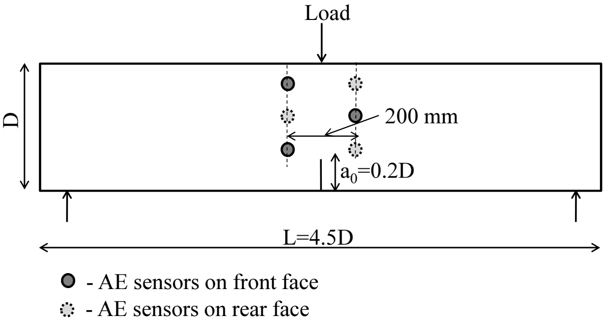

Figure 1: Schematic diagram of the large sized beam with AE sensor location

Table 1: Details of beam dimensions

Designation	Depth (mm)	Width (mm)	Span (mm)	Notch (mm)
Small	75	50	337.5	15
Medium	150	50	675	30
Large	300	50	1350	60

A Physical Acoustic Corporation (PAC) system is used to monitor the acoustic emission throughout the test. Six resonant type R6D AE sensors are mounted on beams as shown in Figure 1. R6D sensors having sensitivity and frequency response over the range of 35 - 100 kHz with a resonant frequency around 55 kHz are used. Due to weak strength of AE signals, preamplifiers with 40 dB gain were set along with a threshold limit of 40 dB for background noise reduction. The sampling rate of 1 MHz was used to ensure good time and signal frequency resolution. Signals below the threshold level are neglected.

4 RESULTS

4.1 Fracture process zone

Development of microcracks around crack tip, known as fracture process zone, depends on the size of the specimen. Numerous studies have shown that the width and length of FPZ varies over crack path influenced by boundary and stress conditions at the point during crack propagation. The relative size of FPZ with respect to specimen dimension dictates its behavior which tends to that of linear elastic fracture mechanics if negligible size of FPZ exists. But quasi-brittle concrete shows significant FPZ size which makes its behave nonlinearly. In geometrically similar beams, in spite of dimensional proportionality, the size of FPZ does not vary linearly proportional to specimen size. Consequently, the nonlinear fracture process zone enroots the size effect [8]. AE is a passive technique used to acquire stress waves generated by cracking in the material. The localization of a crack sources is possible by triangulation method using arrival time difference of the stress waves at multiple sensors location. The localized events provide an overall perspective of FPZ in a tested beam as shown in Figure 2. As crack propagates, the FPZ develops around crack tip with some proportion of the both micro and macrocracks. The distribution of different crack sizes over a crack path also depends on the degree of material heterogeneity which causes strength dispersion locally around crack path. Unlike brittle materials which exhibit single dominant crack leading to failure, the fracturing process of quasi-brittle materials is competitive and interactive process of multi-scale cracks [9]. Microcracking is an useful phenomena which occurs in quasi-brittle material allowing to dissipate applied energy and redistribute stresses. Although numerous microcracks occur ahead of crack tip, some of them remain idle and rest other coalesce to form macrocrack. The relative location of cracks suggests that the macrocracks occur close to main crack while microcracks occur widely dispersed around the main crack. Therefore, macrocracks are mostly constrained to two-dimensional space where two new surfaces of the main crack originate. On the other hand, microcracks are not restricted and can occur in three-dimensional volume as observed in Figure 2. Otsuka *et. al.* [10] observed similar behavior in concrete using AE with X-ray and proposed a sub-zone within the FPZ named fracture core zone (FCZ) as shown in Figure 3. Consequently, maximum width of FPZ increases with increase in size of the specimen as shown in Figure 4. Hence, relatively higher number of microcracks occur as specimen size increases.

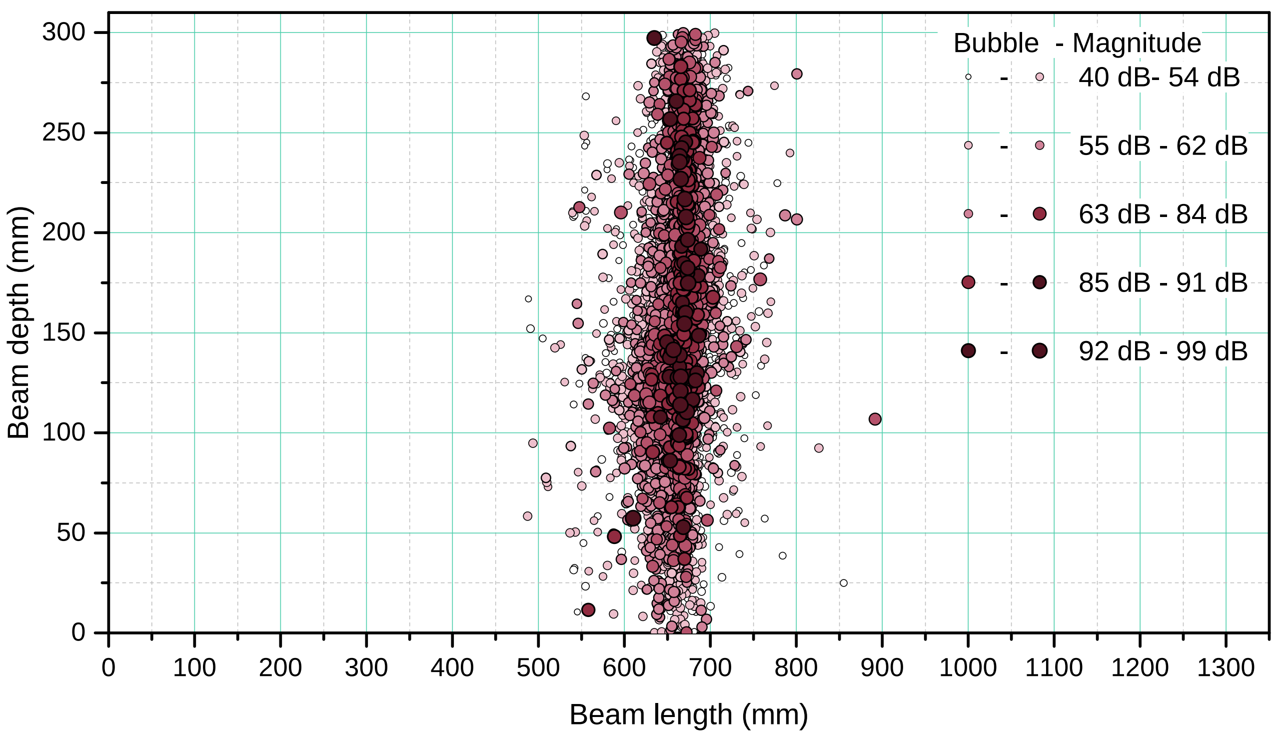

Figure 2: Localized events with color-coded bubbles representing specific magnitude range of events

4.2 AE b-value

In the present work, the b-value of AE events is determined using both equations 1 and 3. The variation in b-value for three beam sizes is shown in Figure 5. The b-value determined using Equation 1 is denoted as b_{LS} (Least Square) and b_{ML} (Maximum Likelihood) denotes b-value evaluated by using Equation 3. Due to the widening of FPZ width as shown in Figure 4, relative number of small magnitude events increases with the increase in size, therefore, the slope of the fitted line increases resulting in higher b-value. It should be noted that the GR relation does not fit well in small magnitude range. The b_{LS} is considered biased towards large magnitudes contrarily b_{ML} is biased towards small magnitudes. In spite of biases, the b-values show the expected variation with size. Although the range of variation in b-value is not small, the average trend is noticeable to consider the size effect on b-value. Range of b-value has been reported from 0.5 to 1.5 for concrete like quasi-brittle material and the presented b-values are well within the specified range. Figure 6 shows the variation of b-value with respect to the maximum width of FPZ. The maximum width of FPZ is determined from standard deviation of localized events although many other methods are available in literature. According to universality implied by the GR law, the b-values should converge to unity near failure irrespective of specimen size. Such convergence to unity appear dubious.

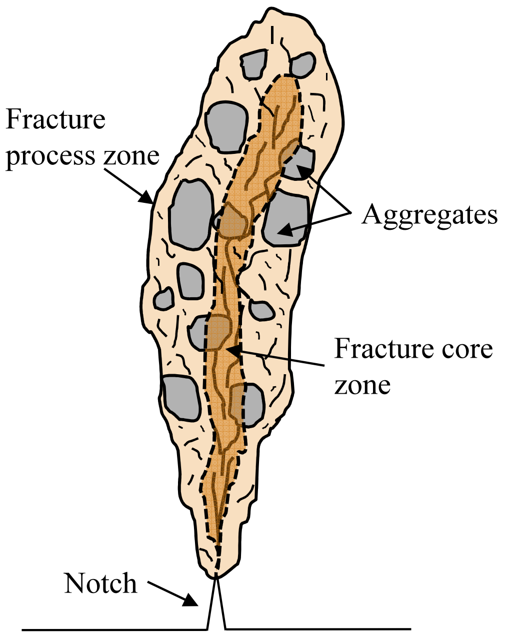

Figure 3: Schematic diagram of FPZ with FCZ

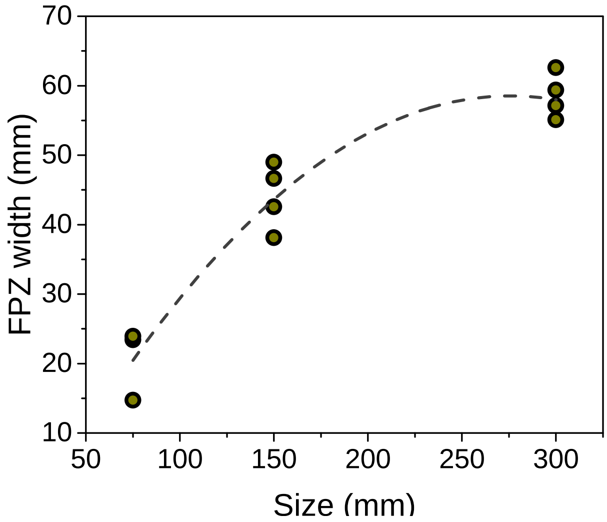

Figure 4: FPZ width vs. specimen size

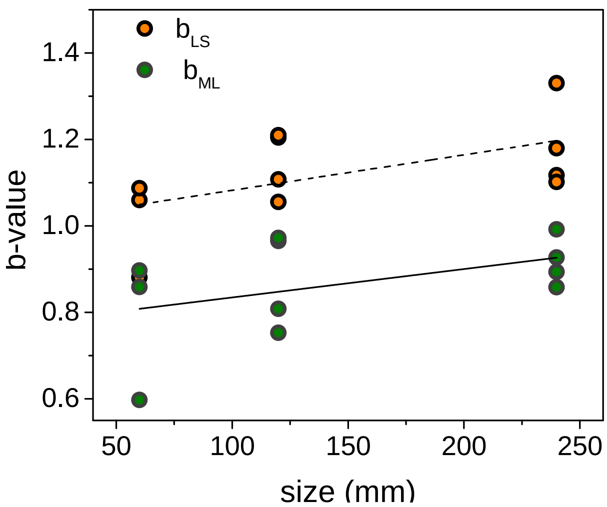

Figure 5: Variation in b-value w.r.t. beam size

5 CONCLUSIONS

The behavior of structural elements is size dependent. Macroscopically observed size-effect is rooted in microstructural composition and behavior of the material. A technique like acoustic emission provides the accounts of micro-structural changes in the material yielded by loading. The existence of FPZ in concrete makes it conducive for energy dissipation and sustaining applied energy flux. The statistical distribution of stress wave magnitudes obtained by AE serves as a proxy for assessment of crack size distribution during damage progress. The Gutenberg-Richter model has been used for the analysis of the AE and earthquake magnitude distribution for decades after its inception.We have followed the same procedure and determined the b-values for three different sizes of beams. We observed widening of FPZ with the increase in the size of the beams. The relative population of micro- and macro-cracks changes with the size which reflects in b-value. The variation of b-value with size shows the different microstates of beams near failure and therefore the microscopic variation is the ultimate cause of macroscopic size-effect in beams. Consequently, the increasing trend of b-value with the size certainly hints at non-universality of the GR law. Least square fitting of magnitude over the number of events on the logarithmic scale itself is a doubtful method which does not warrant existence of power-law distribution therefore the scale-invariance in multi-scale cracking is doubtful. As universality and scaling appear delusional, the existence of criticality near failure is debatable. The macroscopic size-effect is a well-recognized phenomenon in fracture mechanics which originates at the microscopic scale. The existence of universality is considered per se; then the macroscopic size-effect appear paradoxical as universality makes microscopic details irrelevant near failure. The present work emphasizes on this paradoxical viewpoint. Contrary, almost every article in AE literature of concrete equivocally assumes the validity of GR law and acclaims the universality of b-value. Despite of persistent use, the GR law has already eluded the earthquake forecasting aspirations in seismology. In AE damage analysis, the b-value is considered as a qualitative stress meter and its convergence to unity as a critical point near ultimate failure. Regardless of the GR law shortcomings, we believe, it was invented at the time of computational paucity and served as an astonishing tool for the problems which were considered impregnable at the time. Eventually, as a heuristic tool, the GR law has been extensively explored in many natural and artificial complex systems. For further development, the drawbacks of GR law needs to be addressed and the depth of physics involved in emergence of fracture like complex systems should be explored. In this direction, one of the approach currently exercised is the generalization of GR law. Recent works based on generalized logistic equation [11, 12], Bayesian statistics [13] and non-extensive statistical mechanics [14] are promising and needs to be explored further.

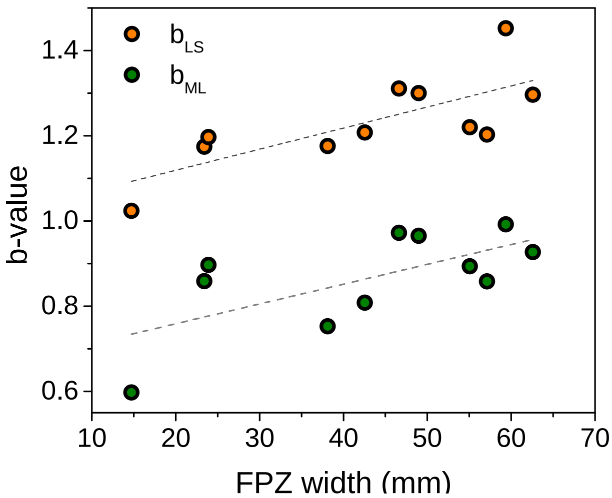

Figure 6: Variation in b-value w.r.t. maximum width of FPZ

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