STUDY OF THE RELATIONSHIP BETWEEN RESIDUAL FLEXURAL AND COMPRESSIVE STRENGTHS IN STEEL FIBER-REINFORCED CONCRETE BY MEANS OF THE RESPONSE SURFACE METHODOLOGY

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Abstract: Annex L draft of the next Eurocode 2 establishes classes of steel fiber-reinforced concrete (SFRC) based on the residual flexural strength for two crack mouth opening displacements, CMOD ($f_{R,1k}$, w_M = 0.5 mm, and $f_{R,3k}$, w_M = 2.5 mm). However, this classification does not consider the residual compressive strength of SFRC, σ_R, defined as the compressive stress for a strain exceeding three times the strain at the compressive peak [*Hormigón y Acero* 2018; 69(S1):75–80]. In this work we apply the Response Surface Methodology on two databases of compression and bending experimental results to correlate the structural classes defined in Eurocode 2 with the compressive residual strength of SFRC. The Analysis of Variance determines which factors have a real influence in the studied responses, namely $f_{R,1k}$, $f_{R,3k}$ and σ_R. Results show that the volume fraction of steel fiber, ϕ_f, the aspect ratio of the fiber, λ, and the compressive strength of SFRC, f_{cuf}, are statistically significant for $f_{R,1k}$. In addition to these parameters, the length of the fiber, ℓ_f, is also significant for $f_{R,3k}$. Finally, a relationship has been found between σ_R, $f_{R,1k}$ and $f_{R3,k}$ /$f_{R1,k}$ so that σ_R could be included as an additional parameter defining the Eurocode 2 classes for SFRC.

1 INTRODUCTION

Inclusion of steel fibers in concrete supposes a gain of flexural residual strength after cracking. This implies an increase of the toughness of the material [1,2]. Thus, steel fiber-reinforced concrete (SFRC) is employed in structures where cracking control is required [3], as industrial pavements [4], tunnel linings [5], pipes [6], etc. Its growing use has made it necessary to include SFRC in structural design codes [7, 8, 9]. Nevertheless, these documents consider only the tensile improvement without taking into account the ability to absorb energy provided by steel fibers in the compressed region of the structural element [10, 11]. Therefore, it is interesting to include the residual compressive strength linked to every class of SFRC defined in the Annex L draft of the next Eurocode 2 (EC2) [9].

Thus, two databases have been created with experimental results (one with 197 compression tests and the other with 484 bending tests) and by means of the Response Surface Methodology [12] and applying the model for compression behavior of SFRC proposed by Ruiz *et al.* [13], compressive residual strengths, σ_R, have been calculated for every SFRC class defined in EC2.

2 MATERIALS AND METHODS

2.1 Response Surface Methodology

The Response Surface Methodology (RSM) is a set of statistical and mathematical techniques that allows the creation of models for data fitting and for searching the relationship between variables that intervene in a problem [14]. Through the Analysis of Variance of the data (ANOVA) it is determined whether the independent variables (factors) have statistical significance to explain the dependent variable (response). Usually, the response is described by means of a polynomial fitting of experimental data; in this study, a polynomial of grade one has been selected:

$$Y = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + \xi \qquad (1)$$

being,

- Y Response or dependent variable
- β_0 Term of constant value
- β_i Term of linear fitting
- X_i Factors or independent variables
- ξ Error observed in the response

The analyses of the responses are made by fitting this linear model to the results in the databases. It is selected as the most appropriate because it provides correct values inside the domain of the data and it does not show multicollinearity (defined as the correlation between variables in the model), since otherwise the correlations could result in wrong estimations of the coefficients and low statistical significance of the factors (this fact is controlled by means of the statistical parameter named Variance Inflation Factor, VIF, which has to be equal or lower than 5). The methodology has been applied through the software Minitab [15].

2.2 Calculus of the residual compressive strength. Model for compression behavior of SFRC

The value of the residual compressive strength, σ_R (defined as the stress supported when the strain is three times higher than the strain at maximum stress), is calculated from the model for compression behavior of SFRC developed by Ruiz *et al.* [13]. This is a simple model in a technological format which has two stretches (Fig. 1): the first is a curve from the origin of the σ-ε curve to the maximum stress, similar to the one described in the EC2 for plain concrete [9]. The second branch is a vertical inverted parabola running from the maximum stress to the intercept with the x-axis. It is obtained so that the energy consumption in the post-peak region is equal to the same energy value measured with the results of the database in compression. It is defined by the following equation:

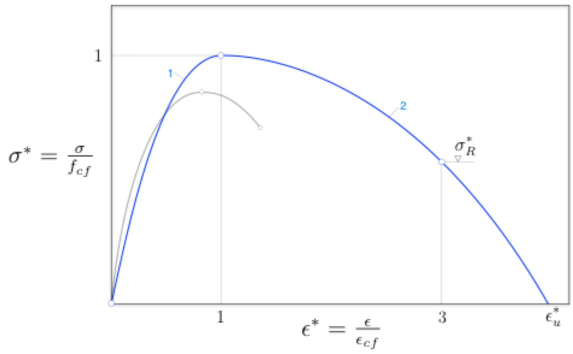

Figure 1: Model for compression behavior of SFRC

$$\sigma^* = 1 - \frac{1}{4}(1 - \sigma_R^*)(\epsilon^* - 1)^2 \qquad (2)$$

being,

- σ^* Non-dimensional stress (= σ/f_{cf})
- ε^* Non-dimensional strain (= $\varepsilon/\varepsilon_{cf}$)
- σ_R^* Non-dimensional residual compressive strength corresponding to $\varepsilon^* = 3$ (= σ_R/f_{cf}, where σ_R is the residual compressive strength for a strain value equal to three times the maximum stress strain, ε_{cf})
- f_{cf} Compressive strength of SFRC (with cylindrical specimen of 150 mm in diameter and 300 mm in height)

Equation 2 is a softening branch which always considers a positive value of σ_R^* lower than the unity, since most of the database curves of compression tests can absorb more energy than the fixed limit $\varepsilon^* = 3$.

3 RESULTS AND DISCUSSION

From the database elaborated with results of bending tests of SFRC (prismatic specimens of dimensions 150 x 150 x 550 mm^3, height x width x length, with a central notch and hooked-steel fibers) and applying RSM, relationships between a set of variables involved in flexural behavior of SFRC has been studied, namely:

- *Responses*: flexural strength of SFRC in the limit of proportionality, $f_{L,k}$, and residual flexural strengths of SFRC for two crack mouth opening displacements, CMOD ($f_{R,1k}$, w_M = 0.5 mm, and $f_{R,3k}$, w_M =2.5mm).
- *Factors*: compressive strength of SFRC in 150 mm cubes, f_{cuf}, volume fraction of fiber, ϕ_f, fiber length, ℓ_f (expressed as ℓ_f^* = ℓ_f/ℓ_0, where ℓ_0 = 30 mm) and aspect ratio, λ (λ = ℓ_f/d_f, being d_f the diameter of the fiber).

Figures 2, 3 and 4 are the Pareto graphs in which statistical significance of factors for every response is shown (bars that cross the dashed red line). For the three responses ($f_{L,k}$, $f_{R,1k}$ and $f_{R,3k}$), f_{cuf} and ϕ_f are significant factors; besides, λ is significant for $f_{R,1k}$ and $f_{R,3k}$, and ℓ_f^* is significant for $f_{R,3k}$ as well. This evidences the importance of the parameters related with the dimensions of the fiber when the crack opening is long. Surfaces for fitting the three responses are also represented in Figs. 2, 3 and 4, with respect to the two main significant factors, i.e. f_{cuf} and ϕ_f. Using the compressive database and the definition of σ_R^* (σ_R^*= σ_R/ f_{cf}) the following expression is obtained:

$$\sigma_R^* = -0.089 + 21.27\phi_f + 0.00407\lambda \qquad (3)$$

Equation 3 allows for the calculation of σ_R^* (and σ_R) for all the types of SFRC in the flexural database. It is assumed, according to Del Viso *et al.* [16], that compressive strength from 150 mm edged cubes is very close to that obtained from Φ150 x 300 mm^2 cylinders .

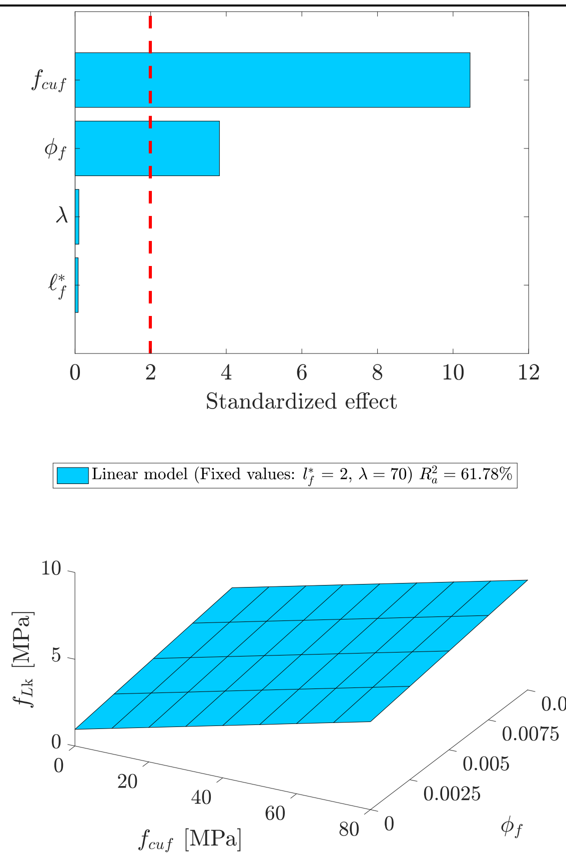

Figure 2: Graph of Pareto and response surface for $f_{L,k}$

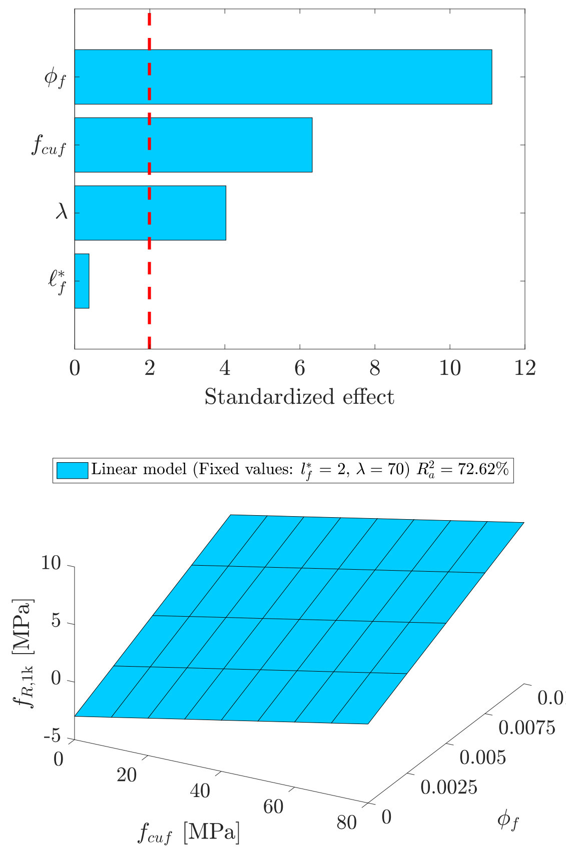

Figure 3: Graph of Pareto and response surface for $f_{R,1k}$

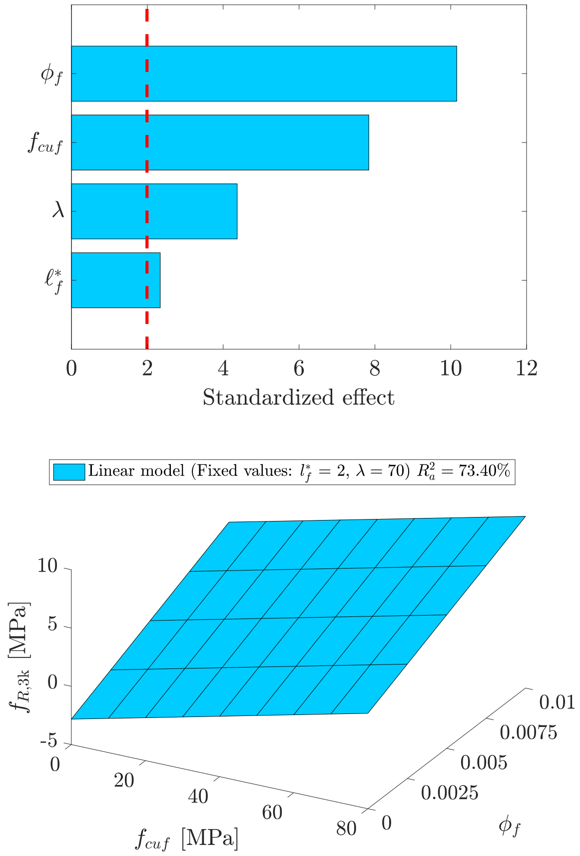

Figure 4: Graphs of Pareto (upper) and response surface (lower) for $f_{R,3k}$

Applying RSM again, σ_R and σ_R^* are calculated considering $f_{R,1k}$ and $f_{R,3k}/ f_{R,1k}$ as factors (both are included in EC2 for classifying SFRC). The results are arrayed in Table 1, which allows characterizing SFRC completely in terms of residual flexural and compressive strengths. This represents a novelty and technological improvement for the analysis and design of SFRC structural elements.

4 CONCLUSIONS

From the Response Surface Methodology, the relationship between flexural and compressive residual strengths, $f_{R,1k}$, $f_{R,3k}$ and σ_R in SFRC (the last defined as the strength supported when the strain is three times higher than the strain obtained under maximum stress) has been studied with the objective of characterizing the flexural/compressive behavior of the material. Through the elaboration of two databases (with compression and bending experimental results, respectively) and applying the cited method, statistically significant factors have been determined. Linear polynomial models have been fitted to the data to describe the responses. Results reveal that volume fraction of fiber, ϕ_f, aspect ratio, λ, and compressive strength of SFRC, f_{cuf}, are significant factors for $f_{R,1k}$. This three factors along with fiber length, ℓ_f (expressed as $\ell_f^* = \ell_f/\ell_0$) are significant for $f_{R,3k}$. By using a simple model for compression behavior of SFRC in a technological format [13], the value of σ_R is determined with the flexural database.

Table 1: Flexural and compressive residual strength class for SFRC (proposed for Annex L of Eurocode 2).

$f_{R,3k}$	$f_{R,1k}$									
f_{cuf} / σ_R	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
Class a:	0.5	0.8	1.0	1.3	1.5	2.0	2.5	3.0	4.0	5.0
$0.5 \leq \frac{f_{R,3k}}{f_{R,1k}} < 0.7$	**27 / 5**	**31 / 7**	**32 / 7**	**36 / 9**	**37 / 9**	**42 / 11**	**45 / 13**	**48 / 15**	**53 / 19**	**57 / 23**
Class b:	0.7	1.1	1.4	1.8	2.1	2.8	3.5	4.2	5.6	7.0
$0.7 \leq \frac{f_{R,3k}}{f_{R,1k}} < 0.9$	**36 / 7**	**40 / 9**	**41 / 9**	**44 / 10**	**45 / 11**	**49 / 13**	**51 / 15**	**54 / 17**	**58 / 21**	**62 / 25**
Class c:	0.9	1.4	1.8	2.3	2.7	3.6	4.5	5.4	7.2	9.0
$0.9 \leq \frac{f_{R,3k}}{f_{R,1k}} < 1.1$	**45 / 9**	**49 / 11**	**49 / 11**	**52 / 12**	**53 / 13**	**56 / 15**	**58 / 17**	**60 / 19**	**64 / 23**	**66 / 27**
Class d:	1.1	1.7	2.2	2.8	3.3	4.4	5.5	6.6	8.8	11.0
$1.1 \leq \frac{f_{R,3k}}{f_{R,1k}} < 1.3$	**54 / 11**	**57 / 12**	**58 / 13**	**60 / 14**	**60 / 15**	**62 / 17**	**64 / 19**	**66 / 21**	**69 / 25**	**71 / 29**
Class e:	1.3	2.0	2.6	3.3	3.9	5.2	6.5	7.8	10.4	13.0
$1.3 \leq \frac{f_{R,3k}}{f_{R,1k}}$	**63 / 13**	**66 / 14**	**66 / 15**	**68 / 16**	**68 / 17**	**69 / 19**	**71 / 21**	**72 / 23**	**74 / 27**	**76 / 31**

Besides, a significant relationship has been found between σ_R, $f_{R,1k}$ and $f_{R3,k}/f_{R1,k}$ so σ_R is proposed to be included as an additional parameter for defining the SFRC classes in the next Eurocode 2 draft for the analysis and design of structural elements.

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