DUCTILE-TO-BRITTLE TRANSITION IN FIBER-REINFORCED BRITTLE-MATRIX COMPOSITES: SCALE AND FIBER VOLUME FRACTION EFFECTS

ALBERTO CARPINTERI* AND FEDERICO ACCORNERO†

* Politecnico di Torino
24, Corso Duca degli Abruzzi, 10129 Torino, Italy
e-mail: alberto.carpinteri@polito.it

† Politecnico di Torino
24, Corso Duca degli Abruzzi, 10129 Torino, Italy
e-mail: federico.accornero@polito.it

Key words: Fiber-reinforced Composites, Scale Effect, Fiber Volume Fraction Effect, Bridged Crack Model, Ductile-to-brittle Transition, Minimum Reinforcement.

Abstract: Knowledge of the structural behaviour of fiber-reinforced brittle-matrix composites can lead to improvements in the material design through an optimization of the components. A Fracture Mechanics approach makes it possible to analyse the composite post-cracking behaviour and, unlike the classical strength theory, to explain certain discontinuous phenomena, that are experimentally verified, such as the size-scale effects or the snap-back and snap-through instabilities. In particular, in the present work the fundamental secondary-phase role played by the fiber volume fraction is investigated by means of the Bridged Crack Model, in order to highlight how the fracture toughness of the brittle matrix is improved by means of the fiber bridging action affecting the matrix micro- and macro-cracks, so as to prevent their coalescence, opening and growth. These bridging toughening mechanisms are due to debonding, sliding and frictional pulling-out between the matrix and the high-resistance fibres, or to yielding of the low-resistance ductile fibres. Moreover, the effect of the size scale is found to be fundamental for the global structural behaviour, which can range from ductile to catastrophic simply with the variation of a dimensionless brittleness number, which is a function of the toughness of the matrix, of the yielding or slippage limit of the reinforcement, of the volume fraction of the reinforcement, and of the characteristic structural size.

1 INTRODUCTION

Brittle matrix fiber-reinforced materials are characterized by enhanced strength, ductility, and fracture toughness thanks to the bridging action exerted by the fiber reinforcements. The Bridged Crack Model [1-6] allows to clearly explore the mechanical behaviour of fiber-reinforced beams subjected to bending, taking into account the elastic-perfectly brittle matrix, and a rigid-perfectly plastic law of the reinforcements, that can represent either yielding or reinforcement slippage. Different versions of this model have been used to describe the microcracking of fiber-reinforced materials, as well as macrocracking of materials reinforced by a rather small number of elements [7-10]. In both cases, the Bridged Crack Model is able to explain and reproduce the constitutive flexural response that is often discontinuous owing to the presence of virtual catastrophic branches, i.e., snap-through branches due to load control, and snap-back branches due to deflection control [1,11-13]. The bridging tractions are assumed as constant plastic forces, whereas the crack runs in an elastic brittle matrix. The effect of the size scale is considered as fundamental for the global structural behaviour, which can range from ductile to catastrophic simply with the variation of a dimensionless brittleness number, N_P, which is a function of the toughness of the matrix [14], of the yielding or slippage limit of the fiber-reinforcement, of the fiber volume fraction of the reinforcement, and of a characteristic structural size [1,15]: $N_P = (\sigma_P\, b^{1/2}/(K_{IC}\, Vf)$.

The model described in the present paper considers a fiber-reinforced brittle matrix rectangular cross-section with an edge crack: beam section, initial crack depth, and the position of the fibers are shown in Figure 1. Only the fibers crossing the crack are considered as active.

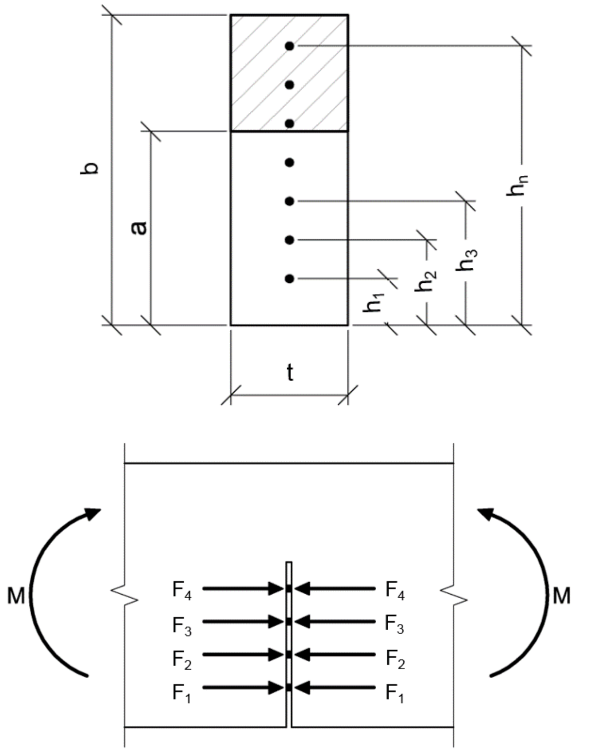

Figure 1. Fiber-reinforced brittle matrix beam model.

The fiber reinforcements exert a bridging action across the crack described by the forces F_i (Fig.1).

The matrix is assumed to be elastic-perfectly brittle and it is characterized by the fracture toughness, K_{IC}, and the Young Modulus, E, whereas the reinforcements are considered rigid-perfectly plastic. In the slippage hypothesis, the reinforcement action is related to the frictional bonding force between the matrix and the reinforcement, whereas in the hypothesis of yielding, it represents the force that makes the fiber plastically flow [1]. The Bridged Crack Model takes into account both equilibrium and compatibility equations [1-6], and the crack propagation condition is evaluated according to Linear Elastic Fracture Mechanics [16,17].

2 NUMERICAL SIMULATIONS AND LAB TESTS

Within the Bridged Crack Model, the behaviour of a fiber-reinforced beam subjected to bending can be evaluated by studying the evolution of the fracturing process [1], and defining for each crack depth the crack-propagation moment, and the corresponding localized rotation [18]. The dimensionless crack propagation moment depends on the crack depth, the fiber reactions, the mechanical and geometrical characteristics of the cross-section. In this way, it is possible to capture the unstable structural behavior which is represented by snap-back and snap-through phenomena: whereas the cusp catastrophe (snap-back) is basically due to brittle fracturing of the cross-sectional matrix [12,13], snap-through is due to plastic deformation of fiber reinforcements [4]. It is worth noting that different structural behaviours are predicted by the Bridged Crack Model by varying the brittleness number, N_P, which represents a synthetic parameter governing the structure ultimate response, and its critical value, N_{PC}, allows to determine the so-called minimum reinforcement condition [19]. In further study, critical values of the brittleness number, N_{PC}, are calculated in order to obtain the asymptotical condition for a number of fibers $n \to \infty$, pointing out how the fundamental secondary-phase asymptote N_{PC} $(n\to\infty)$ arises for $n > 20$.

In the following, the results of a numerical simulation campaign carried out in order to investigate the Three Point Bending (TPB) test of fiber-reinforced concrete beams considering a fiber number n = 20 (transition from a localized to a diffused reinforcement distribution) are discussed. The numerical investigations herein considered take into account 3 different beam sizes (b = 200 mm; 100 mm; 50 mm), and 8 different fiber volume fractions (Vf = 0.05%; 0.10%; 0.15%; 0.20%; 0.30%; 0.40%; 0.50%; 0.60%). In Figures 2-4 the structural behaviours of 24 beams subjected to TPB test are shown, taking into account the above mentioned beam sizes, fiber volume fractions and, in addition: Fracture Toughness of the matrix, K_{IC} = 60 daN/cm$^{1.5}$; matrix cover, a_1 = 5 cm; matrix Young's Modulus, E = 31500 MPa; matrix compressive strength, σ_c = 33 MPa; fiber tensile strength, σ_P = 1100 MPa.

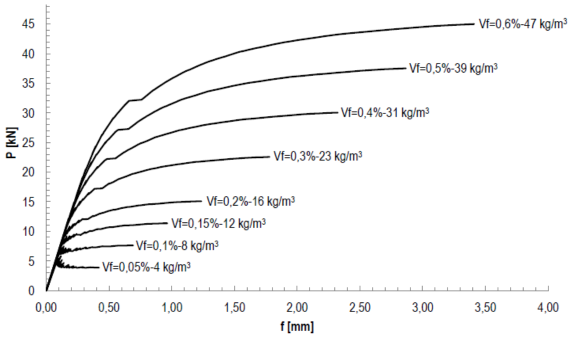

Figure 2. Load vs mid-span deflection of a TPB beam with b = 200 mm: Fiber volume fraction effect.

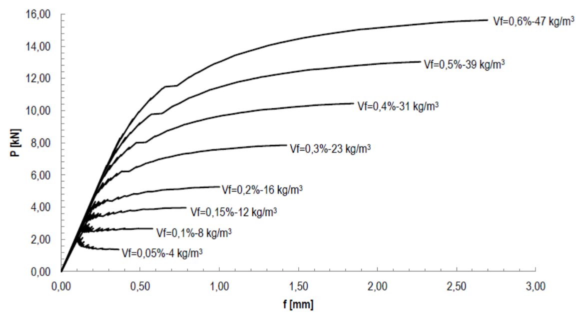

Figure 3. Load vs mid-span deflection of a TPB beam with b = 100 mm: Fiber volume fraction effect.

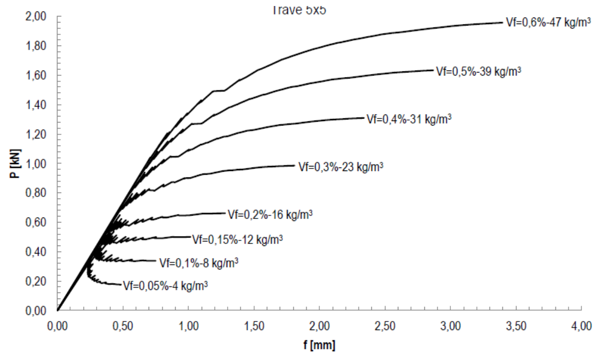

Figure 4. Load vs mid-span deflection of a TPB beam with b = 50 mm: Fiber volume fraction effect.

It is worth noting that a ductile-to-brittle transition is found for each beam scale, and this transition can be effectively described by the brittleness number N_P. Notice that the ratio between the fiber volume fractions corresponding to the three minimum reinforcement conditions only depends on $\sqrt{2}$, as predicted by N_P: the minimum fiber-reinforcement condition for b = 200 mm is Vf = 0.10%; for b = 100 mm, Vf is equal to 0.15%; for b = 50 mm, Vf is equal to 0.20%.

Moreover, the effects of the change in Fracture Toughness, K_{IC}, are shown in Figures 5-7 for three different beam sizes and three different post-peak behaviours: softening, minimum reinforcement, and hardening. In the following diagrams, local and global discontinuous phenomena are clearly shown in terms of snap-back and snap-through instabilities.

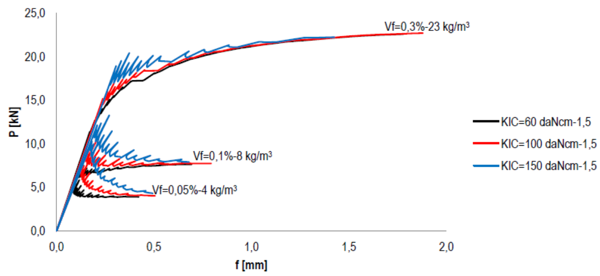

Figure 5. Load vs mid-span deflection of a TPB beam with b = 200 mm. Effect of the change in Fracture Toughness, K_{IC}.

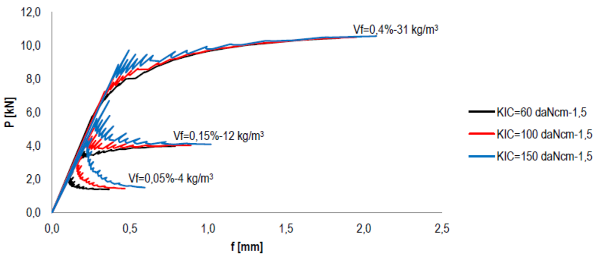

Figure 6. Load vs mid-span deflection of a TPB beam with b = 100 mm. Effect of the change in Fracture Toughness, K_{IC}.

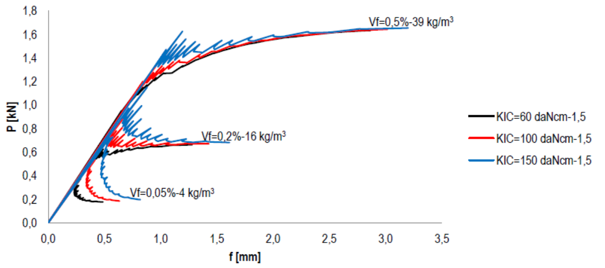

Figure 7. Load vs mid-span deflection of a TPB beam with b = 50 mm. Effect of the change in Fracture Toughness, K_{IC}.

The numerical results here presented will be validated by means of an experimental campaign reproducing beam scales, fiber volume fractions, and material characteristics above discussed. In this way, the flexural behaviour of fibre-reinforced brittle matrix composites will be fully investigated, also confirming the extensive researches which demonstrate its effectiveness in many structural applications.

3 CONCLUDING REMARKS

A Fracture Mechanics approach makes it possible to analyse the fiber-reinforced brittle-matrix composite post-cracking behavior, and to explain certain discontinuous phenomena, that are experimentally verified, such as the size-scale effects or the snap-back and snap-through instabilities. In particular, in the present work the fundamental secondary-phase role played by the fiber volume fraction is investigated by means of the Bridged Crack Model, in order to highlight how the fracture toughness of the brittle matrix is improved by means of the fiber bridging action affecting the matrix micro- and macro-cracks, so as to prevent their coalescence, opening and growth. Moreover, the effect of the size scale is found to be fundamental for the global or local structural behaviour, which can range from ductile to catastrophic simply with the variation of the brittleness number, N_{PC}, which is function of the toughness of the matrix, of the yielding or slippage limit of the fiber-reinforcement, of the fiber volume fraction, and of the characteristic structural size.

REFERENCES

[1] Carpinteri, A., 1984. Stability of fracturing process in RC beams. *Journal of Structural Engineering ASCE*, 110:544-558.

[2] Bosco, C., Carpinteri, A., 1995. Discontinuous constitutive response of brittle matrix fibrous composites. *Journal of the Mechanics and Physics of Solids*, 43:261-274.

[3] Carpinteri, A., Massabò, R., 1996. Bridged versus cohesive crack in the flexural behavior of brittle-matrix composites. *International Journal of Fracture*, 81:125-145.

[4] Carpinteri, A., Massabò, R., 1997. Continuous vs discontinuous bridged crack model of fiber-reinforced materials in flexure. *International Journal of Solids and Structures*, 34:2321-2338.

[5] Carpinteri, A., Massabò, R., 1997. Reversal in failure scaling transition of fibrous composites. *Journal of Engineering Mechanics (ASCE)*, 123:107-114.

[6] Carpinteri, A., Puzzi, S., 2007. The bridged crack model for the analysis of brittle matrix fibrous composites under repeated bending loading. *Journal of Applied Mechanics*, 74:1239-1246.

[7] Cox, B.N., 1991. Extrinsic Factors in the Mechanics of Bridged Cracks. *Acta Metallurgica and Materialia*, 39:1189–1201.

[8] Cox, B.N., Lo, C.S., 1992. Load ratio, notch, and scale effects for bridged cracks in fibrous composites. *Acta Metallurgica and Materialia*, 40:69-80.

[9] Bosco, C., Carpinteri, A., Debernardi,

P.G., 1990a. Fracture of reinforced concrete: Scale effects and snap-back instability. *Engineering Fracture Mechanics*, 35, 665-677.

[10] Bosco, C., Carpinteri, A., 1992. Fracture behavior of beam cracked across reinforcement. *Theoretical and Applied Fracture Mechanics*, 17, 61-68.

[11] Carpinteri, A., Accornero, F., 2018. Multiple snap-back instabilities in progressive microcracking coalescence. *Engineering Fracture Mechanics*, 187:272-281.

[12] Lacidogna, G., Accornero, F., Carpinteri, A., 2019. Influence of snap-back instabilities on Acoustic Emission damage monitoring. *Engineering Fracture Mechanics*, 210:3-12.

[13] Carpinteri, A., 1989. Cusp catastrophe interpretation of fracture instability. *Journal of the Mechanics and Physics of Solids*, 37:567-582.

[14] Lacidogna, G., Accornero, F., Corrado, M., Carpinteri, A., 2013. Crushing and fracture energies in concrete specimens monitored by Acoustic Emission. *Proceedings of the 8th International Conference on Fracture Mechanics of Concrete and Concrete Structures, FraMCoS 2013*, Pages 1726-1736.

[15] Carpinteri, A., 1989. Size effects on strength, toughness, and ductility. *Journal of Engineering Mechanics (ASCE)*, 115:1375-92.

[16] Accornero, F., Lacidogna, G., Carpinteri, A., 2016. Evolutionary fracture analysis of masonry arches: Effects of shallowness ratio and size scale. *Comptes Rendus Mécanique*, 344:623-630.

[17] Lacidogna, G., Accornero, F., 2018. Elastic, plastic, fracture analysis of masonry arches: A multi-span bridge case study. *Curved and Layered Structures*, 5:1-9.

[18] Carpinteri, A., Accornero, F., 2019. Rotation versus curvature fractal scaling in bending failure. *Physical Mesomechanics*, 22:46-51.

[19] Bosco, C., Carpinteri, A., Debernardi, P.G., 1990. Minimum reinforcement in high strength concrete. *Journal of Structural Engineering (ASCE)*, 116:427-437.