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WHY NOMINAL CRACKING STRENGTH CAN BE LOWER FOR LATER CRACKS IN STRAIN-HARDENING CEMENTITIOUS COMPOSITES WITH MULTIPLE CRACKING?

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Abstract: Strain-Hardening Cementitious Composites (SHCC) exhibit multiple-cracking behavior under tension. Theoretically speaking, the distribution of matrix inherent flaws results in variation in cracking strength of SHCC, and the cracking strength decreases accordingly with increasing flaw size. Therefore, for a SHCC specimen under tension, it should show metal-like behavior with a characteristic "yield point" at the end of the elastic stage when the first crack (correlated to the largest flaw perpendicular to the loading direction) appears, and then multiple cracks will form under increasing stress at un-cracked sections with sequentially decreasing flaw sizes. This tensile multiple-cracking process ends once the cracking strength for further cracking in the remaining sections is higher than the fiber-bridging capacity of the weakest section. However, it has been widely observed during tension tests that the nominal cracking strength can be lower for later cracks than earlier ones, which is not consistent with the design theory of SHCC. This paper attempts to explain the aforementioned phenomenon with the consideration of non-uniform stress distribution resulting from inclined cracking. Additionally, a new "90% peak stress" criterion considering this phenomenon for the determination of ultimate tensile strain in SHCC is proposed. The findings in this study offer a new insight in tensile performance of SHCC.

1 INTRODUCTION

To overcome the brittleness of concrete, research in fiber-reinforced brittle matrix has made possible the development of Strain-Hardening Cementitious Composites (SHCC) exhibiting multiple-cracking behavior under tension [1-3]. At the ultimate state under uniaxial tension, the strain of SHCCs can reach 1-8% [4-8], which is hundreds times the tensile strain capacity (around 0.01%) of conventional concrete as well as fiber reinforced concrete. Typically, the crack width of multiple cracks can be self-controlled to less than 100 μm [4], which can made SHCC materials and structures extremely durable [9]. In addition, the compressive strength of SHCC can be designed with the range from 30 MPa to 200 MPa. With excellent mechanical and durability properties, SHCC show great promises over conventional concrete and fiber-reinforced concrete for construction applications [10-12].

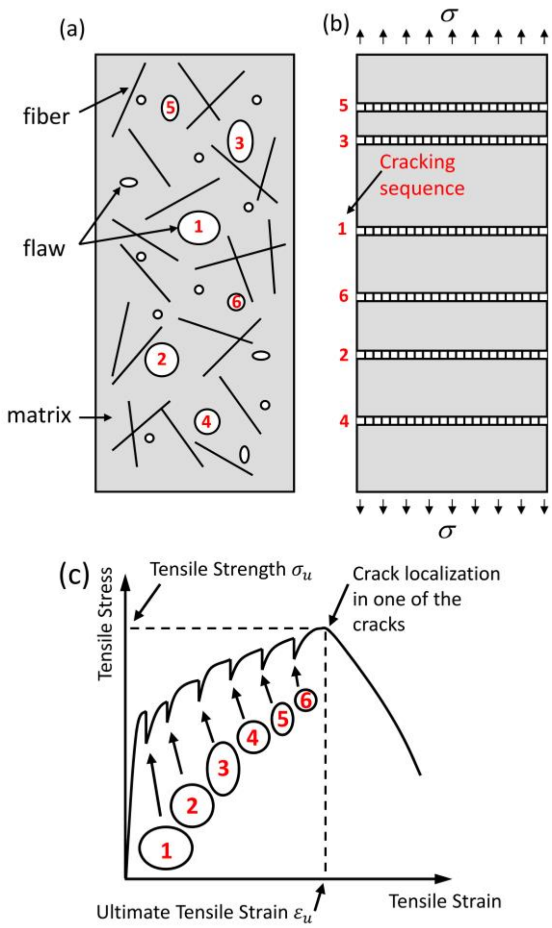


Figure 1: Theory of SHCC: (a) Random distribution of flaws and fibers in matrix; (b) Cracking sequence and crack patterns on the surface; (c) Corresponding tensile stress-strain curve, where each crack is correlated to the flaw in the matrix. (Adapted from Wang [13])

Theoretically speaking, the distribution of inherent flaws in cementitious matrix results in variation in cracking strength of different cross-sections in SHCC, and the cracking strength decreases accordingly with increasing flaw size [1]. In addition, it has been proven that the cracking strength is strongly correlated with the largest flaw size rather than the number of smaller flaws in a particular crack section [14]. Therefore, for a SHCC specimen under tension, it should show metal-like behavior with a characteristic "yield point" at the end of the elastic stage when the first crack appears (correlated to the largest flaw perpendicular to the loading direction – flaw "1" in **Figure 1 a**), and then multiple cracks will form under increasing stress at un-cracked sections with sequentially decreasing flaw sizes (**Figure 1**). This tensile multiple-cracking process ends once the cracking strength for further cracking in the remaining sections is higher than the fiber-bridging capacity of the weakest section. Final failure of the tensile specimen then occurs when the load-bearing capacity of bridging fibers on one of the multiple cracks is exhausted, resulting in fracture localization. However, in some tension tests on SHCCs, it has been observed that the nominal cracking strength can be lower for later cracks than earlier ones, which is not consistent with design theory of the materials.

This paper attempts to explain this phenomenon. A possible mechanism is examined by numerical simulation with finite element method. Additionally, a new criterion considering the aforementioned phenomenon for the determination of ultimate tensile strain (ε_u) in SHCC is proposed, and the rationality of different criteria for determining ε_u is discussed.

2 INCLINED CRACKING AND CRACK CONFLUENCE IN SHCC

Crack deflection widely occurs in concrete materials when the path of least resistance is around a relatively strong particle or along a weak interface [15], which means the crack planes are not perfectly perpendicular to the principle stress and inclined cracking is possible in concrete materials. Since multiple steady-state cracks rather than only one unstable crack appear in SHCC, further cracking near the inclined crack is possible. Once a propagating crack meets an existing crack, the crack tip of this propagating crack can be blunted. This makes possible a kind of "partial" crack - a crack cannot fully propagate over the whole cross-session of the specimen, or we can call this phenomenon "crack confluence".

It should be noted that, we generally monitor/record the crack patterns from only one side of the specimen, as rectangular-section specimens are widely-used for evaluating the uniaxial tension performance of SHCC [16].

The “partial” cracking may not be satisfactorily recorded during the tension test, and two possible cases are shown in **Figure 2**.

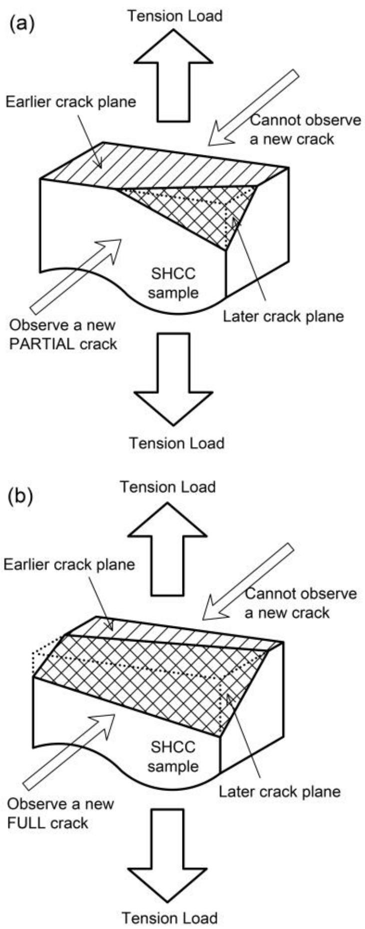


Figure 2: Two cases of crack confluence in SHCC: (a) Crack confluence cannot be observed from the back side; (b) Crack confluence cannot be observed from both front and back sides.

3 FINITE ELEMENT SIMULATION OF TENSILE CRACKING SEQUENCE OF SHCC WITH INCLINED CRACKING

To understand the cracking sequence of SHCC with an inclined crack, the uniaxial tension performance was numerically simulated using a finite element (FE) model.

3.1 Basic assumptions in FE model

The following assumptions were made for the simulation:

(1) Though inclined cracking is a 3-D phenomenon (**Figure 2**), it can be simplified as a 2-D plane stress problem.
(2) The process of crack propagation is not considered.
(3) The cracks can be non-perpendicular to the principle stress, due to the random distribution of weak interfaces.
(4) Instead of considering the randomness of flaws and fibers as well as the resulting cracking positions, a weaker material band with lower cracking strength can be artificially assigned in the FE model to reproduce inclined cracking.
(5) There is a distance to transfer the stress from fibers crossing a crack back to the surrounding matrix. Therefore, a larger flaw shielded by the lower local stress field near a crack may be activated later [17]. This phenomenon is not considered here. In other words, all the elements can crack if the local stress reaches the cracking strength.

3.2 Implementation of FE model

A commercial finite element software ATENA (Version 5.0.3) [18] was employed in this study. In the software, the constitutive behavior of cementitious materials is described by a fracture-plastic model, which is the combination of two individual models for tensile (fracturing) and compressive (plastic) behavior. In tension, the fracture model is based on the classical orthotropic smeared crack formulation and crack band model, in which the Rankine failure criterion is employed. This means that the cohesive traction versus crack opening displacement can be interpreted as a tensile stress versus strain relationship. In compression, the hardening/softening plasticity model is based on the Menétrey-Willam yielding/failure surface.

To satisfactorily reflect the cracking sequence, an individual crack based approach was utilized, in which a crack is eventually smeared into an element, i.e., the crack-bridging stress versus crack-opening curve is translated to the tensile stress versus train curves over an element. As there is only one crack in each element, this approach is element size dependent for the simulation of multiple-cracking process. The size-dependent property for the individual crack based approach is not important for the problem discussed in this study.

A user-defined material model CC3DNonLinCementitious2SHCC for SHCC material in ATENA was used. The experimental compression and single-crack tension results for SHCC in Yu [19] were simplified to multi-linear functions as inputs for the tensile and compressive constitutive relations in the material model, while the default shear constitutive relations developed by Kabele [20] were adopted. Specifically, the single-crack tension constitutive relation is shown in **Figure 3**, where the vertical axis is normalized to the first-cracking strength F_t, and the horizontal axis (crack opening) will be translated to a strain value (over the element size) as input in the material model.

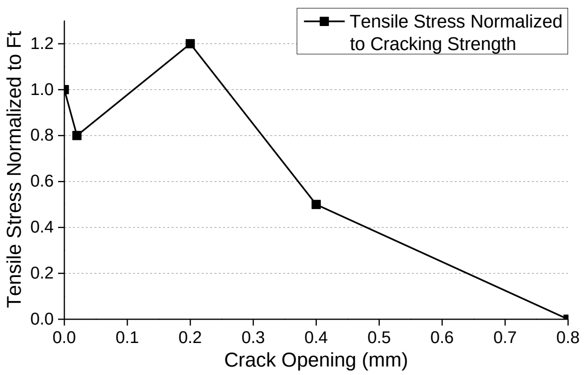


Figure 3: Single-crack tension curve is simplified to a multi-linear function as input for tensile constitutive relation.

The geometry of the tension specimen is 80 mm (length) × 40 mm (width) × 20 mm (thickness). To avoid stress localization as well as bending effect in the model, the tension specimen was perfectly contacted to two pieces of steel blocks in both ends, and the two steel blocks were restricted in the y direction. Then the specimen was fixed at one end, and loaded with a displacement loading rate of 0.002 mm/step from the other end (**Figure 4 a**). Plane quadrilateral elements of size 4 mm were utilized over the whole specimen. The problem was then solved using the Newton-Raphson method.

To evaluate the effect of inclined cracking on the overall tensile stress-strain curve and crack pattern, by setting the cracking strength of the base SHCC material as 4.6 MPa (yellow part in **Figure 4 b**), an artificial weaker material band with lower cracking strength of 4.4 MPa was artificially assigned (green part in **Figure 4 b**) in the FE model. Specifically, the inclination of the artificial weaker material band was controlled by the angle θ, and four different cases with cot(θ) equaled to 0.1, 0.2, 0.3 and 0.4 were explored.

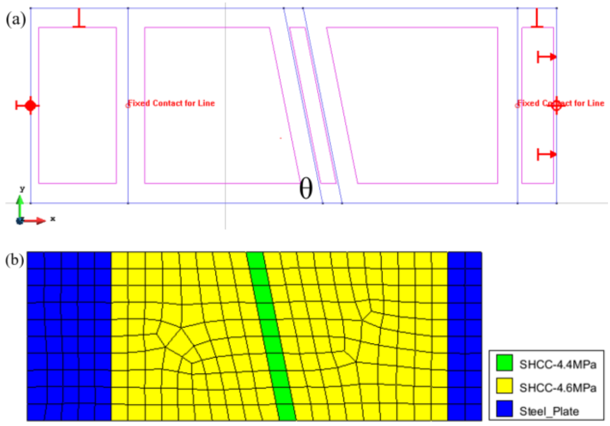
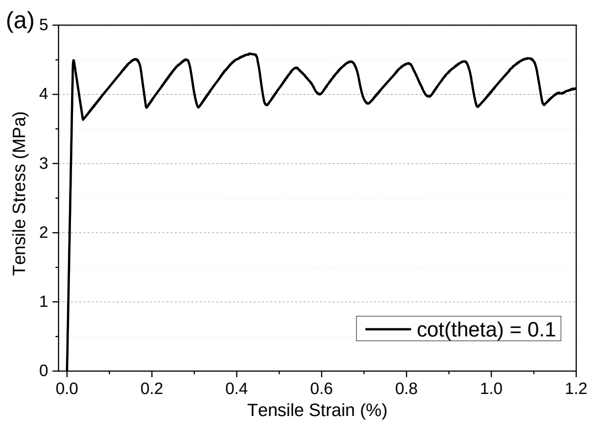


Figure 4: Finite element model for uniaxial tension: (a) boundary conditions; (b) materials, meshing and artificial cracking position (the case with cot θ = 0.2 is shown here)

4.2 Simulation results and discussion

The simulated tensile stress-strain curves for four different cases are shown in **Figure 5**, while the distribution of principle stress σ_{xx} and crack pattern for each tensile stress drop for the case cot(θ) = 0.2 (**Figure 5 b**) are simultaneously shown in **Figure 6**.



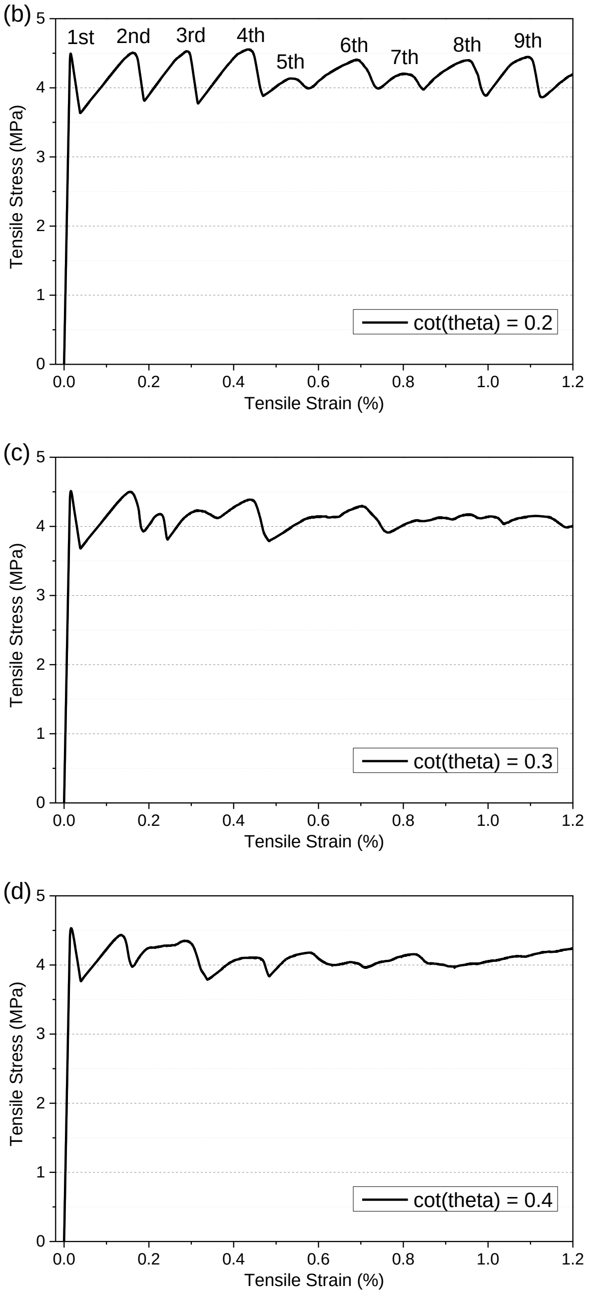
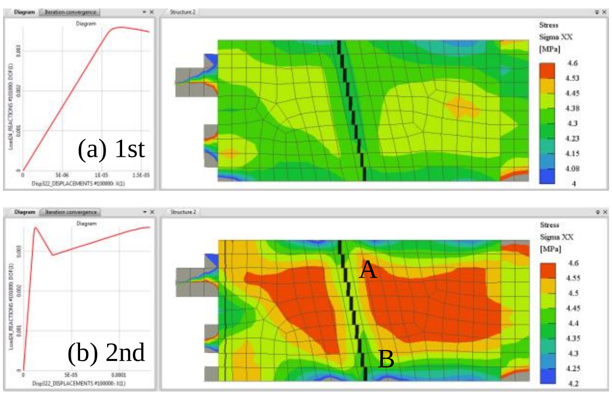


Figure 5: Simulated tensile stress-strain curves for: (a) cotθ = 0.1; (b) cotθ = 0.2; (c) cotθ = 0.3; and (d) cotθ = 0.4.



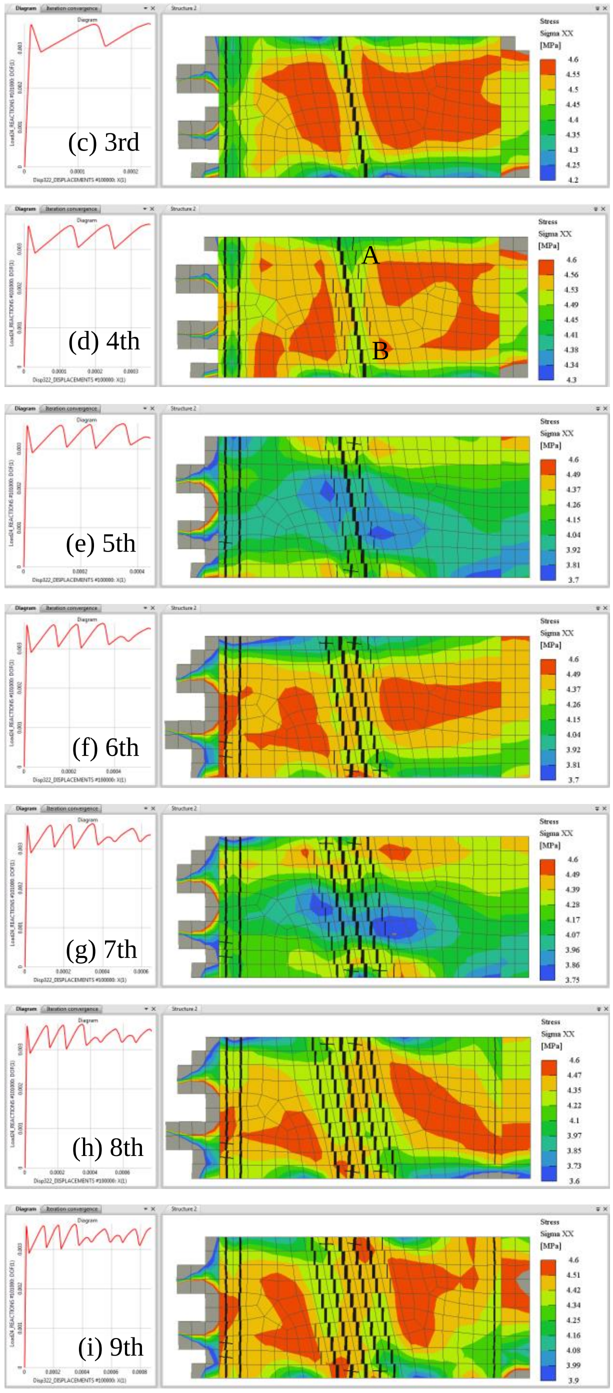


Figure 6: Distribution of σ_{xx} and crack pattern for each tensile stress drop for the case cotθ = 0.2 (Figure 5 b).

All the curves in **Figure 5** have the phenomenon that some of the nominal cracking strengths (tensile load divided by the whole cross-section) for later cracks are lower than those for earlier ones, which is consistent with the experimental observation as discussed previously. A further analysis on the stress field of the specimen indicates that the inclined cracking results in non-uniform distribution of principle stress σ_{xx} (**Figure 6**). Additionally, the cracks for the 2nd, 3rd and 8th stress drops are from stress localization from the boundary restrain, which are not further discussed here.

For the materials near the inclined crack, Region A is under higher local stress and therefore trends to crack earlier than Region B (**Figure 6 b**). As a result, a pair of symmetry "partial" cracks can be found in the crack patterns for the 4th stress drop (**Figure 6 d**), and wider crack width at Region A and narrower crack width at Region B is found. It is interesting that the further cracking from another pair of symmetry "partial" cracks results in the 5th, 6th and 7th stress drops (**Figure 6 e-g**). This kind of "stage-by-stage" cracking should be attributed to the non-uniform distribution of principle stress σ_{xx}.

4 DISCUSSION ON DETERMINATION OF ULTIMATE TENSILE STRAIN IN SHCC

4.1 Different criteria for determination of ultimate tensile strain

The ultimate tensile strain (ε_u) of SHCC is commonly determined as the strain value correlated to the point of peak tensile stress (i.e., "100% peak stress" criterion). Another approach is to define ε_u as the strain value when crack localization occurs, by considering the energy absorption during multiple cracking (i.e., "crack localization" criterion).

Specifically, the ε_u is defined as "the strain at the softening point" in a JSCE recommendation for design and construction of SHCC [16], where the idea case that the tensile stress reaches the peak value just before final failure in SHCC was considered (which is very similar to **Figure 1 c**).

Since the nominal cracking strength can be lower for later cracks than earlier ones, it is possible that the tensile stress reaches a peak value, followed by further multiple-cracking before crack localization and final failure for SHCC under tension (**Figure 7 b-c**). If the ε_u is determined following the "100% peak stress" criterion, the tensile capacity of these SHCC specimens would definitely be underestimated.

To reasonably evaluate the tensile capacity of SHCC, the authors proposed a new "90% peak stress" criterion with the following considerations:

(1) It is quite common that SHCC shows further cracking after the "100% peak stress" under tension.

(2) "90%" is neither too large nor too small. If this value is too large, it would be too close to "100%"; if this value is too small, it is not very reasonable to claim that it reflects the ultimate stage of the specimen.

Having said that, if we keep the commonly-used "100% peak stress" criterion for the determination of ε_u, we are always in the safe side taking into account the scatter of the material's characteristics. On the other hand, the "crack localization" criterion may lead to overestimation of the deformation capacity of SHCC, which will be further discussed in the next section.

4.2 Comparison of ultimate tensile strain determined from the "100% peak stress", "crack localization" and "90% peak stress" criteria

Three typical tensile stress-strain curves are graphically shown in **Figure 7 a-c**, and the values of ε_u determined from the two different criteria are summarized in **Table 1**. In the figures, the values of ε_u following "100% peak stress" criterion (ε_u^{100} , Points A), "crack localization" criterion (ε_u^{loc}, Points B) and "90% peak stress" criterion (ε_u^{90} , Points C) are highlighted in red, green and blue, respectively.

Case 1 (**Figure 7 a**): This is an idea case that the tensile stress reaches the peak value just before crack localization in SHCC, though some sections that crack later exhibit lower cracking strength than sections that crack earlier in the multiple-cracking process. As shown in **Table 1**, the values of ε_u from the three criteria are very close to each other (less than 5% in difference).

Case 2 (**Figure 7 b**): The tensile stress reaches the peak value at the "middle" of the tensile stress-strain curve, and many new cracks appear after Point A. As shown in **Table 1**, the

ε_u from the "100% peak stress" criterion is only half of that from the "90% peak stress" criterion, while this ratio can be even lower in some cases based the authors' experience. Therefore, for cases like Case 2, "90% peak stress" criterion is more reasonable, at least in terms of the energy absorption during multiple cracking.

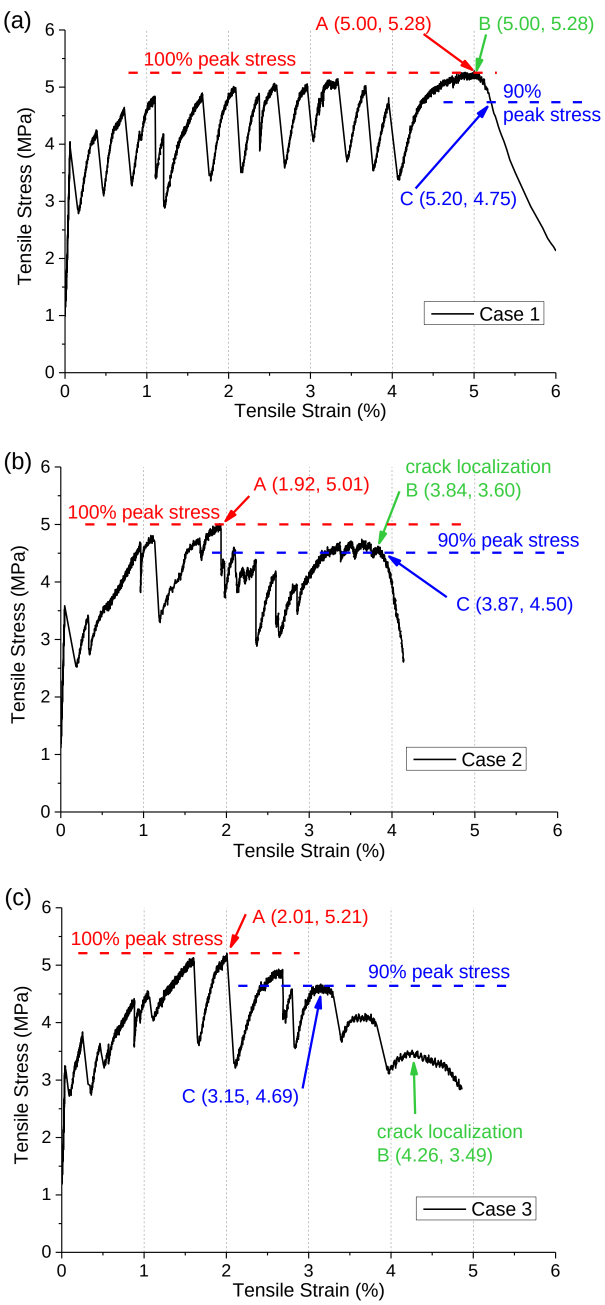


Figure 7: Determination of ultimate tensile strain for three typical cases with different criteria.

Case 3 (**Figure 7 c**): Further cracking with lower cracking strength appears even after Point C (from "90% peak stress" criterion). Additionally, the "crack localization" criterion gives a much larger ε_u (4.26% in Point B). Since the cracking strength for the sections after Point C are too low (e.g., 3.49/5.21=0.67 for Point B), we can just ignore their contributions and we are in the safe side.

Table 1: Comparison of ultimate tensile strain for three typical cases with different criteria.

Case	ε_u^{100} (%)	ε_u^{loc} (%)	ε_u^{90} (%)	$\frac{\varepsilon_u^{100}}{\varepsilon_u^{90}}$	$\frac{\varepsilon_u^{loc}}{\varepsilon_u^{90}}$
1	5.00	5.00	5.20	0.962	0.962
2	1.92	3.84	3.87	0.496	0.992
3	2.01	4.26	3.15	0.638	1.352

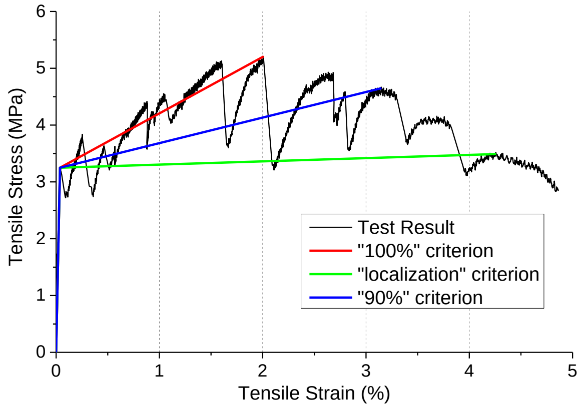


Figure 8: Simplified tensile constitutive relationship from experiment with different criteria for numerical modeling and theoretical analysis.

Additionally, for determining the tensile constitutive relationship from experiments as input for numerical modeling and theoretical analysis, the "90% peak stress" criterion can offer a relationship with slightly lower tensile strength and reasonable ultimate tensile strain (blue curve in **Figure 8**). Compared to the other two criteria (red and green curves in **Figure 8**), the proposed criterion is more reasonable by considering the trade-off for tensile strength and ultimate tensile strain.

In summary, the proposed "90% peak stress" criterion is reasonable for the determination of ε_u in SHCC.

5 CONCLUSIONS

In Strain-Hardening Cementitious

Composites (SHCC) exhibiting multiple-cracking under tension, the nominal cracking strength can be lower for later cracks than earlier ones, which is not consistent with the design theory of the materials. This study physically explains this phenomenon with the hypothesis of inclined cracking and the resulting non-uniform distribution of principle stress as well as “partial” cracking by finite element simulation. Additionally, a new “90% peak stress” criterion considering this phenomenon for the determination of ultimate tensile strain in SHCC was proposed, and the comparison of ultimate tensile strain obtained from different criteria showed that the proposed criterion is more reasonable than the commonly-used ones. The findings in this study offer a new insight in tensile performance of SHCC.

Further study on the theoretical analysis of this phenomenon with the help of fracture mechanics is highly recommended, and using new technologies to record the 3-D cracking patterns of SHCC is worthy to be explored.

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