

INCREASING THE DESIGN SHEAR STRENGTH OF CONCRETE BRIDGE DECKS BY TESTS AND STATISTICAL ANALYSIS OF A SHEAR DATABASE

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Abstract. Most existing deck slabs of concrete bridges show a lack of design shear capacity according to the current european standard EC2-1 [1]. Retrofitting is hardly possible. Fortunately, no structural failures of properly designed bridge decks are known. Thus, increasing the design shear capacity of bridge decks seems to be possible. This paper presents results from slab tests and statistical evaluations of a shear database which justify a significant increase in design shear for concrete bridge decks.

1 INTRODUCTION

Since not all existing bridge decks are designed for today's standard and theoretical load scenarios and the costs of retrofitting are high, research on the structural behaviour of RC slabs without stirrups is of special interest. In different european research labs tests on RC slabs without transverse reinforcement under concentrated wheel loads were conducted and the load-bearing behaviour was analysed. The shear force in the slab is dominated by the wheel loads. The test results showed much higher shear bearing capacities than compared to the theoretical values $v_{Rd,c}$ given by european standard EC2-1 [1].

$$v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} \cdot d \quad (1)$$

with $C_{Rd,c} = 0.12$

The results of the specimens go along with the fact that fortunately no structural failures of properly designed concrete bridge decks are

known. A possible reason for the existing differences in the value of resistance can be found considering the basis of (eq. 1). The shear design of concrete structures without transverse reinforcement according to EC2-1 [1] has been verified by a statistical evaluation of a shear database containing several hundreds of beam tests [2]. While beams are statically determined and show a brittle failure, concentrated loads on RC slabs can be transferred to a greater area in the slab. Thus, it is questionable whether the results of beam experiments are applicable to slabs which show a two-dimensional, statically indeterminate structural behaviour.

The goal of the research presented in the following was to derivate an adjusted shear design model for RC slabs without transverse reinforcement under concentrated loads assisted by testing. A database of RC slabs established by Reissen in 2016 [3] and extended by Henze in 2019 [4] was used. This paper focuses on the

complexity of analysing the gathered database information statistically and with respect to an adequate reliability level. The design assisted by testing method is based on the procedure given in Eurocode 0 Annex D.8 [5].

2 TEST SERIES AT HAMBURG UNIVERSITY OF TECHNOLOGY

According to european standard EC1-2 [6] bridge decks are subjected to uniformly distributed variable loads, which don't exceed a value of $q_k = 12 \text{ kN/m}^2$ combined with wheel loads of $Q_k = 150 \text{ kN}$. The latter dominates the internal shear force. In addition, the distance between the wheel load and the support influences the shear bearing capacity due to a direct load transfer in the support region, which is defined by a shear slenderness of $a_v/d < 2.0$ according to EC2-1 [1]. Thus, in 2017 a test series with 14 full-scale cantilevered slabs ($3.25 \text{ m} \times 4.50 \text{ m}$, thickness $h = 0.25 \text{ m}$, effective depth $d = 0.215 \text{ m}$) without transverse reinforcement has been conducted at the Hamburg University of Technology (TUHH) to study the load bearing behaviour [7]. The slabs were stressed by 1 and 2 block loads with a contact area of $0.4 \text{ m} \times 0.4 \text{ m}$ (figure 1).

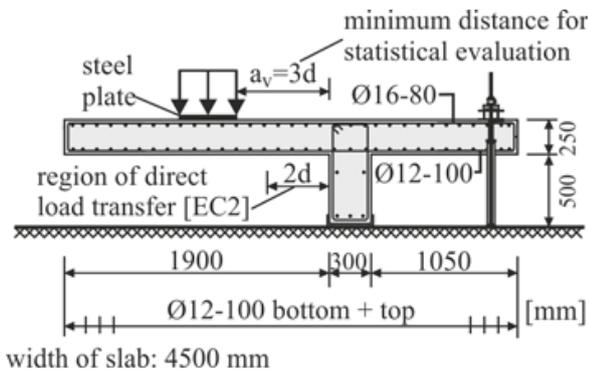


Figure 1: Test specimen at TUHH.

Since the location of the load relative to the support was assumed to have a significant impact on the load-bearing capacity (see figure 1) the most effective approach to analyse the structural behaviour of the slab was to merely vary the ratio a_v/d . Thus, wheel loads with a ratio of $a_v/d = 1 \dots 6$ were subjected to the

cantilevered RC slabs.

All specimens provided significantly greater load bearing capacities than the design value for shear resistance according to the standard [1]. For all single loads outside of the support region the maximum load was approximately $F_u = 500 \dots 600 \text{ kN}$. Moreover the tested slabs didn't fail in the support region, as assumed in common bridge design, but close to the block load. In this way, the structural failure appears to be very similar to punching. Figure 2 shows the crack pattern of a cantilevered slab loaded by a concentrated force with $a_v = 5d$. The area of the fracture is dominated by the oblique cracks. Therefore the area of fracture is significantly higher on the bottom side of the slab than on the upper side.

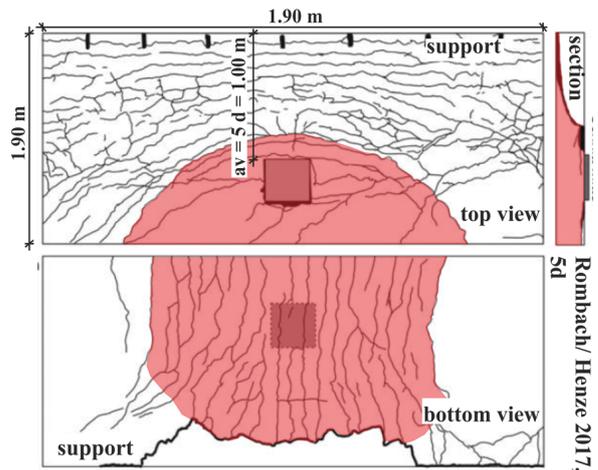


Figure 2: Crack patterns of a cantilever slab loaded by a block load with $a_v/d = 5$ [7].

The test series led to the conclusion that an increase of the theoretical shear resistance according to eq. 1 should be possible. To transfer this assumption to a theoretical resistance model the existing factor $C_{Rd,c}$ is adjusted. Also, since RC slabs fail locally due to shear, the relevant design section should be shifted from the support to a section close to the concentrated load itself. The modified verification method is predominantly based on these two conclusions. Other relevant research results were used as criteria to refine the given database by Reissen [3].

3 STATISTICAL EVALUATION OF A DATABASE

So far there was no method derived that allowed an augmented shear resistance for RC slabs without stirrups under concentrated loads with respect to an adequate reliability level. The analysis below describes the main ideas behind the approach to adjust the resistance model using design assisted by testing method according to EC0 Annex D.8 [5]. Also, difficulties of the procedure and limitations for a general use are outlined.

3.1 Database

Reissen established a database 'KONP1' with 184 slab tests in 2016 [3] containing information about the structural system, measurements of the specimens, subjected loads, concrete properties and ratio of reinforcement. Henze extended this database by 14 additional tests [9]. For the statistical evaluation, in a first step, the extended database is refined by engineering and statistical criteria taking particularly the research results from the TUHH into account.

3.2 Procedure

Before the actual reliability evaluation can be conducted it is necessary to overcome uncertainties by defining criteria from an engineering point of view and by means of statistical tests. The refined database can then be used to statistically determine a resistance model with regard to reliability. Thus, the analysis basically consists of 3 steps:

1. Development of a design model according to eq. 1 based on research results.
2. Collection and evaluation of available data using engineering and statistical criteria.
3. Evaluation of a sufficient reliability level of the modified design method.

The steps 1 and 2 are presented in figure 3. Beside statistical criteria the relevant design

section must be defined. Since the failure appeared locally close to the block load a section in $0.5d$ distance from the edge of the load to the support was chosen [4] whereas d defines the effective depth of the slabs.

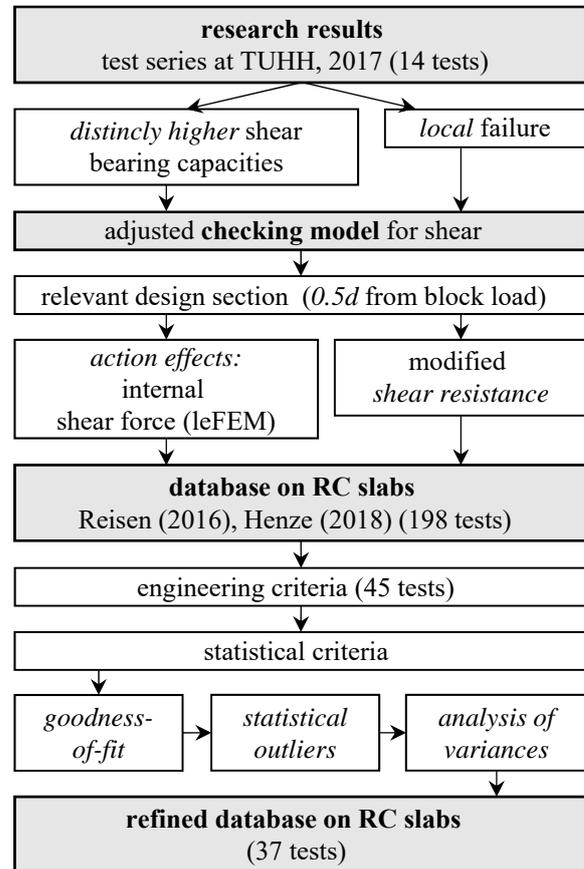


Figure 3: Floatchart for an engineering and statistical evaluation of a database for reliability analysis.

The internal shear force v_{FEM} can be specified using linear-elastic FE-method (shell model) with a finite element size that does not exceed a length of 10 cm or $0.5d$. Non-linear FE calculations are not permitted. For the subsequent evaluation of reliability the ratio γ according to eq. 2 is analysed. $v_{Rd,c}$ is the theoretical shear resistance defined by eq. 1.

$$\gamma = \frac{v_{FEM}}{v_{Rd,c}} \quad (2)$$

3.3 Engineering selection criteria

When deriving a design model assisted by testing several uncertainties have to be considered and diminished by engineering criteria.

First of all, one has to select tests which are well performed and well documented. Further, some uncertainties that are not necessarily captured by objective statistical analysis methods but impact the overall result of the reliability analysis can e.g. derive from different failure modes of the specimens. To overcome such or other systematic failures, research information and other engineering background knowledge is used to refine the given database 'KONP1' [3]. To capture the two-dimensional load transfer the RC slabs must have a minimum width of more than $b = 2.40\text{ m}$. To additionally reduce uncertainties due to size effects only specimens with a slab thickness of $h = 0.20\text{ m}$ are taken into account. Another important factor that influences the load bearing behaviour is the direct load transfer close to support. The research results at TUHH showed a direct load transfer for a ratio $a_v/d < 3.0$ [4]. Thus, tests with a shear slenderness $a_v/d < 3.0$ are excluded. It has to be noted that according to the European standard [1] the region for the direct load transfer is significantly smaller. At last, tests that are influenced by edge distance or have staggered reinforcement are neglected. In table 1 all engineering requirements are listed.

Table 1: Criteria for engineering selection

No.	criteria
KONH1	$b \geq 2.40\text{ m}$
KONH2	$h \geq 0.20\text{ m}$
KONH3	$a_v/d \geq 3.0$
KONH4	no influence of edge distance
KONH5	reinforcement not staggered

The selection criteria were fulfilled by 45 specimens from 6 different test series. Thus, the following research labs were of special interest for the reliability evaluation.

- Muttoni, A.; Rodriguez, R. V.: Lausanne, 2006 ($10\text{ m} \times 4.20\text{ m}$ haunched cantilever slabs; 1 or 2 block loads) [10]
- Rombach, G. A.; Latte, S.: Hamburg, 2011 ($3.0\text{ m} \times 3.0\text{ m}$ single-span slabs

with cantilevers; 1 block load and line load) [11]

- Hegger, J.; Reissen K.: Aachen, 2013, 2016 (single-span slabs and single-span slabs with cantilevers, partly haunched; 1 block load, partly with line load) [3], [12], [13], [14]
- Natario, F.; Muttoni, A.: Lausanne, 2014 ($3.0\text{ m} \times 3.0\text{ m}$ two sided cantilever slabs; 1 block load) [15]
- Rombach, G.A.; Henze, L.: Hamburg, 2017 ($4.50\text{ m} \times 3.25\text{ m}$ cantilever slabs; 1 or 2 block loads) [7]

3.4 Statistical selection criteria

Additionally to engineering criteria an objective statistical examination of the database is conducted (see also [16]). The underlying distribution function of samples of tests is not always distinctly derivable. The EC0 Annex D.8 procedure [5] relies on normal or lognormal distribution for all variables. Thus, the hypothesis whether the specimens of one research lab can be described by the normal distribution is checked using a goodness-of-fit test. Overall the refined database consists of 45 test specimens, while the number of specimens from the same research lab varies from 1 to 11. Goodness-of-fit tests for small samples can be checked using the Kolmogorov-Smirnov test (KS-test) with consideration of the work of Lilliefors or Abdi and Molin [16]. The tables according to Abdi and Molin [18] have to be used to define the critical value Z since mean and standard deviation of the samples are used instead of values of the basic population.

To execute the KS-test the relative bearing capacities $\gamma = v_{FEM,i}/v_{Rd,c,i}$ of a sample are taken. By means of a rank two cumulative frequencies $F_{j,i-1}$ and $F_{j,i}$ (eq. 3, 4) can be estimated.

$$F_{j,i-1} = \frac{(i-1)}{n_j} \quad (3)$$

$$F_{j,i} = \frac{i}{n_j} \quad (4)$$

It is then examined whether the frequency distribution of the normal distribution $\Phi(u_{j,i})$ is in between the range defined by the cumulative frequencies. The null hypothesis can not be rejected if the maximum deviation z calculated according to eq. 5 does not exceed the critical value Z .

$$z = \max \begin{cases} |F_{j,i} - \Phi(u_{j,i})| \\ |F_{j,i-1} - \Phi(u_{j,i})| \end{cases} \quad (5)$$

The critical value Z depends on the sample size and a defined confidence level of $1 - \alpha = 0.95$ for a two-sided test. Table 2 shows the results of the KS-test and the critical values according to Abdi and Molin [18].

Table 2: Statistical data for all research labs

Sample	n_j	μ_j	σ_j	z	Z
Rodriguez 2006	5	2.81	0.56	0.273	0.343
Rombach 2009	1	-	-	-	-
Natario 2014	8	2.46	0.16	0.175	0.288
Hegger 2013	10	2.91	0.45	0.161	0.262
Reissen 2016	11	2.81	0.48	0.176	0.251
Henze 2017	10	2.46	0.38	0.193	0.262

The null hypothesis can not be rejected for any of the samples, hence all 45 tests remain in the database.

Additionally, it is tested whether the maximum or minimum value of a test series differs significantly from the other specimens. The test for statistical outliers by Grubbs with the use of eq. 6 is conducted [19]. Especially outliers caused by measuring errors can be detected with this test [16].

$$z_{j,min} = \frac{\mu_j - x_{j,min}}{\sigma_j}$$

$$z_{j,max} = \frac{x_{j,max} - \mu_j}{\sigma_j} \quad (6)$$

At last, it is statistically tested whether all test samples belong to the same population. Therefore means μ_j and variances σ_j^2 of the samples are tested for significant differences. Here the

Bartlett-test in combination with the F-test is used [16]. While the null hypothesis cannot be rejected for the Bartlett-test, the F-test leads to the result, that the distribution of the test series conducted by Natario [15] shows a significantly different variance σ_j^2 and needs to be excluded from the other samples (see also table 2). As a result, 8 test specimens are not further considered. The remaining 37 tests can be merged to form the database for the reliability analysis. The statistical data is given in table 3.

Table 3: Statistical data of the refined database

$$n = 37 \quad \mu = 2.75 \quad \sigma = 0.47$$

$$v = 0.17 \quad x_{5\%} = 1.98$$

3.5 Evaluation

Subsequent to the before mentioned analysis of the database the reliability analysis can be conducted. The refined database mainly consists of type d) tests in which uncertainties of the resistance model are analysed. For these types of tests the shear design assisted by testing procedure given in EC0 Annex D.8.3 [5] can be applied. Relevant steps are presented in figure 4. The ratio $\gamma_{mod} = v_{FEM}/v_{Rd,c,mod}$ is analysed. Thus in step 1 and 2 the experimental values $r_{e,i}$ are compared to the theoretical values $r_{t,i}$ given by eq. 1. The evaluation of beam tests according to ACI database [2] to validate the shear resistance relies on the fact that the design shear capacity is defined as the 5%-quantile of the mean internal forces of all tests. The 5%-quantile of the modified safety factor then displays the target value of 1.0. Using this information the factor $C_{Rd,c}$ is modified by the 5%-quantile value $x_{5\%}$ to define $r_{t,i} = v_{Rd,c,mod,i}$. The following two equations present the transformation into the further regarded ratio γ_{mod} .

$$x_{5\%} = \mu - 1.645 \cdot \sigma = 1,97 \quad (7)$$

$$\gamma_{mod} = \frac{v_{FEM}}{x_{5\%} \cdot v_{Rd,c}} = \frac{v_{FEM}}{v_{Rd,c,mod}} \quad (8)$$

In [2] another factor x_{Ab} is added to consider that the database of RC beams is not exactly described by the normal distribution.

design assisted by testing Eurocode 0 Annex D.8.3																					
<i>step 1</i>	define theoretical resistance function $v_{Rd,c} = C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} \cdot d$ with $C_{Rd,c} = 0.12$																				
<i>step 2</i>	compare experimental values r_e and theoretical values r_t $r_{e,i} = v_{FEM,i}$ $r_{t,i} = v_{Rd,c,mod,i}$ the theoretical resistance $v_{Rd,c,mod}$ is adjusted by modifying the factor $C_{Rd,c}^* = x_{5\%} \cdot C_{Rd,c}$																				
<i>step 3</i>	estimate mean value correction factor b compare the points $(r_{t,i}, r_{e,i})$ with the trendline defined by $r = b \cdot r_t$																				
<i>step 4</i>	estimate coefficient of variation of the errors V_δ define the error term δ_i $\delta_i = \frac{r_{e,i}}{b \cdot r_{t,i}}$ $\Delta_i = \ln(\delta_i)$ and its mean $\bar{\Delta}$, standard deviation σ_Δ^2 and coefficient of variation V_δ																				
<i>step 5</i>	check for compatibility evaluate the scatter of the points $(r_{t,i}, r_{e,i})$ compared to the trendline r																				
<i>step 6</i>	scatter of basic variables V_{X_i} use prior information to define standard deviations σ_X for V_{X_i} [2], [20]																				
	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th></th> <th>μ_X</th> <th>σ_X</th> <th>V_X</th> </tr> </thead> <tbody> <tr> <td>$f_{cm} =$</td> <td>39.02 MPa</td> <td>4 MPa</td> <td>0.062</td> </tr> <tr> <td>$f_{ck} + 4$ MPa</td> <td></td> <td>1.645</td> <td></td> </tr> <tr> <td>d</td> <td>232.50 mm</td> <td>6 mm</td> <td>0.026</td> </tr> <tr> <td>ρ_l</td> <td>1.03</td> <td>≈ 0</td> <td>0</td> </tr> </tbody> </table>		μ_X	σ_X	V_X	$f_{cm} =$	39.02 MPa	4 MPa	0.062	$f_{ck} + 4$ MPa		1.645		d	232.50 mm	6 mm	0.026	ρ_l	1.03	≈ 0	0
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ρ_l	1.03	≈ 0	0																		
<i>step 7</i>	estimate the characteristic value r_k $r_k^* = \exp(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\delta Q_\delta - 0.5 Q^2)$ for a small number of tests ($n < 100$)																				
<i>step 8</i>	estimate the design value r_d $r_d^* = \exp(-k_{d,\infty} \alpha_{rt} Q_{rt} - k_{d,n} \alpha_\delta Q_\delta - 0.5 Q^2)$ for a small number of tests ($n < 100$)																				
<i>step 9</i>	estimate the safety factor γ_r and the final modification factor $C_{Rd,c}^*$ $\gamma_r = r_k / r_d$ $C_{Rd,c}^* = \frac{x_{5\%} \cdot C_{Rd,c}}{\gamma_r}$																				

Figure 4: Flowchart for design assisted by testing method given by EC0 Annex D.8.3. [5].

Exactly 5% of the test specimen should display failure. This factor is not needed for the research presented here since it was made sure by hypothesis testing, that the test results can be displayed by the normal distribution. If the tests conducted by Natario [15] were included, a factor $x_{Ab} = 1.025$ would be calculated. A linear regression is conducted only for the factor $C_{Rd,c}^*$ of the existing function according to eq. 1. It is not examined whether other basic variables are presented correctly in the function for $v_{Rd,c}$. While in step 3 and 4 the variation of the test results are estimated, another important step during the EC0 reliability analysis is the evaluation of compatibility of the theoretical function compared to the test results. Deviations between test points and the function $r_e = b \cdot r_t$ define the scatter. In figure 5 it can be seen that the test results scatter around the modified theoretical value with a variance of $V_\delta = 0.171$, a usual value.

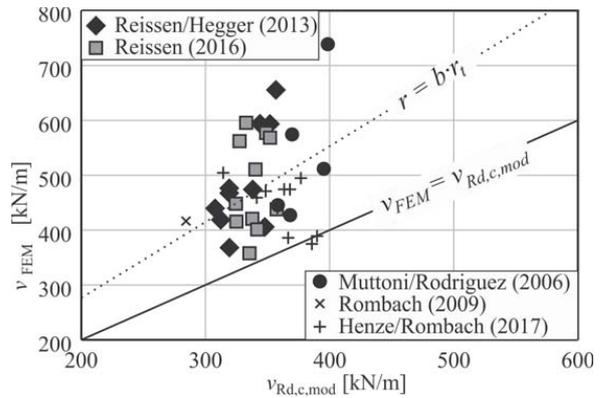


Figure 5: Scatter of the test points $(r_{e,i}, r_{t,i})$.

This scatter arises mostly from inconsistencies in the empirical formula eq. 1 for shear. (German Committee for Reinforced Concrete vol. 617 [2]: $V_\delta = 0.198$; vol. 515 [16]: $V_\delta = 0.174$) In the following step 6, the coefficient of variation for the basic variables needs to be defined. Here the use of prior knowledge is reasonable, since the basic variables used in the database do not properly display the statistical population. The table in step 6 (figure 4) shows classical standard deviations according to [2] and [20]

that are used for the reliability evaluation. At last, in step 7 to 9, the characteristic and design values are calculated using the coefficients of variation of the errors V_δ and the basic variables $V_{X,i}$ which for small values can be combined after their linear regression as shown in eq. 9.

$$V_r^2 = V_\delta^2 + \left[\left(\frac{1}{3} \cdot V_{fcm} \right)^2 + \left(\frac{2}{3} \cdot V_d \right)^2 \right] \quad (9)$$

Based on the Probabilistic Model Code of the JCSS [21] a safety index of $\beta = 4.4$ for brittle failure is used to calculate the design value r_d^* . In the last step 9, a specific safety factor for shear resistance is calculated and used to define the finally modified factor $C_{Rd,c}^*$.

The reliability analysis results in a modified factor $C_{Rd,c}^* = 0.161$ which is 34% greater than the standard value $C_{Rd,c} = 0.12$. Including the test series of Nataro [15], a slightly higher factor of $C_{Rd,c}^* = 0.168$ was calculated due to a smaller standard deviation for this test series.

4 CONCLUSIONS

The reliability analysis of a database of RC slabs without transverse reinforcement under concentrated single loads consists of three major steps: gathering of research data, refinement of the test data by means of engineering and a statistical criteria concluded by a reliability analysis.

By means of well-defined engineering criteria (153 tests sorted out) combined with additional statistical methods (further 8 tests sorted out) the amount of data can be selected for a reliability analysis. The function $v_{Rd,c}$ according to EC2 [1] (see eq. 1) was analysed using the design assisted by testing method given in EC0 Annex D.8.3 [5]. As a result, the factor $C_{Rd,c} = 0.12$ according to the standard can be increased by 34 % to a modified value $C_{Rd,c}^* = 0.16$.

As in all statistical evaluations the approach should only be considered valid for the parameters covered by the database, which are amongst others defined by the criteria in Table 2. Additionally other loads on the RC slabs must be low compared to the single load. The slabs should have a reinforcement ratio of $\rho_l = A_{sl}/(b \cdot d) \approx$

1%.

The shear resistance of concrete bridge decks under wheel loads could be enhanced significantly by means of statistical investigations and consideration of reliability. The presented investigations led to a more economic and more realistic design rule.

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SYMBOLS**Latin letters**

a_v	distance between the edge of the wheel load and the support
A_{sl}	cross sectional area of reinforcement
b	width of the cross section
d	effective depth of the cross
f_{ck}	characteristic compressive cylinder strength of concrete
$F_{j,i-1}$	lower cumulative frequency
$F_{j,i}$	upper cumulative frequency
F_u	ultimate load
h	depth of the cross section
i	rank
k	coefficient
k_n	characteristic fractile factor for a sample
$k_{d,n}$	design fractile factor for a sample
n	number of specimens
q_k	characteristic uniformly distributed variable action
Q_k	characteristic variable single action
Q	$= \sqrt{\ln(V^2 + 1)}$
r	function for the resistance
r_e	experimental resistance value
r_t	theoretical resistance value
v	variance
v_{FEM}	internal shear force
V	coefficient of variation
V_r	overall coefficient of variation for reliability analysis
x_j	test value of the sample
$x_{5\%}$	5%-quantile
x_{Ab}	factor to achieve 5% failure for all test specimens
z	calculated value of the regarded sample for different statistical tests
Z	critical value for different statistical tests (KS-test, Grubbs, F-test)

Greek letters

α_x	weighting factor
β	reliability index
γ	relative load-bearing capacity
γ_r	corrected partial factor for resistance
δ	error term
Δ	logarithm of the error term δ
$\bar{\Delta}$	estimated value for Δ
μ_j	mean of sample results
ρ_l	$= A_{sl}/(b \cdot d)$ reinforcement ratio
σ_j	standard deviation from sample results
$\Phi(u_{j,i})$	cumulative distribution function of the standard normal distribution

Indices

d	design value
$i \dots n$	number of specimens
$j \dots n$	number of test series
k	characteristic value
min	minimum value
max	maximum value
mod	modified
rt	resistance
X	basic variable
δ	error term