

MESOSCALE INVESTIGATION OF THE FPZ LENGTH-CRACK LENGTH CORRELATION IN QUASI-BRITTLE MATERIALS LIKE CONCRETE

N. AISSAOUI^{*} AND M. MATALLAH[†]

^{*} RiSAM, University of Tlemcen, Algeria.
e-mail: nassima2307@hotmail.fr

[†] RiSAM, University of Tlemcen, Algeria.
e-mail: mohammed.matallah@gmail.com

Key words: Fracture Process Zone, Crack length, Concrete, Mesoscale modeling, R-curve.

Abstract: Owing to its central role in describing the fracture behavior of concrete, the fracture process zone has been studied both experimentally and theoretically. This paper presents a numerical approach to describe the fracture process zone of notched concrete beams subjected to three point bending. The FPZ length is determined numerically during the fracturing process by evaluating the tangential stress along the crack path with a mesoscopic finite element modeling framework. The link between the fracture process zone and crack length in concrete structure is investigated. The ratio between the crack length and FPZ length is not constant throughout the cracking process but is varying into three stages of crack propagation. It has been also proved that the FPZ length increase with increasing the crack length and then decreases gradually after that. The numerical crack extension is therefore used to investigate the R-curves.

1 INTRODUCTION

Fracture in quasi-brittle materials like concrete is characterized by the existing of a non-linear zone of micro-cracking around the crack tip. Due to the existence of this characteristic property (called Fracture Process Zone (FPZ)) during the fracture process, the material behavior becomes soft and LEFM cannot correctly reproduce the stress field within this zone. In concrete, the size of the FPZ is relatively significant compared to the specimen size, this leads to size dependency of the strength and fracture toughness. The existence of the FPZ may be the intrinsic cause of the size effect. So, the features of the FPZ are important to be known for the engineering community, specially its evolution during crack propagation of concrete.

Numerically, to characterize the fracture process of concrete, damage and/or fracture based models have been proposed (fictitious

crack model [9], Crack band model [6] ...). On the other hand, some models have been developed based on the modification of the LEFM using the equivalent crack length concept.

For quasi-brittle material, the stress field in the zones away from the FPZ is essentially elastic. The stress field for the whole material including the FPZ zone could be approximate by LEFM with an equivalent crack length. Irwin [10] introduced the term “equivalent crack length” to describe a fictitious increase of crack when a new distribution of stress is considered within the FPZ. Within the framework of *Equivalent-LEFM*, the increase of the specimen compliance due to the fracture process zone (FPZ) development is attributed to the propagation of an effective crack or in other terms, an equivalent elastic crack of length a_{eq} [5]. The correlation between the FPZ length

and the crack length was suggested by many researchers in the literature [4][13].

In the concept of the equivalent linear elastic fracture mechanics, any diminution of the apparent compliance (or stiffness) of the specimen will be accompanied by the propagation of an equivalent elastic crack, in other words, the crack which, in a specimen considered perfectly elastic, produced in accordance with the LEFM, the same compliance (or stiffness) as the actual specimen cracked with its damaged zone. The tip of this equivalent crack is not located at the beginning of the FPZ, but at a certain distance such that $a_{eq} = a_0 + \Delta a$ with a_0 is the length of the initial crack and Δa is the increment of the equivalent elastic crack.

Due to the important role of the FPZ in understanding the size effect phenomenon and determining various fracture parameters in quasi-brittle materials, it is crucial to be able to measure accurately the evolution of the FPZ during the fracture process. However, there are different consensuses among the researchers on the evolution of the FPZ of structural concrete. Some experimental and theoretical studies ([18][17]) affirmed that the length of the FPZ increases before the FPZ is fully developed and decreases after that while other studies affirmed that the length of FPZ keeps constant after FPZ is fully open [19] [14].

From the perspective of the numerical simulation, various techniques have been employed to track the FPZ extent. There is no widely accepted conclusion on features of variation of FPZ in concrete; the debate is still ongoing as: how to exactly define and measure the size of FPZ?

Numerically speaking, the most well-indicator to describe the overall FPZ size is the normal stress profiles along the crack path as described in the fictitious crack model proposed by Hillerborg [9]. The principle of this method has been recently used by the authors in [1] to compute successfully the length of the FPZ by studying the evolution of the tangential stress using the mesoscopic approach.

This paper presents a numerical investigation on the evolution of the Fracture Process Zone (FPZ) length in concrete using the mesoscopic approach. The three point bending beams tested by Laura Rojas Solano [16] are considered for the numerical investigation. An energetic regulation method based on the crack band approach ([2][6][11][12]) was adopted to control the localization process. To explore the variation of the FPZ extent during the whole fracture process, the tangential stress profile obtained numerically on the crack path is evaluated at each calculation step for two series of notched concrete beams under three point bending. The crack length/FPZ length ratio is therefore investigated.

2 MESOSCALE INVESTIGATION OF THE GLOBAL BEHAVIOR

In the present paper, the three point bending beams tested by Laura Rojas Solano [16] are considered for the numerical investigation. Mesoscale modeling of four different sizes of geometrically similar notched beams of size range $50 < D < 400$ mm with notch-to- depth ratio of 0.2 and 0.5 are conducted.

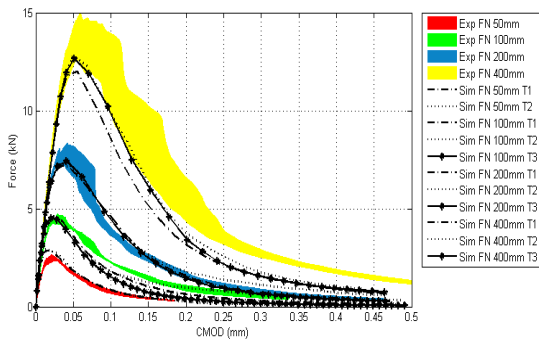
Numerical simulation of concrete at mesoscale leads to a realistic description of the concrete behavior. The meso-scale permits an explicit representation of concrete constituents ([15][8]). Concrete is considered as a biphasic material. The mortar and the aggregate phases are described by their own characteristic behavior. A softening damage law is used both for the aggregate and the mortar constituents with different characteristic. The Interfacial Transition Zone (ITZ) is not considered. Damage creation is the result of stress concentrations occurring at the aggregate-matrix interface. The influence of the ITZ has been extensively discussed by Grondin and Matallah in [8].

Computation modeling is driven in 2D under stress plan condition with displacement control. Only the central part of the beam where damage is expected to occur is considered with two constituents. The left and right ends of the beam are considered as

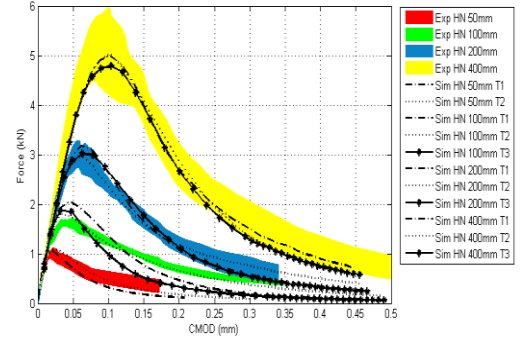
monophasic with a gradual mesh. Smooth transition between the different mesh zones is adopted to avoid stress concentration. A linear elastic model is considered for the macroscopic parts of the mesh to reduce the computation time. For both aggregate particles and the matrix we use an isotropic damage model proposed by Fichant [7]. An energetic regulation method based on the crack band approach ([2][6][11][12]) was adopted to control the mesh dependency in concrete in tension induced by the localization phenomenon. The parameter controlling the descending branch of the softening curve is adjusted to the size of the finite element used. Dissipation is thus governed by the fracture energy. The characteristics of the aggregates and the mortar are respectively ($F_t = 6\text{MPa}$, $G_f = 85\text{N/m}$, $E = 70\text{MPa}$) and ($F_t = 3.5\text{MPa}$, $G_f = 55\text{N/m}$, $E = 30\text{MPa}$).

2.1 Analysis of the global behavior of beams

Figure 2 (a) and (b) show a comparison of the load-CMOD curves obtained from the experimental and numerical simulation with the mesoscopic approach for the different beam sizes with two geometries (fifth-notched specimens and half-notched specimens) respectively.



(a)



(b)

Figure 1: Numerical curve Force-CMOD: a) beams notched at 20% of the height of specimens, b) notched beams at 50% of the height of specimens.

The comparison shows that the overall behavior is well reproduced numerically; fracture and damage of concrete are correctly reproduced for all specimen sizes and for the both geometries. The results show non-significant difference on the global behavior regarding the random distribution of the aggregates because the dissipation is governed by the fracture energy.

3 NUMERICAL CHARACTERIZATION ON THE VARIATION OF THE FPZ LENGTH IN CONCRETE

In this section we propose to follow numerically the evolution of the FPZ length during the whole fracturing process of the notched beams at 3 points which will be used later for the construction of the resistance curve (R curve).

3.1 The fully developed FPZ length

Thus, on the basis of the enrichment stress field-based criterion ([1][3]), the length of a fully developed FPZ corresponds to an evolution of the tangential stress equal to tensile strength f_t at the crack tip until this stress gradually decrease to zero at the notch tip.

Figure 2 illustrates the evolution of the tangential stress along the crack path for the different beam geometries for fully opened FPZ. This plot reveals clearly that the evolution of the tangential stress $\sigma_{\theta\theta}$ (equal to

the normal stress) is nonlinear along the fracture process zone.

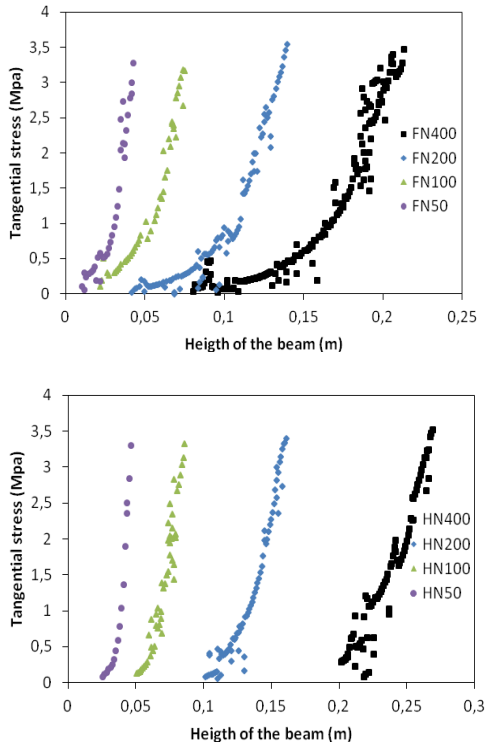


Figure 2: Evolution of the tangential stresses for fully opened FPZ.

3.2 Estimation of the FPZ extent

As aforementioned, there is a contrast among the researchers regarding the extent of the FPZ length after its fully development; if it remains constant or decreases! Numerically speaking, the most well-indicator to describe the overall FPZ size is the tangential stress profiles along the crack path as described above. As for the cohesive models, the FPZ length is the zone on which the stress distributes nonlinearly where softening cohesive behavior occurs, i.e., the distance between the stress at the crack tip equal to the tensile strength and equal to zero at the initial tip.

To explore the variation of the FPZ extent during the whole fracture process, the tangential stress obtained numerically at both the notch tip and the crack tip is plotted in Figure 3 at each computation step for beams FN400 and HN 400.

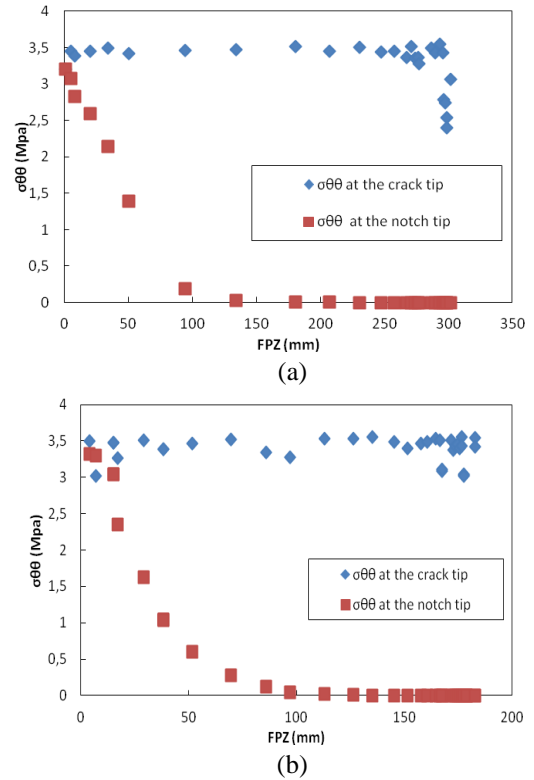


Figure 3: FPZ length progress and corresponding tangential stresses values at both the crack tip and notch tip.

Initially, it can be seen from Figure 3 that the length of the FPZ increases progressively. With further progress of cracking, the tangential stress at the crack tip reaches its maximum value equal to the tensile strength and vanishes at the notch tip. The first fully developed FPZ is then formed at this time and a stress-free length occurs in front of the notch beam behind the FPZ.

The entire fracture process of concrete is illustrated by a histogram in Figure 4 presenting the formation and progress of the FPZ length and crack length along the ligament for beams sizes 400 mm during the entire loading process.

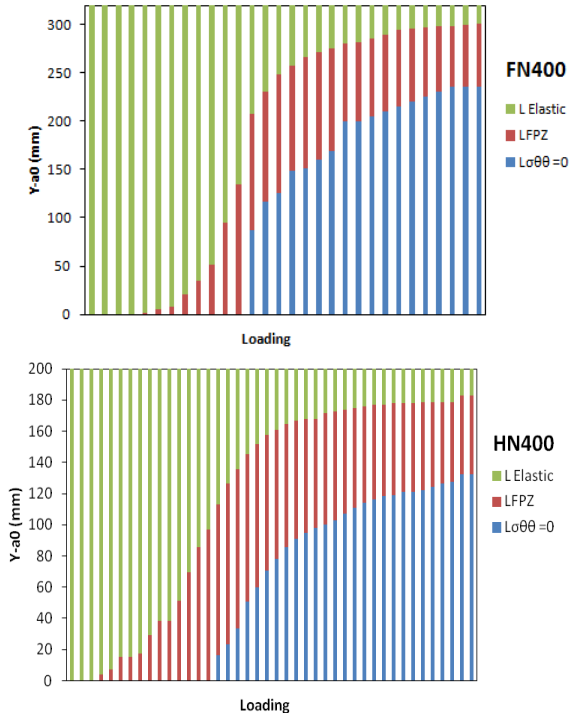


Figure 4: FPZ evolution and crack growth for beams size 400mm.

According to this plot, the evolution of the FPZ extension can be divided into two parts: in the first part, the length of the FPZ increases gradually ones started from the notch tip until reaching its maximum length L_{FPZ} whereas crack has not started yet. The second part is characterized by the crack initiation-development but the FPZ length starts to shrink. From this perspective, the restriction of the FPZ can be attributed to the fact that during the crack progression, the available part of the ligament for the FPZ to develop fully becomes increasingly small in a way they prevent the free growth of the FPZ.

Another interesting plot is given in Figure 5 in which the changes of FPZ size along the ligament according to various sizes are normalized. The FPZ length increases linearly with increasing the crack length until it reaches its maximum value denoted the fully FPZ length (LFPZ) and then decreases gradually after that. The fully FPZ length is size dependent. As shown in Figure 5, the full FPZ length increase with the increase of the specimen size, however, the relative crack length (a/D') (D' is the ligament) corresponding to the full FPZ length decreases.

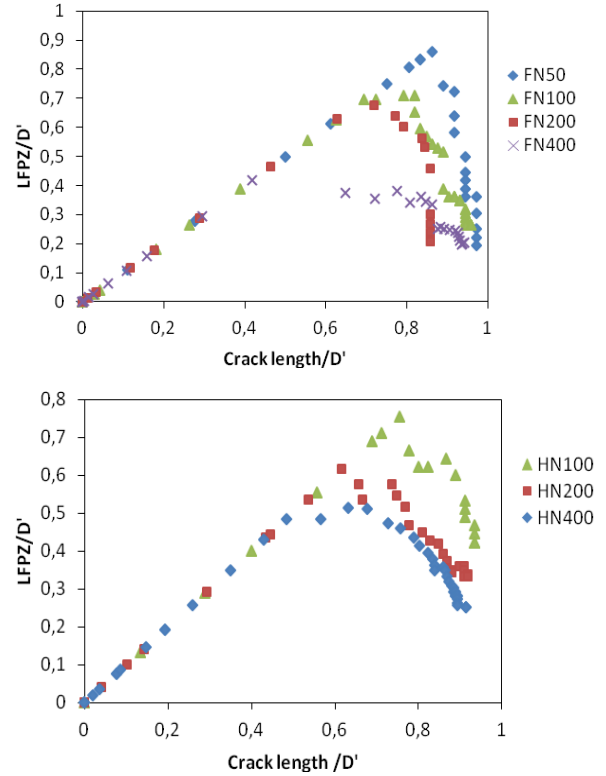


Figure 5: variation of the relative FPZ length for notched beams FN and HN.

4 CONSTRUCTION OF R-CURVES VIA THE MESOSCOPIC APPROACH

In order to estimate the resistance curve for the notched beam, two parameters must be determined numerically which are: the energy release rate and the crack extension during the entire fracturing process.

The main steps for the construction of such a curve are presented as follows.

4.1 The equivalent elastic crack length

The objective is to numerically determine the crack extension for notched beams within the framework of the equivalent elastic linear fracture mechanics, ie, to find the value of the extension of the crack which will allow to find the same stiffness as the beam cracked with its FPZ length.

Figure 6 shows the F-displacement curve for the beam FN200.

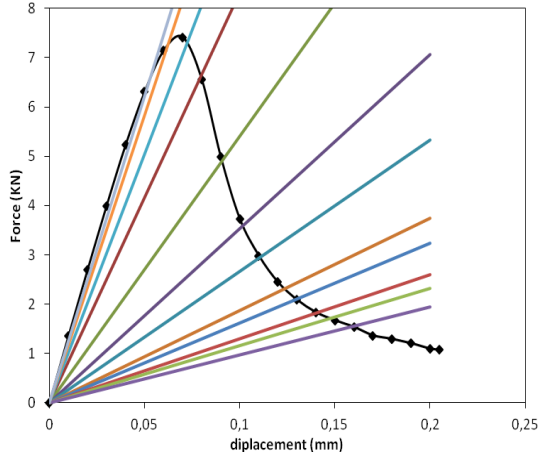


Figure 6: Force-displacement curve- FN200 beam.

Knowing the crack length evolution, corresponding to the initial length (or notch length) a_0 with the length of the damaged area (the FPZ) (section 3.2), the stiffness of the notched beam is estimated at each point of the F-displacement curve. The second step is to simulate the same beam with different crack lengths until we find the crack length that makes it possible to approach (reproduce) the stiffness estimated in the first step (see Figure 6). This process could be automated by an inverse analysis approach.

The optimization process shows that the ratio between the crack length and FPZ length is not constant throughout the cracking process but is varying into three stages of crack propagation (Figure 7):

-stage 1 $L_{FPZ} < L_{FPZ_{max}}$

$$L_{eq} = a_0 + 0.45 \times L_{FPZ} \quad (1)$$

-stage 2 $L_{FPZ} = L_{FPZ_{max}}$

$$L_{eq} = a_0 + (L_{FPZ} / 2) \quad (2)$$

-stage 3 $L_{FPZ} < L_{FPZ_{max}}$ with crack propagation

$$L_{eq} = a_0 + a_{\sigma\theta=0} + (L_{FPZ} \times Coef) \quad (3)$$

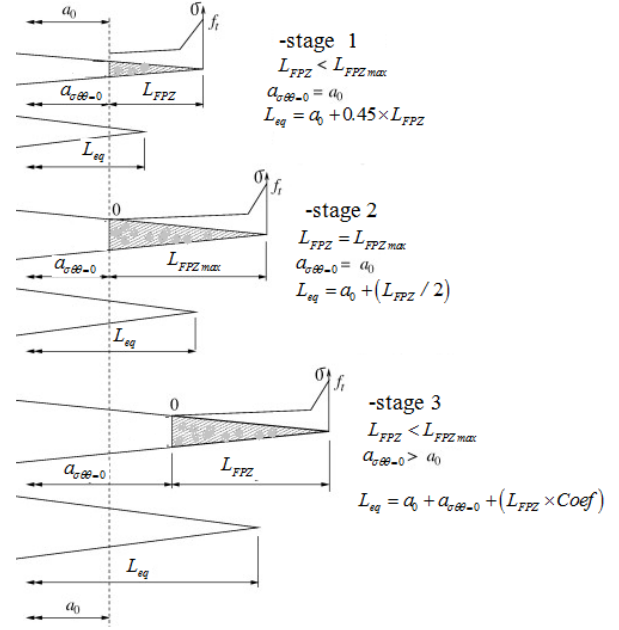


Figure 7: Stages of FPZ evolution.

A closer examination shows that at the full development of the FPZ, the crack extension is the half of the FPZ length.

This ratio is close to that found by the authors in [1] where the length of the FPZ estimated numerically following the evolution of the tangential stress along the crack was compared to the extension of the crack proposed by Bazant [4]. The *Coef* in Equ. (3) is variable.

The equivalent crack evolution is shown in Figure 8 for beam FN200. This evolution is not linear and increases as the crack propagates.

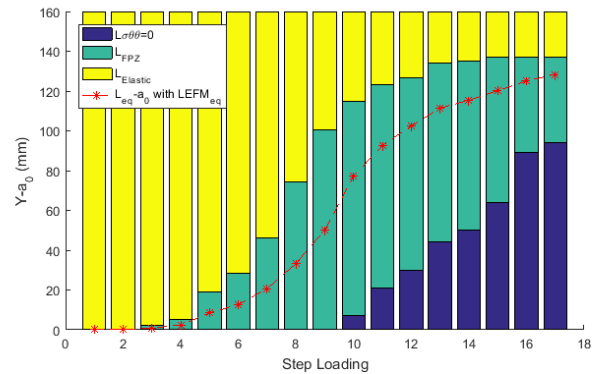


Figure 8: Correlation between crack length and FPZ length.

4.2 Energy release rate estimation

In accordance with the linear elastic fracture mechanics, the resistance to crack growth $G_R(a)$ can be expressed from the energy release rate $G(a)$. This energy release rate expresses the rate of energy change for each increase of small crack δa . Theoretically, the energy release rate can be estimated by several ways.

Among the existing methods in the literature, there is an expression of the energy release rate is a function of the adimensional energy release rate is done as:

$$G(a) = \frac{F^2}{Eb^2D} g(\alpha) = G_R(a) \quad (4)$$

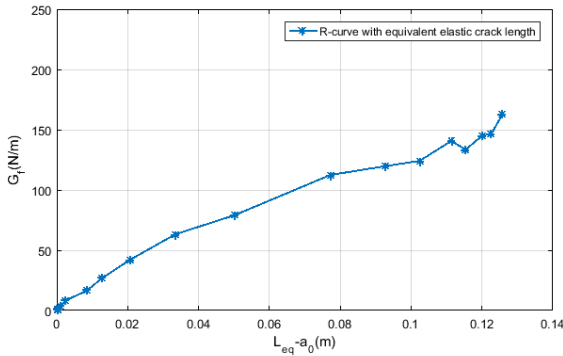


Figure 9: R-curve for notched beam FN200.

The resistance curve shown in Figure 9 is obtained with the numerical crack extension. The energy restitution is estimated as a function of the adimensional function of the energy release rate (equation (4)). An inverse analysis procedure is used to estimate equivalent elastic crack lengths.

The ascending part of the R-curve is well reproduced. The first upward part relating to the development of the fracture process is accompanied with a significant increase in resistance to crack growth G_f . The second phase of the plateau value is not represented. This phenomenon would be the consequence of an early confinement of the FPZ.

6 CONCLUSIONS

In this study a mesoscopic approach is employed to investigate the evolution of the fracture process zone during the entire fracture

process of concrete members. The evolution of the tangential stress along the crack path is used to identify the length of the FPZ. It has been proved that the FPZ length increase with increasing the crack length until FPZ is fully developed and then decreases gradually after that.

The crack extension has been determined numerically based on the equivalent linear elastic fracture mechanics.

The numerical results show that the crack length-FPZ length ratio is not constant throughout the cracking process. The crack extension is the half of the fully maximum FPZ length.

The resistance curve (R-curve) for the notched beam is investigated based on the numerical crack extension obtained with a correlation of the mesoscopic approach and the equivalent linear elastic fracture mechanics. The R-curve is well reproduced.

REFERENCES

- [1] Aissaoui, N. & Matallah, M., 2017. Numerical and analytical investigation of the size-dependency of the FPZ length in concrete. *International Journal of Fracture*, 205(2), pp.127–138.
- [2] Aissaoui, N. & Matallah, M., 2015. Sources of error and limits of applying the energetic-based-regularization method. In *COMPLAS XIII : proceedings of the XIII International Conference on Computational Plasticity : fundamentals and applications*. International Center for Numerical Methods in Engineering (CIMNE), pp. 916–921.
- [3] Ayatollahi, M.R. & Akbardoost, J., 2012. Size effects on fracture toughness of quasi-brittle materials – A new approach. *Engineering Fracture Mechanics*, 92, pp.89–100.

- [4] Bažant, Z.P., 2002. Concrete fracture models : testing and practice. *Engineering Fracture Mechanics*, 69, pp.165–205.
- [5] Bazant, Z.P. & Kazemi, M.T., 1990. Determination of fracture energy , process zone length and brittleness number from size effect , with application to rock and concrete. *International Journal of Fracture*, 44(2), pp.111–131.
- [6] Bažant, Z.P. & Oh, B.H., 1983. Crack band theory for fracture of concrete. *Materials and Structures*, 16, pp.155–177.
- [7] Fichant, S., 1996. Endommagement et anisotropie induite du béton de structures: modélisations approchées. Ph.D thesis, ENS Cachant, France.
- [8] Grondin, F. & Matallah, M., 2014. How to consider the Interfacial Transition Zones in the finite element modelling of concrete? *Cement Concrete Res*, 58, pp.67–75.
- [9] Hillerborg, A., Modéer, M. & Petersson, P.E., 1976. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and Concrete Research*, 6(6), pp.773–782.
- [10] Irwin, G.R., 1957. Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate. *J. Applied Mechanics*, 24, pp.361–364.
- [11] Jirásek, M. & Bauer, M., 2012. Numerical aspects of the crack band approach. *Computers & Structures*, 110, pp.60–78.
- [12] Matallah, M. et al., 2013. Size-independent fracture energy of concrete at very early ages by inverse analysis. *Engineering Fracture Mechanics*, 109, pp.1–16.
- [13] Morel, S. et al., 2010. Bilinear softening parameters and equivalent LEFM R-curve in quasibrittle failure. *International Journal of Solids and Structures*, 47(6), pp.837–850.
- [14] Morel, S. & Dourado, N., 2011. Size effect in quasibrittle failure : Analytical model and numerical simulations using cohesive zone model. *International Journal of Solids and Structures*, 48(10), pp.1403–1412.
- [15] Nguyen, D. et al., 2010. A mesoscopic model for a better understand understanding of the transition from diffuse damage to localized damage. *Eur J Environ Civ En*, 14, pp.751–775.
- [16] Grégoire, D., Rojas-Solano, L.B. & Pijaudier-cabot, G., 2013. Failure and size effect for notched and unnotched concrete beams. *Int. J. Numer. Anal. Meth. Geomech*, 37, pp.1434–1452.
- [17] Wecharatana, M. & Shah, S.P., 1983. Predictions of Nonlinear Fracture Process Zone in Concrete. *Journal of Engineering Mechanics*, 109(5), pp.1231–1246.
- [18] Wu, Z. et al., 2011. An experimental investigation on the FPZ properties in concrete using digital image correlation technique. *Engineering Fracture Mechanics*, 78(17), pp.2978–2990.
- [19] Xu, S. & W. Reinhardt, H., 1998. Crack Extension Resistance and Fracture Properties of Quasi-Brittle Softening Materials like Concrete Based on the Complete Process of Fracture. *International Journal of Fracture*, 92(1), pp.71–99.