10^{th} International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-X
G. Pijaudier-Cabot, P. Grassl and C. La Borderie (Eds)
https://doi.org/10.21012/FC10.235469

FRACTURE PROPAGATION IN NUCLEAR GRAPHITE

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Key words: Bittle fracture, material forces, nuclear graphite, finite elements

Abstract. This paper presents an enhanced theoretical formulation and associated computational framework for brittle fracture in nuclear graphite within the context of configurational mechanics. A new condition for crack front equilibrium is exploited that leads to an implicit crack propagation formulation. This paper focuses on an extension of our previous work, whereby the complex internal stress state in a nuclear reactor is the primary driver for crack propagation in individual graphite bricks. The resulting crack path is resolved as a discrete displacement discontinuity, where the material displacements of the nodes on the crack front change continuously, without the need for enrichment techniques. Performance of the formulation is demonstrated by means of a representative numerical simulation, demonstrating both accuracy and robustness.

1 INTRODUCTION

Understanding the behaviour of Advanced Gas-Cooled Reactor (AGR) graphite cores with multiple cracked bricks is paramount to the assessment of structural integrity, safe operation and life extension. In this paper, the latest developments in the finite element modelling and simulation of crack propagation in graphite bricks are briefly presented.

Configurational mechanics (CM) provides the theoretical basis for our work on crack propagation. This approach has a strong physical motivation, exploiting the 1st and 2nd laws of thermodynamics to establish crack front equilibrium and the crack path direction. The authors have also developed the numerical techniques to implement this theory within a finite element analysis software framework (MoFEM [9]). This provides the ability to simulate propagating cracks in 3D solids that are discretely and continuously resolved by adapting the FE mesh in a smooth manner (exploiting the crack front equilibrium condition), thereby avoiding the need for enrichment.

CM dates back to the original work of Eshelby and his study of forces acting on material defects [1]. The concept of configurational (or material) forces is now a well established method to evaluate defects in a material providing a unified framework for the analysis of material imperfections and has been adopted by, amongst others, Maugin [2]. Steinmann [3] developed a computational strategy for the assessment of fractured bodies. Miehe et al. [4, 5] and Kaczmarczyk et al. [6, 8] built on this work to establish a finite element methodology for crack propagation.

2 CRACK PROPAGATION PROBLEM

To formulate the crack propagation problem within the framework of configurational mechanics, two related kinematic descriptions are defined in the spatial and material settings. In the former, the classical conservation law of linear momentum balance is described, where Newtonian forces are work conjugate to changes in the spatial position, at fixed material position (i.e. no crack propagation). In the material setting, which represents a dual to the spatial setting, an equivalent conservation law is described, where configurational forces are conjugate to changes in material position but with no spatial motion. This decomposition of the behaviour is proven to be a simple but powerful methodology for describing crack propagation. The authors' previous paper [6] describes the mathematical formulation for crack propagation and a methodology for resolving the evolving crack path within the context of the finite element method, and represented an advancement of the work of Miehe et al. [4, 5]. The current paper briefly explains how this previous work has been extended for internal stresses as the driver for crack propagation.

Key features of our work to date include:

- Griffith's fracture criterion is expressed correctly in terms of configurational forces.
- An expression for equilibrium of the crack front is established, balancing the configurational forces on the crack front with the resistance of the material. This is exploited so that the crack front can advance continuously.
- To maintain mesh quality, a mesh smoothing strategy, with surface constraints, is presented as a continuous process as part of a problem-tailored Arbitrary Lagrangian Eulerian formulation.
- The spatial and material displacement fields are both discretised using the same finite element mesh, although we adopt different levels of approximation for the two fields.
- The resulting discretised weak form of the two conservation equations represent a set of coupled, nonlinear, algebraic equations that is solved in a monolithic manner using a Newton-Raphson scheme.
- An arc-length method is adopted to trace the dissipative load path for brittle fracture propagation, using crack area rather than displacements as a control.

The current material coordinates $\mathbf{X}$ are mapped onto the spatial coordinates $\mathbf{x}$ via the familiar deformation map $\boldsymbol{\varphi}(\mathbf{X}, t)$. The physical displacement is:

$$\mathbf{u} = \mathbf{x} - \mathbf{X} \tag{1}$$

$\Xi(\boldsymbol{\chi}, t)$ maps the reference material coordinates $\boldsymbol{\chi}$ on to the current material coordinates $\mathbf{X}$, representing a configurational change, i.e. extension of the crack due to advancement of the crack front. $\boldsymbol{\Phi}$ maps the reference material coordinates $\boldsymbol{\chi}$ on to the spatial coordinates $\mathbf{x}$. The current material and spatial displacement fields are given as:

$$\mathbf{W} = \mathbf{X} - \boldsymbol{\chi} \quad \text{and} \quad \mathbf{w} = \mathbf{x} - \boldsymbol{\chi} \tag{2}$$

Finite element approximation is applied to the displacements in both the current material and physical spaces. Three-dimensional domains are discretised with tetrahedral finite elements. In the spatial domain, hierarchical basis functions of arbitrary polynomial order are applied, following the work of Ainsworth and Coyle [7]. This enables the use of elements with variable, non-uniform orders of approximation, with conformity enforced across element boundaries. In the material domain, linear approximation is adopted, as this is sufficient for describing the crack front.

Kaczmarczyk et al. [8] derived a new expression for equilibrium of the crack front:

$$\dot{\mathbf{W}} \cdot (\gamma \mathbf{A} - \mathbf{G}) = 0 \tag{3}$$

where γ is the surface energy, $\mathbf{A}$ is a dimensionless kinematic state variable that defines the current orientation of the crack front [6,8] and G is the configurational force calculated as the integral of the Eshelby stress $\boldsymbol{\Sigma}$ around the crack front:

$$\mathbf{G} = \lim_{|\mathcal{L}_\mathrm{n}| \to 0} \int_{\mathcal{L}_\mathrm{n}} \boldsymbol{\Sigma} \mathbf{N} \, \mathrm{d}L \tag{4}$$

where This crack front equilibrium condition balances the configurational forces on the crack

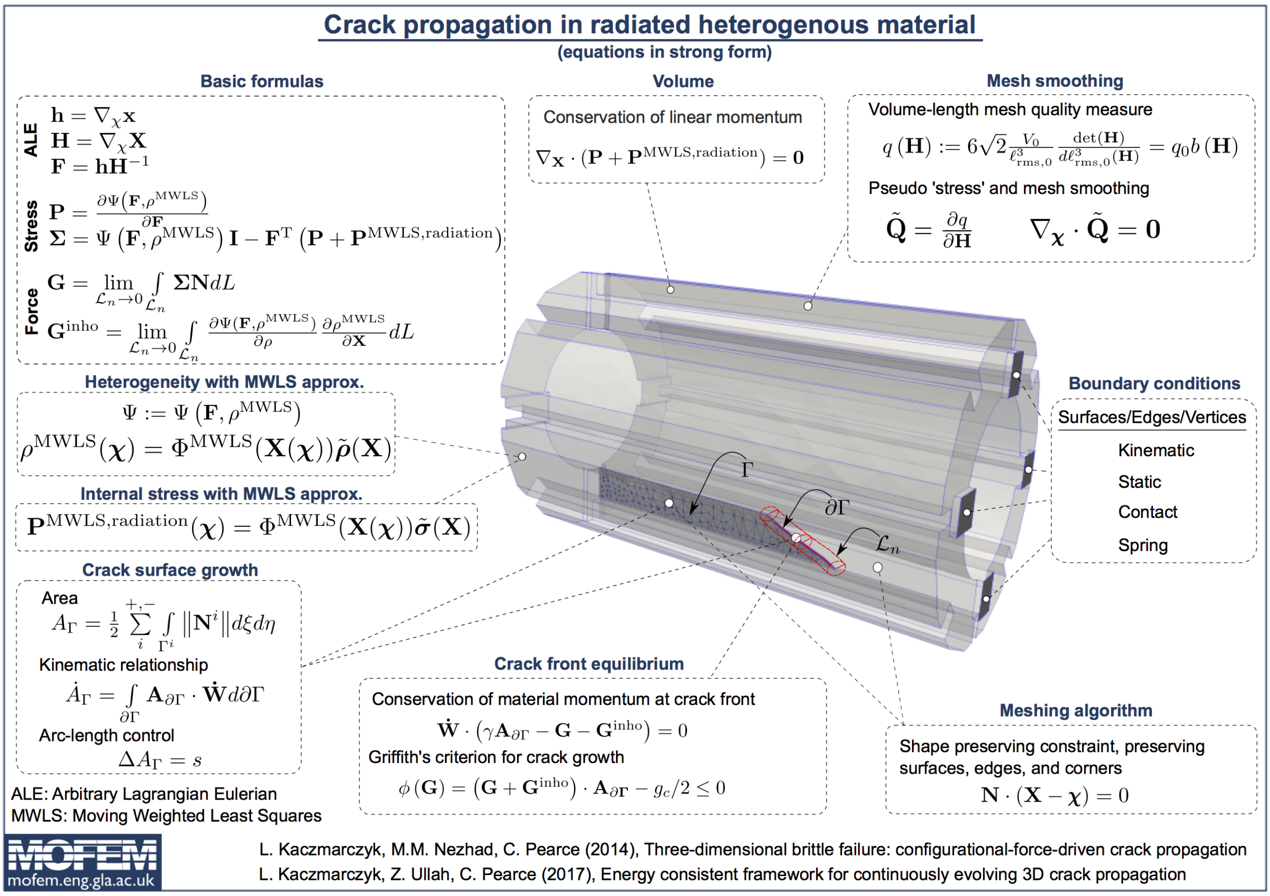

Figure 1: Equations for crack propagation in the radiated graphite, i.e. brittle heterogenous material with internal stresses.

front with the resistance of the material. This is exploited so that the crack front can advance continuously, re-establishing the crack front to the physically correct position each load step, without recourse to any kind of crack tracking algorithm, and without influence from the finite element mesh.

3 INTERNAL STRESS DRIVEN CRACK PROPAGATION

To account for the influence of internal stresses, the residual, discretised equations for spatial and material equilibrium are modified to become:

$$\mathbf{r}_{\mathrm{s}} = \mathbf{f}_{\mathrm{s,int}} - \mathbf{f}_{\mathrm{s,ext}} + \lambda \mathbf{F}_{\mathrm{r}} \quad (5)$$

$$\mathbf{r}_{\mathrm{m}} = \mathbf{G} - \mathbf{f}_{\mathrm{m,res}} + \lambda \mathbf{G}_{\mathrm{r}} \quad (6)$$

$\mathbf{f}_{\mathrm{s,int}}$ and $\mathbf{f}_{\mathrm{s,ext}}$ are the standard vectors of nodal spatial internal and external forces. $\mathbf{f}_{\mathrm{m,res}}$ is the material resistance, which is a function of the surface energy and the crack front orientation. λ is the load factor. $\mathbf{F}_{\mathrm{r}}$ and $\mathbf{G}_{\mathrm{r}}$ are additional terms that are included in the equilibrium equations to account for the internal stresses

$$\mathbf{F}_{\mathrm{r}} = \int_{\beta_t} \mathbf{B}^{\mathrm{T}} \mathbf{P}^{\mathrm{MWL,radiation}} dV \quad (7)$$

$$\mathbf{G}_{\mathrm{r}} = \int_{\beta_t} \mathbf{B}^{\mathrm{T}} \mathbf{F}^{\mathrm{T}} \mathbf{P}^{\mathrm{MWL,radiation}} dV \quad (8)$$

$$(9)$$

where $\mathbf{F}$ is gradient of deformation, defined with other quantities on Fig. 1.

4 NUMERICAL EXAMPLE

An example of a keyway root crack in a nuclear graphite brick, driven by internal stresses, is shown in the figure. The stress state is the result of operating at full power of 30 years. The crack is initiated at one end of the brick at a keyway root. The brick is only restrained to remove rigid body motion. The crack front advances simultaneously inwards to the free surface of the bore and along the length of the brick. Young's modulus, $E = 9600$ MPa, Poisson's ratio, $v = 0.2$, and fracture energy, $\gamma = 145$ J/m^2. The analysis was undertaken using MoFEM [9].

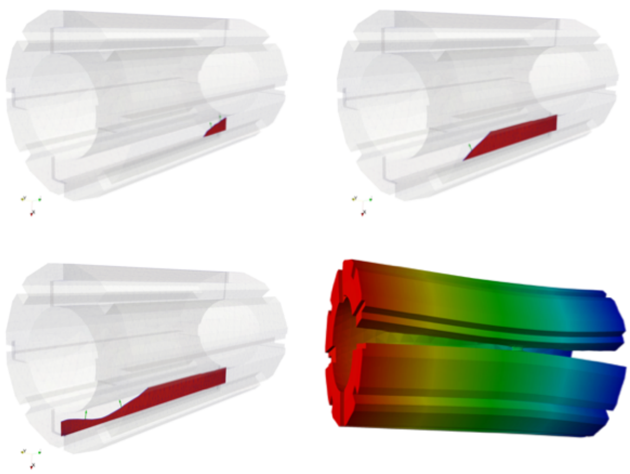

Figure 2: Progression of keyway root crack driven by internal stresses.

5 CONCLUSIONS

A novel formulation for brittle fracture in elastic solids within the context of configurational mechanics has been presented for the prediction of crack paths in nuclear graphite. The previous formulation [6, 8] has been extended to account for internal stresses as a driver for crack propagation. The formulation has been tested on single graphite brick subjected to internal stresses.

ACKNOWLEDGEMENTS

This work was supported by EDF Energy Nuclear Generation Ltd and The Royal Academy of Engineers. The views expressed in this paper are those of the authors and not necessarily those of EDF Energy Nuclear Generation Ltd.

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