CRITICAL OVERVIEW AND NEW CONFORM MATCHING LAW TO ASSESS PERMEABILITY OF CONCRETE AS A FUNCTION OF STRAIN ACCOUNTING FOR UNLOADING PATH

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Abstract: This contribution aims at improving the finite element modeling of concrete’s damage-permeability coupling using continuous and elastic-based damage approaches. Accordingly, a review of existing laws is achieved showing how they lack physical representativeness in terms of (a) permeability evolution from a microcracked (homogeneous) state towards a macrocracked (discrete) one and (b) permeability evolution considering mechanical loading-unloading paths. To resolve those issues, a new strain-based (rather than damage based) permeability law is suggested. The validation of this new law is achieved through two applications: (a) at the specimen scale and based on analytical developments where the permeability of the BIZEDE tensile test is considered for one loading-unloading cycle (b) at the structural scale and using FE analysis considering the permeability of a cracked VeRCoRs RSV (Representative Structural Volume of a 1:3 scale Containment Building) under pressurization-depressurization loads. The obtained results clearly demonstrate the enhancement of the permeability evaluation in concrete structures which should lead to a better assessment of the structural durability and serviceability.

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1 INTRODUCTION

The durability of concrete structures in general and the tightness of some of them such as the confinement vessels of nuclear power plants are completely driven by the transfer properties of concrete. The migration of a fluid in a porous media, possibly cracked, may happen in the connected porous network itself (homogeneous state) and/or along the cracks (discontinuous state). This conditions the durability of concrete structures and their serviceability.

The continuous part of permeability (denoted $k_D$) may be handled thanks to one of the numerous empirical models fitted on experimental campaign [1]. Since they are only valid for small damage level, they are usually in the form of exponential or power damage-dependent laws. Thus, if they are used in a matching law for higher damage corresponding to the flow localization in cracks, their increase must be mathematically limited using a development truncated to the few first polynomial terms. Besides, only the experimental results in [1] propose an analysis of permeability in both loaded and unloaded states. Unfortunately, the work in [1] does not propose a model which accounts for the decrease of permeability upon unloading, for instance when cracks are closed upon the prestressing of the material in confinement vessels.

As for the discontinuous part of permeability through localized cracks (denoted $k_F$), one should note that the damage variable $d$ (by definition bounded to a fixed value, in general 1) can no longer be used to define a permeability model where the flow is dependent on a crack opening which is physically unbounded. So, it is more accurate to consider a strain-dependent or crack opening dependent permeability instead [2]; usually, the Poiseuille’s law is used.

One can notice that both flow modes have separate validity domains, the first (continuous) for small damage values and the second (discontinuous) for highly localized strains. However, for the sake of structural modelling, a single permeability law is required through a so-called matching law [3]:

\[ k_{eq} = f(k_D, k_F) \]  

$k_{eq}$ is the equivalent permeability, $f$ the function which tends to $k_D$ and $k_F$ when $d$ tends to zero and one respectively.

However, being damage-dependent, in terms of formulation and weighing, this kind of matching laws show several drawbacks:

- They do not allow a proper simulation of permeability during the unloading path. Indeed, as the damage (thermodynamically irreversible quantity) tends to 1, the equivalent permeability $k_{eq}$ tends (irreversibly) to $k_F$. As the crack is closed upon unloading or under compressive loads, such permeability $k_{eq} = k_F$ tends to zero. Whereas in reality, $k_{eq}$ should tend towards a residual permeability $k_D^{res}$ higher than $k_0$ to account for the partial closure of cracks.

- They do not ensure a physical evolution of permeability which should be strictly monotonously increasing with respect to an increasing applied strain.

Eventually, in this contribution, our aim is to define a new permeability law which overcomes the previous limitations and allow a more physical numerical description of concrete’s permeability under loading-unloading cycles from the continuous state towards the localized one. To do so, this article is structured following three main parts:

- Part 1 recalls the main hypotheses behind damage-permeability coupling and presents a comparative analysis of existing laws and demonstrates their limitations.
- Part 2 introduces a new strain-based permeability law to alleviate those drawbacks.
- Part 3 explores two applications for
validation purposes: one at the specimen scale using analytical developments and the other at the structural scale using FE analysis.

2 DAMAGE-PERMEABILITY COUPLING: CRITICAL REVIEW

Considering a staggered framework, the damage-permeability coupling consists of solving a given mechanical problem, which provides damage and crack opening outcomes, before resolving a flow equation using a resultant permeability.

It is worth mentioning that, for any permeability law, the used damage model plays a major role since it allows the quantification of damage and strain localization used to define the permeability. In this work, the validation of damage models will not be tackled (one is referred to the appendix in [4]). For the sake of illustration and to achieve permeability law analysis and validation, a local unilateral energy-regularized damage model is considered as a reference model. One should note the importance of the unilateral effect (or the distinction of compressive and tensile damage variables) to allow cracks’ closure under loading-unloading cycles.

2.1 Damage model

For an isotropic damage formulation, the general behaviour law writes:

$$\sigma = (1 - d)\mathbf{C} : \varepsilon$$  \hspace{1cm} (2)

with $\sigma, \mathbf{C}, \varepsilon$ the stress, the initial stiffness and strain tensors respectively.

The damage variable $d$ increases from 0 to 1 according to a given loading function (1D case in eq. 3 and see [5] for more details) and a given damage criterion (strain based criterion in eq. 4 and see [6] for more details).

$$d = \begin{cases} \frac{\varepsilon - \varepsilon_{eq}}{\varepsilon_{eq}} & \text{if } \varepsilon \leq \varepsilon_{do} \\ \varepsilon_{eq} & \text{if } \varepsilon \geq \varepsilon_{do} \\ 0 & \text{if } \varepsilon \leq \varepsilon_{do} \end{cases}$$  \hspace{1cm} (3)

$$\varepsilon_{eq} = \frac{\varepsilon_{eq}}{\varepsilon_{eq}} = \frac{1}{2} \left( \frac{\varepsilon_{eq}}{\varepsilon_{eq}} + \sqrt{\left( \frac{\varepsilon_{eq}}{\varepsilon_{eq}} \right)^2 + 4\varepsilon_{do}^{2}} \right)$$  \hspace{1cm} (4)

$\varepsilon_{do} = \frac{B_1}{E}$  \hspace{1cm} (5)

$\mathbf{C}$ is a norm to define the maximal value of $X$ over time, $\varepsilon_{eq}$ and $I_X$ are the two first invariants of a tensor $X$, $\varepsilon_{eq}$ is the strain damage threshold, $\varepsilon_{eq}$ is the equivalent strain, $B_1$ is the softening parameter defined considering energetic-based regularization criterion [7] (depending on the tensile strength $R_t$, the Young’s modulus $E$, the fracture energy $G_f$), $\nu$ is the Poisson ratio.

As for the crack opening evaluation, and given the continuous nature of the used damage model, a post-processing phase of the strain and damage fields is required [8]. The crack opening then writes (accounting for elastic unloading):

$$w_{ck} = dD_\varepsilon < \varepsilon_1 >_+$$  \hspace{1cm} (5)

$D_\varepsilon$ is the characteristic size over which energy dissipation occurs (size of the Fracture Process Zone), $< x >_+ = \max(x; 0)$ and $\varepsilon_1$ the maximal principal strain.

2.2 damage-based permeability laws

As mentioned in the introduction, two distinctive permeability modes are usually defined and a matching law between the two is of numerical interest:

- For the continuous one $k_D$, several damage-dependent models do exist [1][9-10] (eq. 6-8). They all require empirical fitting based on a damage analysis of a specimen under loading. Being dependent on the experimental protocol (damage under compressive loads, tensile test, ...) and the way damage is evaluated (empirical global loss of rigidity, numerical inverse analysis, ...), such fittings might lead to different results even if the same concrete design mix is used. Indeed, it is worth mentioning that, experimentally, the definition of damage based on experimental results remains – at best – in a mean sense (not a local one) and still requires the definition of a damage model which eventually affects the permeability estimation. For instance, models in [1] and [9] have been developed based on a compressive test at low damage values. However, for the same concrete design mix, considerably lower
permeability values have been measured in [1] compared to [9] given that the performed analysis covered the whole specimen in [1] instead of the central part where damage is concentrated in [9]. In that sense, results from [9] are more reliable since they have more local sense than the ones in [1]. One should note that, for both studies: (a) the equivalence between compressive and tensile damages (low values) has been considered and (b) the use of a polynomial development limited to the first terms (eq. 7) is favoured compared to an exponential one (eq. 6) to prevent an overestimation of permeability at high damage values. Along the same line, the model suggested in eq. 8 by [10] is questionable especially given its supposed validity domain for damage values up to 80%. Indeed, for such high values, it is more accurate to consider a localized permeability mode rather than a continuous one; or better yet an intermediate state which should be provided by the matching law.

\[ K_D = k_D e^{(\alpha d)\beta} \quad 0 \leq d \leq 0.15 \]  
\[ k_D = \begin{cases} k_0 \left(1 + \sum_{n=1}^{\infty} \frac{(ad)^n}{n!}\right)(1 + (\alpha' d)\beta'); & \text{if loaded} \\ k_0 \left(1 + \sum_{n=1}^{\infty} \frac{(ad)^n}{n!}\right); & \text{if unloaded} \end{cases} \quad 0 \leq d \leq 0.15 \]  
\[ k_D = k_0 10^{\alpha d} \quad 0 \leq d \leq 0.8 \]  
(\(\alpha, \beta, \alpha', \beta'\)) are fitting parameters identified using an empirical permeability test under loading.

Another issue raised by the experimental work in [1] and [11], it is related to the existence of irreversible \(k_D^i\) and reversible \(k_D^r\) parts of the continuous damage-induced permeability (eq. 9); without defining any path-dependent model to theorize such observations.

\[ K_D^i = k_0 \left(1 + \sum_{n=1}^{\infty} \frac{(ad)^n}{n!}\right) \]  
\[ K_D^r = k_0 \left(1 + \sum_{n=1}^{\infty} \frac{(ad)^n}{n!}\right)(1 + (\alpha' d)\beta') - k_D^i \quad 0 \leq d \leq 0.15 \]

At low damage values, such irreversibility is due to the partial closure of the microcracks’ network [1]. And at high damage values [11], the irreversible part of permeability might be linked to the additional residual opening of the macrocrack. This partial crack closure at all scales is also the source of permanent strain when the specimen is load free.

![Figure 1: Comparison of low-damage-permeability laws in eq. 6-8 for (a) \(0 \leq d \leq 1\) (b) \(d \leq 0.2\)](image-url)

So, based on such conceptual critical analysis (Fig. 1), the model suggested in eq. 9 is considered for further developments with the experimental results from [9] (\(\alpha = \alpha' = 11.3, \beta = \beta' = 1.6\)).

For the discontinuous term \(k_F\), Poiseuille’s law is used to describe permeability through a macrocracked domain with an opening \(w_{ck}\). However, given the imperfect state (in terms of roughness, shape irregularity, local bridging, in-depth tortuosity, ...) of the crack lips, and compared to the Poiseuille’s hypothesis where the flow is created between two infinitely long parallel plates with a constant pressure gradient and no slip, a correction factor \(0 \leq \xi \leq 1\) is required not to overestimate the flow through cracked concrete [12-13].
Based on the experimental work in [13], where a split test is considered and the air flow through an increasing residual crack opening up to 200 $\mu$m is measured, the coefficient $\xi$ varies depending on the concrete type and formulation (Ordinary: 3%, High performance: 10%, Fiber reinforced: 1%) and is not dependent on the crack opening values. Though, the obtained results in [13] show good accuracy of the model in eq. 10, a constant $\xi$ remains a strong hypothesis which is unsupported by physical and theoretical developments [14]. Indeed, such roughness and shape effects are expected to be less influential as the crack opening increases. So, ideally, $\xi$ is increasing with $w_{ck}$. From that point of view, similar experimental work in [12] – based on water flow measurements during active split test loading (opening values up to 160 $\mu$m) – led to the definition of a physically representative correction factor.

\[
\xi = \min\left(\alpha_w w_{ck}^{\beta_w}; 1\right)
\]

with $(\alpha_w, \beta_w) = (3726.71 \text{ m}^{-1}, 1.19)$

However, in terms of amplitude (Fig. 2), and for $w_{ck} \geq 20 \mu$m, the identified values of $\xi$ are considerably higher than 3% shown in [13]. Amongst the reasons of such non negligible difference, there is of course the experimental protocol (boundary conditions, flow measuring device, …), fluid nature (air vs. water) and the type concrete; and mostly, the loading configuration (residual vs. active) during which permeability is measured. Indeed, under active loads, and given the same crack opening in the center, one expects the volume of the opened crack to be higher than in the residual case leading to higher flow and more important $\xi$ factor at identification. There are also the unresolved issues of the representativeness of the surface crack opening values of the opening through the concrete thickness.

Not to be restrained by the complexity of the experimental process, and based only on a physical representativeness criterion, the model in eq. 11 will be considered from this point on.

![Figure 2: Effect of parameter $\xi$ on Poiseuille's permeability](image)

- Matching laws between the continuous $k_D$ and localized $k_F$ states are of crucial interest for structural damage modelling. Indeed, from a mechanical point of view, concrete evolves from a non-damageable state towards a low damage one before strain localization. Accordingly, the permeability needs to be described continuously from one state to the other. In the absence of experimental investigation of permeability at intermediate states, the matching can be achieved differently [3] (linear function in eq. 12, weighted linear function in eq. 13, log-type weighted function in eq. 14 – Fig. 3).

\[
k_{eq} = k_D + k_F
\] (12)

\[
k_{eq} = (1 - d)k_D + d k_F
\] (13)

\[
k_{eq} = k_D^{1-d} k_F^d
\] (14)
For the linear function, and in the absence of damage weights, a limitation of the continuous part \( k_D \) is required \( (k_D^{\text{max}} = k_D(d = 0.15)) \); otherwise, at high damage values, the permeability is largely overestimated. Accordingly, damage pondered functions were considered to allow the correct limits \( \left( \begin{array}{l} k_{eq} \rightarrow_{d<1} k_D \\ k_{eq} \rightarrow_{d>0} k_F \end{array} \right) \). One can notice that the log-type matching allows a smoother transition from one mode to the other compared to the linear one. For all those laws, the monotony of the equivalent permeability is not ensured, especially when the reduction factor is applied. The main reason is due to the overestimation of the weight of continuous part at intermediate damage states (between 0.15 and 0.9). So, the damage variable is not suited to describe permeability increase upon loading, at least not in an absolute way. Another issue, raised here above, is the non-conformity of those matching laws upon unloading cycles. A part of the problem is due to the absence of a loading path associated with the permeability \( k_D \) but also to the fact the defined weights are dependent on irreversible damage quantity. As a result, the residual permeability when the crack is closed is underestimated since it tends to zero (if \( d=1, \ w_{ck} \rightarrow 0 \) then \( k_{eq} = k_F \rightarrow 0 \)).

So, the matching laws need to be revisited especially in terms of weight distributions to allow permeability reversibility and monotony with the induced loading. For that, and given its smoothness, the log-type matching will be considered hereafter for further developments.

3 NEW MATCHING LAW

To sum up, the foreseen law should:

- Ensure a monotonously increasing permeability with loading
- Distinguish reversible and irreversible parts of permeability
- Account for loading-unloading cycles

To that aim, the model in eq. 9-14 is revisited.

3.1 Revisiting the continuous damage-permeability law

To allow reversibility, the continuous permeability \( k_D^c \) in eq. 9 cannot be damage-dependent. Instead, a new quantity is introduced which represents a sort of “reversible” damage quantity \( d^r \). \( d^r \) is post-processed directly from the strain field and writes (using the definitions in eq. 3-4):

\[
\tilde{\epsilon}_{eq} = \frac{1}{2} \left( \frac{1}{1-2\nu} + \frac{\sqrt{\pi}}{1+\nu} \right)
\]

which leads to the new formulation of \( K_D^c \) as:

\[
k_D^c = k_0 \left( 1 + \sum_{n=1}^{N} \frac{(s_d)^n}{n!} (\sigma d^n)^\beta \right)
\]

One should note that, in the absence of experimental results, no hysteresis is considered for the reversible permeability \( k_D^r \). As for the irreversible permeability \( k_D^i \), and
to prevent any overestimation at high damage values, it needs to be bounded according to a certain damage threshold \( d_{lim} \) from which the residual permeability remains constant. The new formulation of \( k_b^D \) (path-dependent) becomes:

\[
k_b^D = k_0 \left( 1 + \sum_{n=1}^{\infty} \left( \frac{d_n}{d_{lim}} \right)^n \right) \quad (17)
\]

\[
d^I = \frac{d_t}{d_{eq}} d + \left( 1 - \frac{d_t}{d_{eq}} \right) \min(d; d_{lim})
\]

As shown in Fig. 5, and compared to results in Fig. 3, the new strain-based permeability law is more accurate and physically acceptable for both the loading and unloading phases compared to the original logarithmic one. In particular:

- The permeability transition from Darcy’s to Poiseuille’s mode is successfully achieved with respect to the monotonously increasing permeability with damage.
- The permeability transition from the loaded to the residual (unloaded) configuration is properly defined using a reversible path with no hysteresis.
- At macrocrack’s closure, a non-null residual permeability is guaranteed to be fitted based on experimental observations.

4 CASE STUDIES FOR VALIDATION

Experimental results covering concrete’s permeability under loading-unloading cycles for low damage values (besides the work in [1]) are not numerous in the literature.

In this work, two applications are studied:

4.1 BIPEDE tensile test

The BIPEDE tensile test [11] consists of applying uniaxial tensile loads to a disc glued to steel plates. Tension is applied to the plates and is transmitted to the disc until cracks develop (Fig. 6a).

The disc is 40 mm thick and has a diameter of 110 mm. The plates are holed so as to allow fluid (water) transfer through the concrete disc (hole’s diameter is 55 mm). Test results are provided in terms of water flow through the concrete specimen \(q \text{m/s})\). The specimen’s equivalent permeability can be estimated using eq. 19 supposing that the stationary state is constantly achieved.

\[
k(m^2) = \frac{\Delta P}{\frac{\mu}{8 \Delta r} q}
\]

with \(\Delta P = 0.25 \text{ MPa}, e = 40 \text{ mm}, \mu = \mu_{\text{water}} = 10^{-3} \text{ Pa.s}, S = \pi \frac{a^2}{4} = 2375 \text{ mm}^2\).

It is worth mentioning that, experimentally; one or two cracks were obtained (not forcibly centred). Herein, one crack is supposed to develop and local strains at the point of localization are directly used to define – according to the softening law in eq. 3 – the resultant damage and crack opening values (explicit analytical scheme). For the numerical application, the considered concrete properties are the following: \(D_c = 6 \text{ cm}, R_t = 4.5 \text{ MPa}, G_f = 80 \text{ N/m} \) and \(E = 30 \text{ GPa}\). In Fig. 3b the adjustment of eq. 18 to experimental results is performed using the least squared method.

For this test, the roughness effect on the computed Poiseuille’s permeability is less pronounced compared to the one in [12]. As shown in Fig. 3, the drawback of damage-based approaches is more pronounced as the roughness and tortuosity effects increase. However, thanks to the new matching law, problems of monotonous increase of permeability are alleviated. As for the unloading phase, it is clear that the new strain-
based law leads to a more physically admissible result as a non-null residual permeability is obtained. One should note that the strong scattering of experimental data as concrete is fully unloaded does not allow a clear understanding of whether the loading and unloading permeability paths are superimposed or not. With that regards, an in-depth experimental investigation of permeability evolution of an unloaded concrete is still lacked. Nevertheless, our main interest is served since the suggested strain-based permeability law in eq. 18 allows appropriate fitting of realistic experimental results.

4.2 VeRCoRs Representative Structural Volume

One of the main roles of Nuclear Containment Buildings is to protect the environment from radioactive substances inside the reactor building. For double walled NCBs such as the VeRCoRs mock-up (an experimental 1:3 scale NCB [15]), the structural tightness is ensured by the inner wall made out of reinforced and prestressed concrete (Fig. 7a). To test the conformity of the air tightness, EDF (nuclear fleet operator in France) proceeds to periodic pressurization tests under controlled overpressure and temperature (Fig. 7b-Fig. 8) to verify that the air leakage rate through the building is less than the regulatory threshold. Due to ageing effects (drying, creep, prestressing losses, etc.), the structural tightness decreases over time leading to costly maintenance and repair operations. Accordingly, the main challenge for operators is to predict such evolution and its effect on the overall structural performance.

In this part, the VeRCoRs gusset (1st lift of the NCB in Fig. 7a) is considered for illustration. This part of the wall is of particular interest as its cracking risk at early age is quite high due to restrained thermal and chemical shrinkages. Those cracks (Fig. 7c), and as the operational phase goes on, lead to an increasing leakage rate. Indeed, because of drying shrinkage and concrete creep, the initially applied prestressing decreases over time; so early age cracks reopen more and more (even if the applied pressurization load is constant). Considering the gusset’s structural volume, simulations are performed at the Representative Structural Volume (RSV) scale considering several cracking scenarios (details about the results of the mechanical analysis can be found in [16] and are supposed valid and physically representative – Fig. 7c). The leakage rate is then computed using the original eq. 18 and the former eq. 14 permeability laws for a 6-year long operational phase.
The obtained results of such numerical analysis are presented in Fig. 8. With the new law, in Fig. 8b–c, the continuous part of permeability $k_D$ and the matching law weighing function $f^r$ are time-dependent; which is more realistic compared to the former law where both are constant and overestimated. One should note that the increase of $k_D$ in time (between 1 year and 6 year) is mainly due to the increase of the relative permeability due to drying. As for the evolution of $f^r$ over time, it is associated to the variation of the $d^r$ variable as the crack opening increases between 1 and 6 years from 50 $\mu m$ to 120 $\mu m$.

Finally, in terms of the computed leakage rate through cracks (Fig. 8d), one should note that both laws lead to the same maximal leakage rate. However, they differ in terms of the time evolution with the former law underestimating the resultant permeability. Indeed, with the damage variable as a weighting function, and as the crack reopens or closes, the residual permeability tends to zero whereas with the new law it tends to $k_D^{res} > 0$. But, as the load increases, both laws lead to the same results as both show $k_{eq} \to k_F$. Nevertheless, if the interest is directed towards the cumulative air flow over time, instead of the maximal leakage rate, the difference between the two laws becomes non-negligible and, clearly, the new law is more physically representative, especially for intermediate loading levels between the validity domains of $k_D$ and $k_F$. So, the new law enhances the reliability of permeability description in large concrete structures with a containment role, and, by extension, the decision making process based on a leakage rate criterion.
5 CONCLUSIONS

In this contribution, a new strain-based permeability log-type matching law is suggested to allow a continuous numerical modeling of concrete’s permeability from a diffuse towards a localized state. Compared to existing damage-based laws, and as demonstrated through the BIPEDE tensile test and the VeRCoRs test simulations, the new law allows better description of permeability evolution during the loading-unloading path – thanks to the distinction of reversible and irreversible contributions – and ensures a monotonously increasing permeability as the loading advances.

As convincing experimental works of permeability during the transition from the tensile towards the compressive domains are lacking, our future work shall investigate concrete hydraulic properties more thoroughly for more than one loading-unloading cycle in order to better characterize the residual permeability and its evolution under active and variable mechanical loadings.

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