

## FULL NORMALIZATION OF THE CYCLIC CREEP CURVE OF STEEL-FIBER REINFORCED CONCRETE

E. POVEDA<sup>\*†</sup>, S. BLASÓN<sup>1††</sup>, G. RUIZ<sup>1†</sup>, H. CIFUENTES<sup>†††</sup>, A. FERNÁNDEZ CANTELI<sup>2††</sup>

<sup>†</sup> University of Castilla-La Mancha  
Ciudad Real, Spain  
e-mail: \*Elisa.Poveda@uclm.es, <sup>1</sup>Gonzalo.Ruiz@uclm.es

<sup>††</sup> University of Oviedo  
Gijón, Spain  
e-mail: <sup>1</sup>BlasonSergio@uniovi.es, <sup>2</sup>afc@uniovi.es

<sup>†††</sup> University of Sevilla  
Sevilla, Spain  
e-mail: Bulte@us.es

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**Abstract:** The cyclic creep curve represents the evolution of the maximum strain,  $\varepsilon$ , versus the number of cycles to failure,  $N$ , and it can be used to predict fatigue failure and its scatter. In this kind of curves, two random phenomena are observed. The first one is related to the evolution of  $\varepsilon$  as the number of cycles increases. It represents the damage progress identified as fatigue creep. Meanwhile, the second phenomenon is referred to the scatter of  $N$ , commonly used in fatigue life assessment. From this curve and the phenomena involved, this work proposes a new probabilistic failure criterion for strain-based fatigue in plain and steel-fiber reinforced concrete. It is based on a double normalization, from which these curves fit accurately to a Weibull distribution function of minima. The first normalization is referred to  $N$ , meanwhile the second one is related to  $\varepsilon$ . The proposed methodology allows the entire cyclic creep curve and the definition of the ultimate limit state to be predicted. It is validated with a series of low-cycle fatigue tests of concrete mixes with different amounts of fiber sharing the same concrete matrix.

### 1 INTRODUCTION

Fiber-reinforced concrete (FRC) is a composite material in which fibers are randomly distributed in a concrete matrix. These fibers control the growth and propagation of cracks by the *stitching effect*, improving its tensile and fracture properties. Nevertheless, in compression no consensus is found in the bibliography on the benefits of the fiber presence and its volume fraction [1-3], especially as far as fatigue is concerned. Moreover, the existing bibliography is scarce, involving a limited variety of mix compositions [4,5] or a very

limited number of tests [6-7]. It is important to emphasize that the scatter in the number of cycles to failure increases in case of concrete fatigue [4,8-10] and fibers further increase this scatter [5,9]. For that reason, a considerable number of tests [11] is required to face satisfactorily this topic from a statistical point of view.

Different criteria have been developed to predict fatigue failure in concrete. The most precise are those based on strain criteria, in particular, the so-called cyclic creep curve criterion [12]. The cyclic creep curve repre-

sents the evolution of the maximum strain,  $\varepsilon$ , as a function of the number of cycles to failure,  $N$ , exhibiting three different phases that can be easily identified. Specifically, the intermediate part or secondary branch is characterized by an almost linear increase of the strain with respect to the number of cycles. Its slope is referred to as the secondary strain rate per cycle [8,9,12]. This parameter permits fatigue life to be predicted, due to a remarkable relationship between the logarithm of the secondary strain rate and the logarithm of the number of cycles to failure [5,8,12]. Although this relation is well known, the strain failure criteria for concrete are really scarce. In this respect, the models of Holmen [13] and Huang *et al.* [14] are highlighted, both based on single normalization of the number of cycles, besides the strain failure criterion developed by Poveda *et al.* [9] where failure is considered when the strain reaches a critical value. Recently, Blason *et al.* [15] fitted the entire experimental cyclic creep curves through a triparametric Weibull distribution, which leads to more precise criteria concerning strain failure. This paper summarizes that former investigation.

The article is structured in the following manner: Section 2 presents the failure criterion for FRC. Section 3 presents the validation of the model and a brief description of the concretes mixes. Finally, in section 4 the present research and the most relevant conclusions are summarized.

## 2 TWOFOLD NORMALIZATION

In this section, a new fatigue failure criterion is presented based on the evolution evidenced by the cyclic creep curves for FRC. This criterion assigns a probabilistic character to the deformation process due to accumulated micro-fractures of the “primary elements” in the concrete matrix, hitherto unknown.

Although all the specimens were shape similar and were tested under the same conditions, their cyclic creep curves evidence two kinds of scatter owing to two different random phenomena acting during the damage

process: that related to lifetime [8,10,11], and that related to strain evolution, identified as fatigue creep. As a result, a double normalization of such curves referring to the final number of cycles and strain, respectively, is proposed in order to ensure a reliable lifetime prediction.

The 0 to 1 range normalization with respect to the final number of cycles at failure for the fatigue lives of each specimen, together with the strain growth sigmoidal shape, suggest that the evolution of the strain during fatigue tests can be described as a cumulative distribution function (CDF) of the generalized extreme value distributions for minima, as given by Eqn. (1).

$$F(x; \lambda, \delta, \kappa) = 1 - \exp \left\{ - \left[ 1 + \kappa \left( \frac{x - \lambda}{\delta} \right) \right]^{1/\kappa} \right\}; \quad (1)$$

$$1 + \kappa \left( \frac{x - \lambda}{\delta} \right) \geq 0, \kappa \neq 0$$

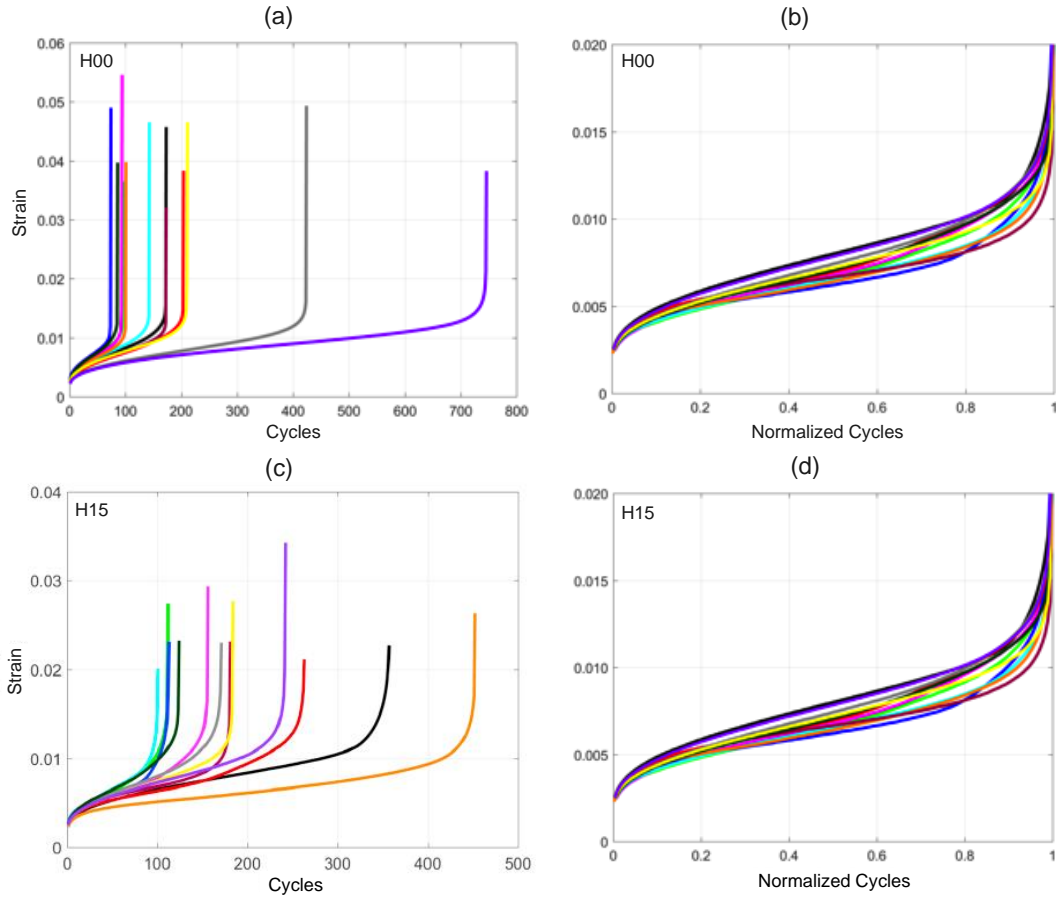
being  $\kappa$ ,  $\lambda$  and  $\delta$  the shape, scale and location parameters of the Weibull distribution function, respectively, that are fitted from the experimental results.

On the other hand, a second normalization is applied to the strain axis as a function of the normalized number of cycles to failure by taking  $\varepsilon^* = (\varepsilon - \lambda) / \delta$ .

Once the variability of the test data is reduced by using the twofold normalization proposed, all the strain curves practically coincide in a unique one. Consequently, the aforementioned procedure allows predicting the fatigue failure, as well as, reducing the duration of the fatigue test when a critical strain value is reached, without finishing the test.

## 3 VALIDATION

Poveda *et al.* [9] study the fatigue behavior of five different self-compacting steel-fiber reinforced concretes, SCSFRC. These concretes were designed and manufactured maintaining the same concrete matrix but only changing the fiber composition. The mixing proportions by weight were 1:2.71:0.55:1.47:0.62:0.018 (cement:limestone



**Figure 1:** a) Original cyclic creep curves for H00, b) normalization of H00 curves, c) original cyclic creep curves for H15 and its normalization curves in d).

sand: siliceous sand: coarse aggregate:water: superplasticizer). The hooked-end fibers are made of steel from ArcelorMittal (HE 55/35), with an aspect ratio of 64 and 35 mm length. The dosages of steel fibers vary from 0 to 45 kg/m<sup>3</sup>, being H00 the plain concrete or the base matrix and H15, H30, H45 and H60 the steel-fiber reinforced concretes with fiber contents of 15, 30, 45 and 60 kg/m<sup>3</sup> respectively. To exemplify the application of this methodology, only the results of H00 and H15 are presented in this work, although in [15] all the concretes are included.

Figures 1 a) and c) show the original experimental curves of the H00 and H15 examples as recorded in [9, 15] under compressive fatigue. Specifically, the strain vs the number of cycles during the fatigue tests in cubic specimens for H00 and H15 at the same stress level (87% of the compressive strength and a load rate of 0.27) are showed. Although all the tests were carried out under the same

conditions, these curves present a high scatter. The high deviation between Figs. 1 a) and c) gives us an idea about the probabilistic character of the phenomenon to be considered in the lifetime prediction based on the cyclic creep curves.

Figure 1 b) shows the  $\varepsilon-N$  curves for H00 after normalizing the number of cycles of each specimen from the original Fig. 1 a), meanwhile Fig. 1 d) represents the  $\varepsilon-N$  curves of H15 after normalizing the number of cycles for each specimen of Fig. 1 c). Further examples of the former concretes H30, H45 and H60 can be found in [15].

After the normalization, the cyclic creep curves can be fitted as cumulative distribution function pertaining to the generalized extreme value family, see Eqn. (1). The estimated location, scale and shape parameters, given in Table 1 for the H00 and H15 samples, evidence they pertain to Weibull distribution for minima.

**Table 1:** Fitting parameters for H00 and H15: shape,  $\kappa$ , scale,  $\lambda$  and location,  $\delta$  parameters of Eqn. (1) and correlation coefficient,  $R^2$ .

<b>H00</b>	$\lambda$	$\delta$	$\kappa$	$R^2$
1	0.00751	0.00210	0.40445	0.99205
2	0.00784	0.00247	0.42622	0.99106
3	0.00762	0.00242	0.42590	0.99009
4	0.00765	0.00235	0.42304	0.99530
5	0.00742	0.00210	0.39439	0.99353
6	0.00838	0.00257	0.40495	0.99460
7	0.00864	0.00265	0.40989	0.99868
8	0.00795	0.00221	0.37885	0.99720
9	0.00847	0.00274	0.41443	0.99825
10	0.00875	0.00271	0.39867	0.99511
11	0.00877	0.00267	0.38083	0.99739
12	0.00966	0.00284	0.35760	0.99527
<b>H15</b>	$\lambda$	$\delta$	$\kappa$	$R^2$
1	0.00742	0.00233	0.43658	0.99507
2	0.00768	0.00271	0.49682	0.99589
3	0.00726	0.00224	0.45397	0.96963
4	0.00815	0.00276	0.44921	0.99792
5	0.00796	0.00280	0.46835	0.99519
6	0.00851	0.00292	0.43974	0.99710
7	0.00738	0.00196	0.36017	0.98655
8	0.00803	0.00251	0.41242	0.99842
9	0.00884	0.00285	0.41383	0.99636
10	0.00829	0.00266	0.42797	0.98504
11	0.00898	0.00281	0.39501	0.99308
12	0.00737	0.00204	0.37532	0.97966

As an explanatory example, Figs. 2 a) and b) show the fitting procedure applied to two specimens of H00 and H15. As can be observed from the high correlation coefficients of Table 1, the experimental curves practically coincide with the analytical expression of Eqn. (1).

Finally, Figs. 2 c) and d) show the second normalization, this time carried out with respect to the strain by considering the normalized strain  $\varepsilon^* = (\varepsilon - \lambda) / \delta$ . Apparently, all the curves fit fairly well to a unique one while the scatter of Figs. 1 a) and c) is practically suppressed. Accordingly, it can be concluded that the proposed methodology allows the fatigue failure and the entire cyclic creep curve for any specimen to be reliably predicted. Furthermore, once the normalized

strain curve for the particular concrete is estimated, a premature interruption of the test can be envisaged while ensuring a good prospective estimation of the specimen lifetime. In this way, substantial time and cost saving in the fatigue tests can be achieved.

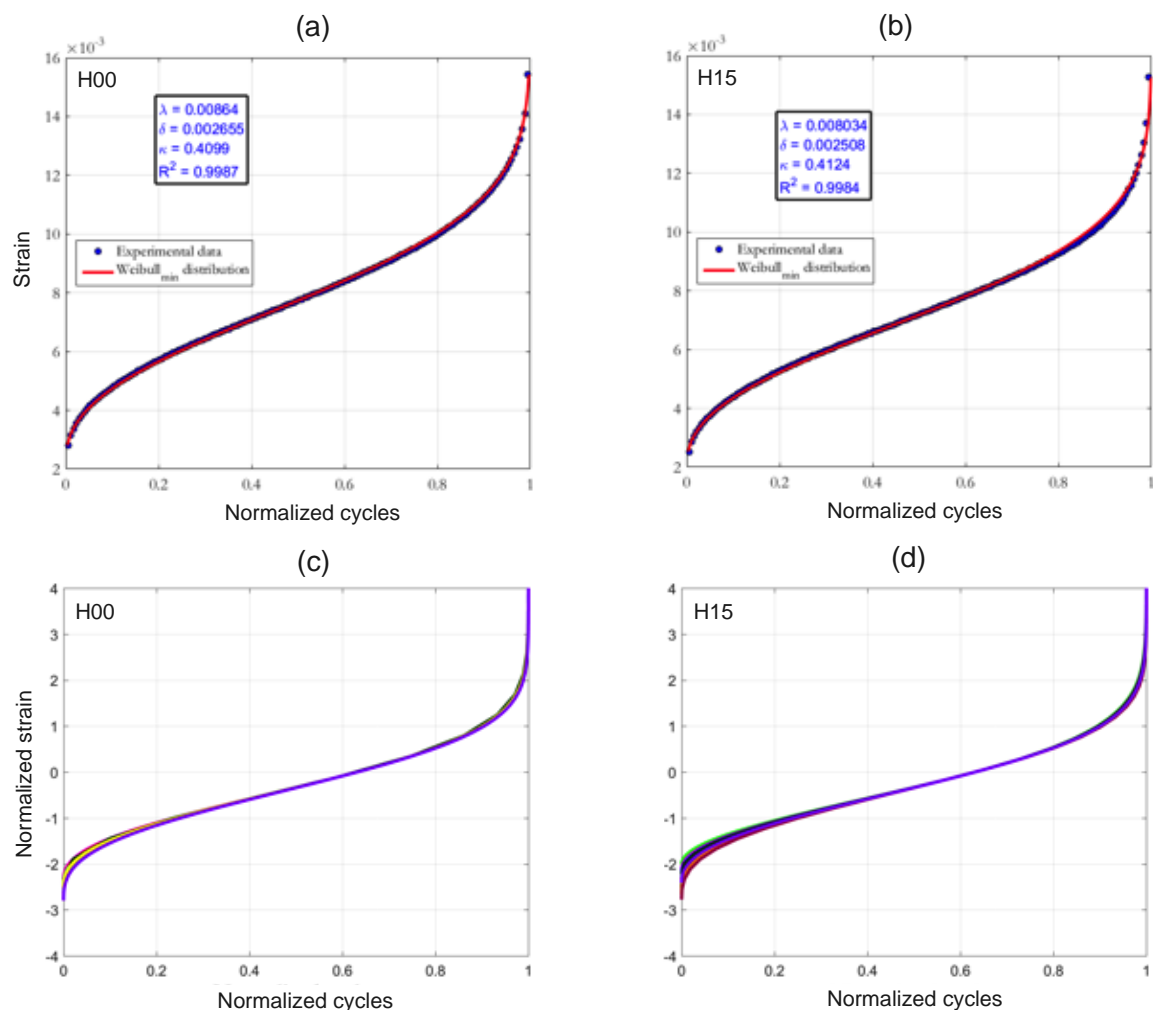
The well-known method of the secondary strain rate [5,8,9,12] developed by Sparks and Menzies [12] represents a partial derivation of the proposed methodology. In fact, unlike the first method, this proposal allows obtaining the entire cyclic creep curve with a satisfactory agreement, including both the initial and the final phases of the strain curve (being the last one particularly relevant). On the contrary, Sparks and Menzies' approach reduces all the information from the cyclic creep curve to a single point, represented by the secondary strain rate, so it implies a loss of information of the fatigue results.

#### 4 CONCLUSIONS

A new fatigue failure criterion is presented based on the cyclic creep curves for plain and steel-fiber reinforced concrete.

Two random phenomena owning to the fatigue damage process are identified. The first one provides lifetime scatter, i.e. of the total number of cycles to failure, while the second one induces variability of the strain shape evolution, identified as fatigue creep.

By applying twofold normalization to the strain curves, a unique strain-lifetime master curve can be derived. In this way, the entire cyclic creep curve for any specimen can be identified as a cumulative distribution of the generalized extreme value family, particularly of the Weibull type for minima allowing reliable prediction of the fatigue failure to be achieved.



**Figure 2:** a) Exemplary fit for a H00 specimen, b) exemplary fit for a H15 specimen, c) twofold normalization of all the cyclic creep curves for H00 and d) twofold normalization of all the cyclic creep curves for H15.

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