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MODELLING OF HIGH-CYCLE FATIGUE CRACK GROWTH IN CONCRETE

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Abstract: The paper presents an efficient approach for modelling high-cycle fatigue in concrete structures without the need to model each individual cycle. The three-dimensional fracture-plastic model has been extended to capture fatigue damage in tension. The material model is based on the classical S-N or Wöhler curve. The S-N criterion is translated into a material damage, which is introduced into the material model based on stress increments at each material point and the number of cycles. The fatigue damage evolution is integrated per number of cycles without the need to model each individual cycle. This approach allows to consider the additional crack growth that may result due to force redistribution during the fatigue damage evolution process. The current paper presents the improved version of the fatigue model published previously by the authors as well as its application to the modelling of fatigue crack growth in concrete structures strengthened by FRP laminates.

1 INTRODUCTION

The paper presents an efficient approach for modelling high-cycle fatigue in concrete structures without the need to model each individual cycle. Fatigue crack propagation in concrete is an important problem in structures subjected to cyclic loading. However, there are very few fatigue models available for use in conjunction with advanced concrete material models and nonlinear finite element analysis. The available models that are published in the literature (for instance [1]) usually deal with the low cycled fatigue, when it is necessary to perform the numerical nonlinear analysis for all investigated cycles. Such an approach is definitely not applicable, when it is necessary to consider thousands or millions of cycles. The available models for high cycle fatigue [2] are usually based on linear elastic fracture mechanics, and are usually not used for tracing the actual crack propagation. Such method cannot be easily extended for the finite element analysis using the smeared crack approach, which is by far the most used approach in commercial applications for practical analysis of concrete structures.

The three-dimensional fracture-plastic model [3] has been extended to capture fatigue damage in tension. The first version of the model has been presented in [4] and [5]. The current paper presents the improved version of the fatigue model as well as its application to the modelling of fatigue crack growth in concrete structures strengthened by FRP laminates.

The material model is based on the classical S-N or Wöhler curve. The S-N criterion is translated into a material damage, which is introduced into the material model based on stress increments at each material point and the number of cycles. The fatigue damage evolution is integrated per number of cycles without the need to model each individual cycle. This approach allows to consider the additional crack growth that may result due to force redistribution during the fatigue damage evolution process.

The fatigue strength is important factor in the design of infrastructures such as building, bridges, or motorways subjected to repeated loads for instance by high speed railways. Another typical structures, where fatigue is an important design criterion, are wind turbine support structures.

The fatigue model validation is demonstrated on an example of three point bending experiment from VUT Brno [6] and using an example of concrete strengthening.

Existing structures are often strengthened by various strengthening measures. One of the common solutions that widely used is the strengthening technique with FRP composite material. This is because of its performance of superior tensile strength, light weight, and ability to resist harsh environmental conditions. The characteristics of FRP composite materials have significant influence on sustainability of infrastructures as it provided low emission to air and water when it compared to traditional material such as steel. The laminates can be glued to the external surfaces of the strengthened structure. This process can be very fast with minimum disruption to its usage. Also the application of FRP laminates has a minimum affect on the structural dimensions.

Nonlinear simulation can be used to verify the strengthening effect of the FRP laminates. The presented fatigue model is used in this paper to simulate the fatigue strength of CFRP laminate bonded to concrete. The simulation is compared with experimental results performed at Swinburne University [8]. A finite element model (FEM) was developed to verify the experimental result and obtain further insight of fatigue damage behavior and related parameters. The calibrated model showed successful correlation with the experimental result in terms of the failure number of cycles and mode of failure.

2 MATERIAL MODEL FOR FATIGUE

The material model for the crack fatigue growth extends the fracture-plastic material model [3]. This models assumes for tensile fracture the total strain ε decomposition into the elastic ε_e and fracturing ε_f part.

$$\varepsilon = \varepsilon_e + \varepsilon_f \tag{1}$$

In the fracture-plastic material model, the crack growth is controlled by the internal parameter $\varepsilon^f_{\max}$, which represents the maximal fracturing strain reached during the cracking process at each crack.

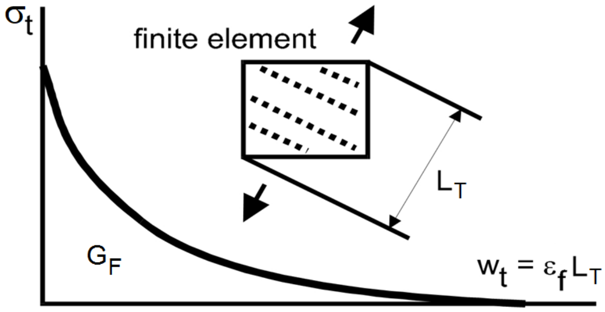

Figure 1: Softening law in tension.

This means that the current tensile strength at any time during the fracturing process can determined using the tensile softening law (Figure 1) as:

$$\sigma_t = f_t(w_{\max}),\ w_{\max} = \varepsilon^f_{\max}\ L_t \tag{2}$$

Where σ_t represents the current tensile strength, $w_{\max}$ is the maximal crack opening w_t reached during the loading process and L_t is the crack band size, which is based on the finite element size as shown in Figure 1. For the above softening law it is possible to define the following inverse relationship:

$$w_{\max} = f_t^{-1}(\sigma) \tag{3}$$

The maximal crack opening displacement reached during the cracking process defines the amount of damage. Using the strain decomposition (1) it can be easily related to the classical damage parameter D by the following relationship:

$$D = \frac{E\,w_{\max}}{E\,w_{\max} + f_t(w_{\max})\,L_T} \tag{4}$$

Where E is the elastic modulus. For the purpose of transparency the proposed fatigue model uses the value of the maximal crack opening, i.e. $w_{\max}$ as the measure of damage.

The fatigue strength of concrete structures in practice is usually verified using the S-N (Wöhler) curve:

$$\sigma_u = \sigma_u(N) = f_t\left(1-\beta_{fat}\left(1-R\right)\log N\right) \quad (5)$$

or its inverse form:

$$N = N(\sigma_u) = 10^{\left(\left(1-\frac{\sigma_u}{f_t}\right)/\left(\beta_{fat}(1-R)\right)\right)} \quad (6)$$

Where σ_u is the upper stress level during the cycling loading, β_{fat} is the fatigue parameter defining a linear slope of the fatigue strength in the relation to the logarithms of the number of load cycles N. The parameter R defines the ratio between the base stress σ_b and the upper stress σ_u:

$$R = \sigma_b / \sigma_u \quad (7)$$

The fatigue crack growth modelled by increasing the maximal crack opening $w_{\max}$ by the fatigue contribution. The main model assumptions are summarized in the XXXXX. The figure shows a general situation when a material point with already existing crack, which is partially unloaded, i.e. point 1, is subjected to a cyclic loading between two stress levels σ_b, σ_u. During the cycles, the damage will gradually accumulate until the point 3 is reached when the failure occurs. It is asumed that this point is reached when the number of cycles corresponds to the Wöhler formula (5). If less cycles is applied at the given point the accumulated crack opening will remain between points 2 and 3. If more cycles is applied and the Wöhler limit (6) is exceeded. The softening regime is reached and it is assumed that the cracking progress should stop at the stress level given by the Wöhler law corresponding to the total number of applied cycles.

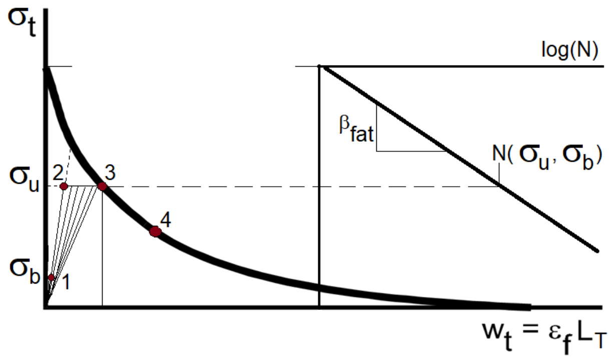

Figure 2: Assumptions of the proposed fatigue model.

The additional crack opening w_{fat} applied during the fatigue loading can be described as:

$$\begin{aligned} & n \le N(\sigma_u,\sigma_b) - N_{ini} \\ & \qquad w_{fat} = \frac{n}{N(\sigma_u,\sigma_b)} f_t^{-1}(\sigma_u) \\ & n > N(\sigma_u,\sigma_b) - N_{ini} \\ & \qquad w_{fat} = f_t^{-1}(\sigma_u(n)) \end{aligned} \quad (8)$$

This represents a general function $w_{fat}(n,\sigma_u,\sigma_b)$ of the number of applied cycles n, upper σ_u and lower σ_b stress levels. N_{ini} represents the initial damage at a given material point represented by the equivalent number of cycles at the given stress level.

$$N_{ini} = \frac{w_{\max}}{f_t^{-1}(\sigma_u)} N\left(\sigma_u,\sigma_b\right) \quad (9)$$

During the load cycles this additional fatigue contribution can be generally expressed as:

$$dw_{fat} = \frac{\partial w_{fat}}{\partial n} dn + \frac{\partial w_{fat}}{\partial \sigma_u} d\sigma_u + \frac{\partial w_{fat}}{\partial \sigma_b} d\sigma_b \quad (10)$$

The derivatives can be evaluated only for simple expressions of the softening law or the fatigue law. This means that numerical derivation is used. The increment of the maximal crack opening due to the fatigue loading is then integrated numerically for a given number of fatigue intervals $k = n/\Delta n$.

$$\Delta w_{fat} =$$

$$\sum_{i=1}^{k-1} \left\{ \frac{w_{fat}(\Delta n(i+1),\sigma_u^i,\sigma_b^i) - w_{fat}(i\,\Delta n,\sigma_u^i,\sigma_b^i)}{\Delta n} \Delta n + \right.$$

$$\frac{w_{fat}(i\,\Delta n,\sigma_u^i+\Delta\sigma_u^i,\sigma_b^i) - w_{fat}(i\,\Delta n,\sigma_u^i,\sigma_b^i)}{\Delta\sigma_u^i} \Delta\sigma_u^i +$$

$$\left.\frac{w_{fat}(i\,\Delta n,\sigma_u^i,\sigma_b^i+\Delta\sigma_b^i)-w_{fat}(i\,\Delta n,\sigma_u^i,\sigma_b^i)}{\Delta\sigma_b^i}\Delta\sigma_b^i\right\} \tag{11}$$

The fatigue algorithm involves the following steps in the nonlinear finite element analysis:

Algorithm 1:

(1) Mark the start of the fatigue increment, save $\boldsymbol{\sigma}_b$ at all material points.
(2) Perform fatigue loading up to the required upper load level. Then save $\boldsymbol{\sigma}_u$ at all material points.
(3) Calculate and gradually apply damage due to n cycles in k steps as shown in (11).

After the application of the damage in step (3) of the above algorithm, the stresses at each material point are checked with respect to the fracture-plastic material model. At this point additional damage and cracking may be introduced due to the violation of the material cracking criteria [3]. The proposed algorithm is sensitive to the selected steps size Δn, since due to the redistribution of internal forces during the fatigue loading, the levels of the upper σ_u and base stress σ_b will be always changing. The change of the upper stress level σ_u is taken into account in the step (3) of the above algorithm. To consider the gradual changes of the lower stress σ_b levels, it is recommended to unload the model to the base load levels during the fatigue loading process, and repeat the steps (1) and (2) in the above algorithm.

3 DIRECT TENSION EXAMPLE

This section presents one of the example validation problems used to verify the proposed model. This example is a three-point bending beam tested at VUT Brno [6]. The photo of the experimental setup is shown in Figure 3. The specimen dimensions were 400 mm (length) × 100 mm (height) × 100 mm (depth), supports distance 300 mm, notch depth 10 mm. The finite element model used to simulate the fatigue strength is shown in Figure 4. Two sets of specimens were tested made of C30/37 and C45/55 class concrete.

The static response of the experiments was first used to calibrate the main material parameters such as tensile strength, fracture energy, etc.

Figure 3: Experimental setup of the three-point bending experiments from VUT Brno [6].

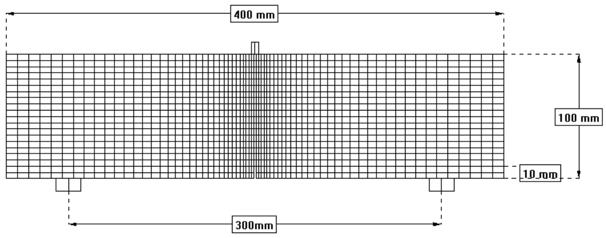

Figure 4: The finite element model for the validation three-point vending example.

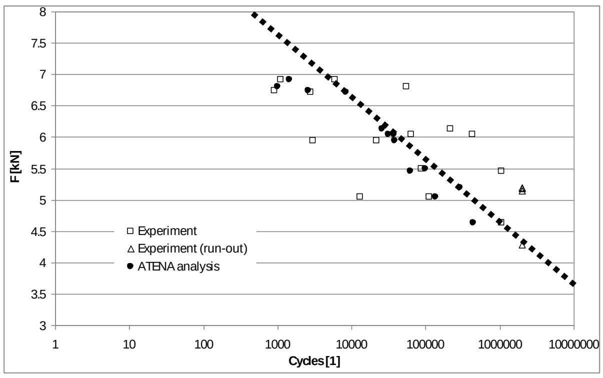

Figure 5: The comparison of the fatigue strength for the three-point bending validation example for C30/37.

The fatigue analysis was performed using the following approach:

1. loading up to the base load level (2 analysis steps)

2. storing the base stress $\boldsymbol{\sigma}_b$

3. increasing the load to the upper stress level $\boldsymbol{\sigma}_u$ (18 analysis steps)

4. calculating the fatigue damage due to 10 cycles based on the difference between the stored and curent stress and crack fields

5. introducing the damage into the material (10 analysis steps). The damage results in a displacement increase.

6. repeating steps 4 and 5 for increasing cycle counts until the model fails or 2 millions total cycles are reached. The 2 millions cycles were divided into 37 sets of increasing size (10, 10, 20, 30, 50, ..., 250 000, 300 000, 400 000, 400 000). In total, there were 380 analysis steps.

7. determining the number of cycles survived from the number of the last converged analysis step.

A parametric study was performed to determine the optimal values of β_{fat}. The optimal value was determined to be 0.08 for class 30/37 and 0.11 for class 45/55 concrete. These results are shown in Figure 5and Figure 6 for class 30/37 and 45/55 concretes, respectively. The samples that did not fail in tests before reaching 2 000 000 cycles ("run-out") are marked by empty triangles. The comparison shows that the numerical model very well captures the mean values of the cycles to failure, observed in the experiment.

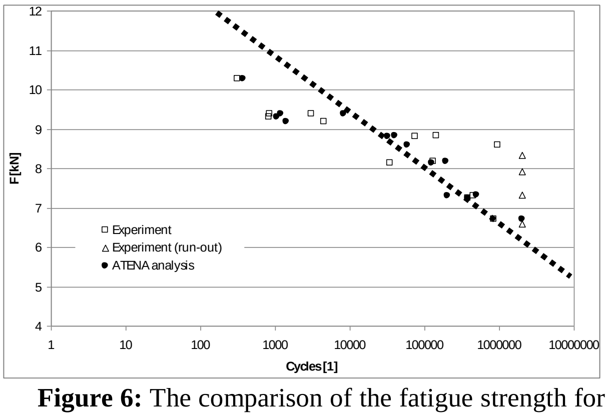

Figure 6: The comparison of the fatigue strength for the three-point vending validation example for C45/55.

4 FRP STRENGTHENING

FRP have become a popular material for the strengthening and rehabilitation of concrete bridges and other concrete structures. The material can be applied to structural elements as externally-bonded reinforcement to enhance their flexural, shear, axial and torsional strength.

The FRP material shows many advantages for strengthening purposed, but its weakest point is the bond between the FRP and the concrete. In order to prevent the occurrence of debonding failure, various techniques have been explored such as the concept of FRP anchorage systems, which are summarized by Kalfat et al [7].

Such an anchorage system is selected as the second validation and demonstration problem for the presented fatigue model. This system was developed and experimentally tested at Swinburne University in Australia [8].

Figure 7 shows the test specimen with the developed anchor system. The experimental work [8] contained concrete blocks with dimensions of 400×400×250mm as shown in Figure 8. The blocks are vertically placed into the testing machine as shown in Figure 9. The blocks are reinforced with four 12mm closed ties in each direction. The reinforcement has the spacing of of 120mm and a cover of 20mm. The two larger faces of the concrete specimen (400x400mm) were carefully sandblasted to achieve a good bond quality between concrete and FRP. The bi-directional fibre patch anchors were placed in two layers of bidirectional fabric sheets with dimensions of 400 x 300mm applied to the concrete surface, with the CFRP laminate sandwiched between each fabric layer. The first fabric layer was saturated with resin and applied to the concrete using a wet lay-up technique. The next step involved applying the FRP laminate over the first layer of bidirectional fabric sheet using laminate adhesive. Lastly, the second layer of bidirectional fabric sheet was applied over the top of the FRP laminate. This arrangement is replicated in the FEM model as shown in Figure 10.

Figure 11 shows the finite element mesh used in the numerical simulations. First the static tests were analyzed. In these tests the specimens are subjected to static uniformly increasing load up to failure to establish a reference strength and structural response of the anchored system. The load-displacement curves obtained by this investigation are shown and compared in Figure 12. The static tests are also used to determine and validate the material to be used in the numerical simulations. The fitted material parameters are listed in Table 1. The calculated strength of the static test is about 148kN. Very brittle failure was obtained similar to the experimental observations, where it was not possible to have a stable control of the load after the peak was reached. The observed failure mode was similar in the experiment as well as in the numerical simulation. Figure 13 to Figure 15 shows the evolution of the bond failure at various load levels. In the proposed numerical model the bond failure is modeled by crack propagation in the surface layer of concrete right below the first fabric layer.

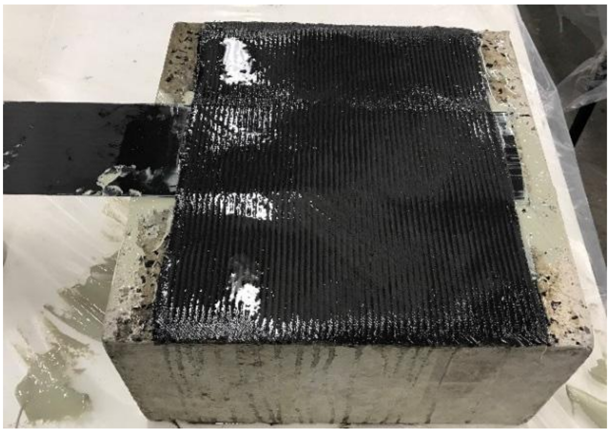

Figure 7: The view of the specimen preparation before the test.

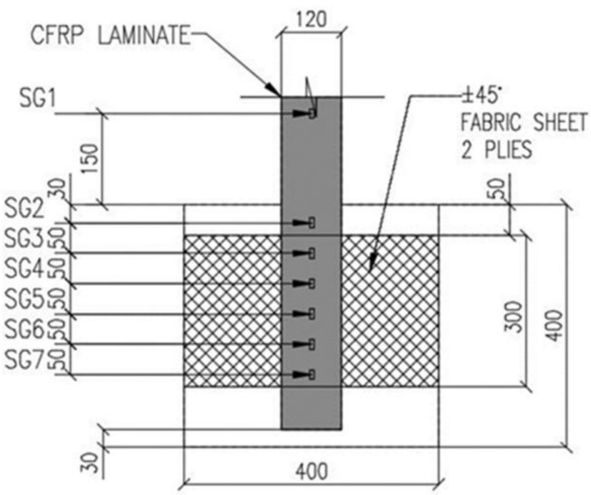

Figure 8: The geometry of the test setup.

Figure 9: The photo of the specimen in the testing machine.

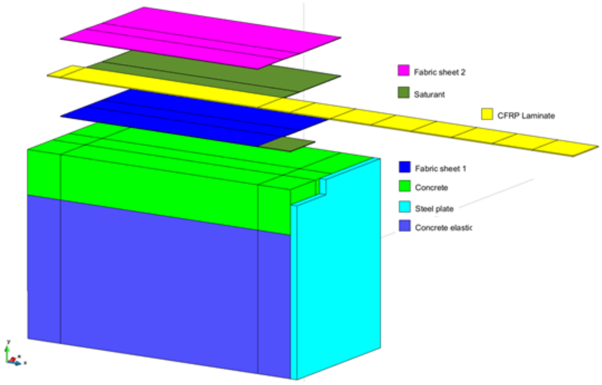

Figure 10: The view of the various elements and layers in the numerical model.

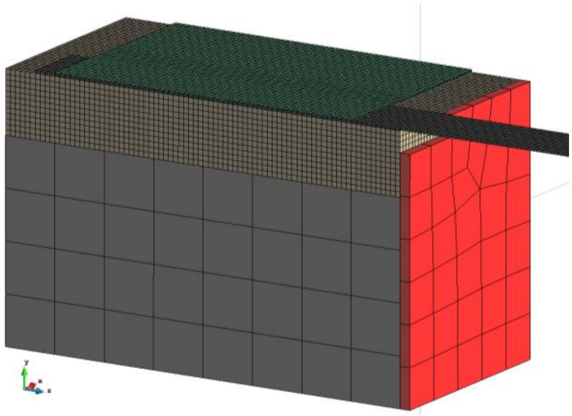

Figure 11: The finite element model for the test specimen with the anchored FRP laminate.

This assumption is in good agreement with the observed behavior, where the failure is observed in the surface concrete layer and not in the glue or fabric layers. The static tests are used to validate this assumption. This assumption enables to apply the proposed fatigue model, which is developed for fatigue tensile cracking of concrete, to the simulation of the fatigue strength of the FRP anchored strengthening systems. In this approach, the fatigue failure of the bonding mechanism is simulated by the fatigue cracking of the concrete top surface layer.

The fatigue simulation was performed using the fracture-plastic material model and the fatigue model described in Section 2. The fatigue analysis follows the process described in Section 2 in Algorithm 1. The model is first loaded up to the base level as indicated in Table 2. After that the upper stress level in all integration points is stored.

Table 1: Material parameters for FRP strengthening example.

Property	Concrete model	CFRP Lamin.	Satur.	Bi-dir. FRP
Comp. str. f_c [MPa]	31.6	-	-	-
Tensile str. f_t [MPa]	2	2906	68	3790
Elastic m. E [GPa]	33.3	181	>3.0	230
Poisson ν	0.2	0.42	-	-
Fract. engr. G_F [N/m]	80	-	-	-
β_{fat}	0.15	-	-	-

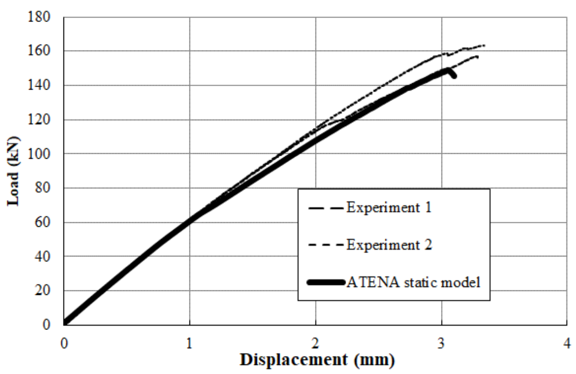

Figure 12: The comparison of load-displacement curves for Static análisis.

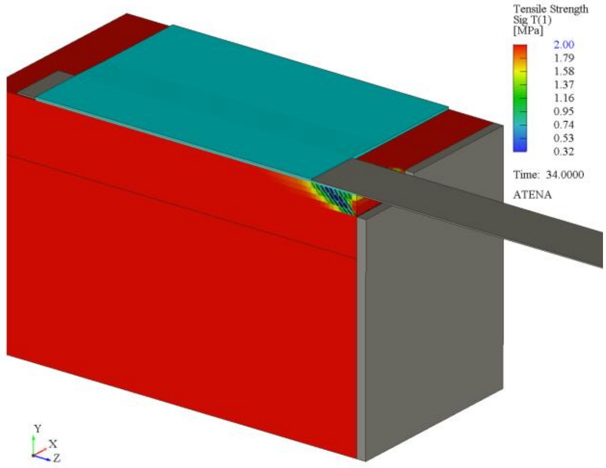

Figure 13: Static test at 51.5kN, cracks > 0.01 mm

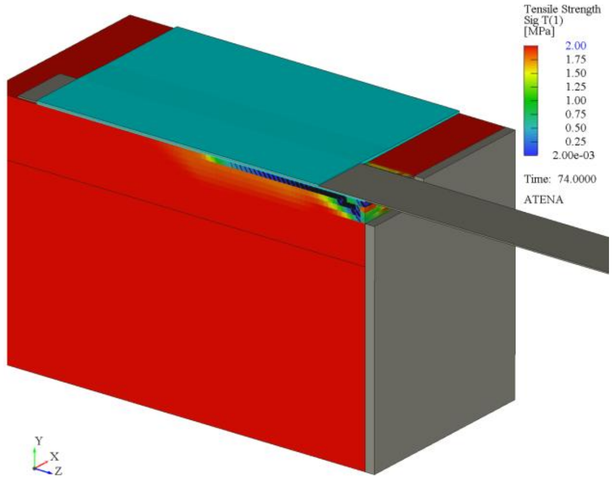

Figure 14: Static test at 100kN, cracks > 0.05 mm

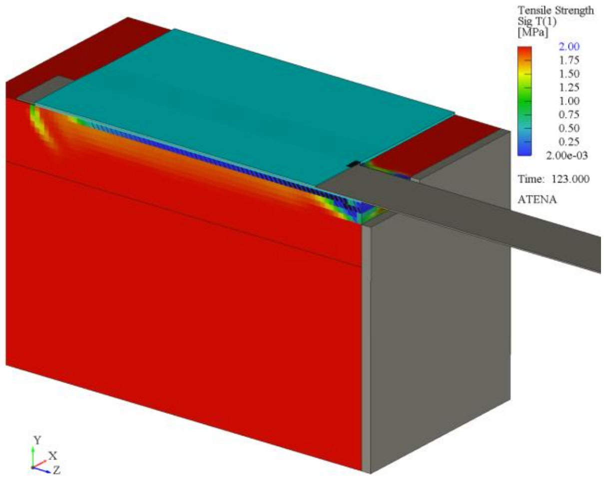

Figure 15: Static test at peak 148 kN, cracks > 0.05 mm

Then the fatigue damage is evaluated and applied gradually using the equation (11). The fatigue calculation and application follows in a number of predefined steps, which allows the force redistribution to take place and upper and bottom stress levels σ_b and σ_b are gradually updated.

Table 2: Summary of fatigue results for the FRP anchored example.

Test #	Stress Ratio [%]	Load range [kN]	Experiment		FEM
			no of cycles at failure (mil.)	no of cycles stopped no fail. (mil.)	β_{fat} 0.15 (mil.)
AF1	20-70	32-112	1.355		1.0
AF1-R			0.607		
AF2	20-60	32-96		5.6	7.6
AF2-R				4.0	

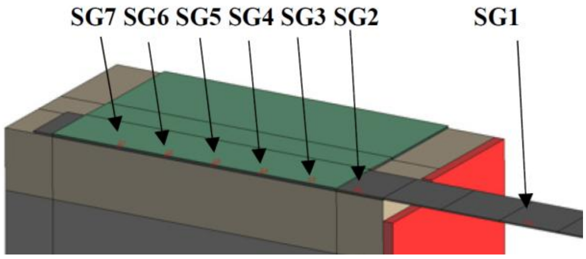

Figure 16: Location of strain sensors.

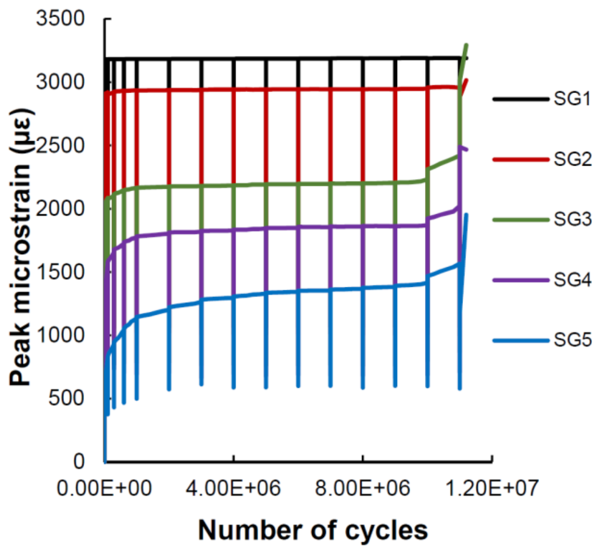

Figure 17: Strain evolution in the strain sensors during the fatigue loading.

The main results from the fatigue analysis are summarized in Table 2. The main parameter to compare is the number of cycles to failure. Overall four fatigue tests have been performed for two load level intervals: 20-70% of the failure load and 20-60%. For each load interval two tests have been performed.

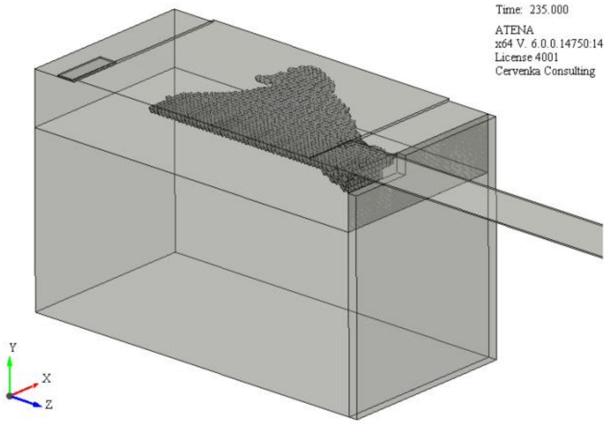

Figure 18: Crack pattern before failure in the fatigue test.

For the first interval 20-70% the fatigue failure was observed after 1.355 mil. load cycles and 0.607 cycles respectively. For the second load interval 20-60% no fatigue failure was observed, and the test was terminated after 5.6 and 4 million cycles. The fatigue failure loads obtained from the numerical simulations are listed in the last column of Table 2. Good agreement was obtained when using the fatigue parameter $\beta_{fat} = 0.15$. This represents a realistic value, since for typical concrete β_{fat} values are in the range of 0.06-0.08. In FRP strengthening, the bond strength is governed by the thin surface layer of concrete, which can be expected to have much lower fatigue strength, i.e. higher value of β_{fat}. The numerical simulation can provide interesting insight into the system behavior during the fatigue crack propagation such as for instance the evolution of strains at selected locations (see Figure 16 and Figure 17). Typical crack pattern at the time of failure is shown in Figure 18.

5 CONCLUSIONS

The paper presents a fatigue model that was developed for high cycle fatigue assessment of concrete structures by finite element method. The objective of the model is to enable the assessment of structures for high cycle fatigue when thousands and millions of cycles are taken into account without the need to analyze each cycle. On the other hand the model enables to consider the redistribution of forces, which must occur during the crack propagation process.

Several validation problems are presented in Section 3 for three point bending fatigue concrete tests with a notch and in Section 4 for fatigue bond failure of concrete structure strengthened by CFRP laminates.

The model shows good agreement with experimental results, it is easy to use, it is based on standard fatigue criterion, and it provides interesting insight into the crack propagation and structural behavior during the fatigue loading process.

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