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NUMERICAL SIMULATIONS OF THE TRANSITION FROM FLEXURAL CRACKING TO SHEAR TO FLEXURAL CRUSHING FAILURES IN PRE-NOTCHED REINFORCED CONCRETE BEAMS

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Abstract: This study predicts the transition between flexural cracking, shear, and flexural crushing failure modes in reinforced concrete (RC) beams through a series of four-point bending simulations on 48 pre-notched specimens. The upcoming experimental setup explores the influence of key parameters, including: (1) longitudinal reinforcement ratios ranging from 0.13% to 2.26%; (2) slenderness ratios (λ) from 3 to 24; (3) beams of varying size-scale with a constant reinforcement ratio; (4) beams of varying size-scale with a constant reinforcement area. The bridged crack model is applied to assess crack propagation and failure mechanisms, providing insight into the mechanical response, ultimate load capacity, and the critical cracks leading to failure. Dimensionless numbers N_P and N_C are used to capture the brittleness transitions between different failure modes, offering a unified theoretical framework for evaluating these behaviors. The results provide valuable data for understanding the scale effects on brittleness in RC beams.

1 INTRODUCTION

The determination of the shear strength of reinforced concrete (RC) beam elements without transverse reinforcement has been deeply investigated in literature since the beginnings of the development of RC technology. Another remarkable line of studies is due to Carpinteri and co-workers who proposed an approach to the problem based on fracture mechanics and dimensional analysis background and performed several experimental tests. They initially studied the problem of minimum reinforcement in RC beams and the related brittle or ductile bending failure mechanism [1–7]. These studies were then extended to include the effect of fibres reinforcement [8–10], shear and concrete crushing failures [11–13,15], influence of axial force [14] and size effects, which were introduced through dimensionless brittleness numbers. Structural elements having the same brittleness number show the same mechanical behavior. The works by Carpinteri and coauthors share the common idea that failure is caused by the formation and propagation of a critical crack. This concept has been used both for the early works on bending failure and specifically in mixed bending/shear works where the shape of the critical crack was reported from experimental testing [12,15].

The aim of this work is to provide the scientific community with the numerical simulations data of a series of four-point bending on pre-notched specimens. In total 48 beams will be tested， we use numerical simulations to predict some relative data in advance. The outline of the paper is as

follows:Section 2 is devoted to the description of the experimental program, by detailing the reinforcement amount and the beam size. Section 3 shows the load carrying capacity of each beam simulated numerically. Section 4 predicts the failure form of each beam based on the dimensionless analysis. Finally, Section 5 draws conclusions.

2 EXPERIMENTAL PROGRAM

An extended experimental program was carried out in the Structures Laboratory of Shantou University to investigate the failure mechanisms and cracking modes of concrete beams reinforced only with longitudinal reinforcement. This experiment consists of four sets of beams, and the effects of these variables on concrete beams can be obtained by varying reinforcement ratio, length-to-depth ratio, and the size-scale. The experimental setup explores the influence of key parameters, including: (1) longitudinal reinforcement ratios ranging from 0.13% to 2.26%; (2) slenderness ratios (λ) from 3 to 24; (3) beams of varying size-scale with a constant reinforcement ratio; (4) beams of varying size-scale with a constant reinforcement area.

2.1 Specimen geometry and test setup

The test specimen has span S, depth h, width b, and notch depth a_0. The reinforcement ratio ρ is equal to A_S / (hb). The beams are simply supported and loaded in four-point bending (see Fig. 1).

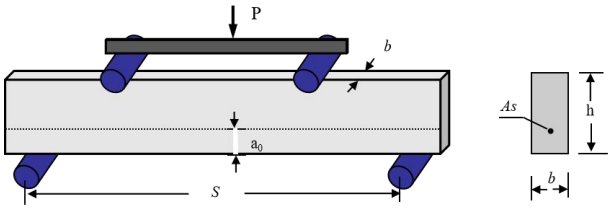

Figure 1: Test setup.

Four series of specimens will be tested, totaling 16 different types of beams. Table 1 lists the geometric parameters and longitudinal reinforcement amounts for all beams. It is worth noting that, in order to be consistent with the definitions of the brittleness numbers N_P and N_C [2,15,16], all parameters are normalized by the beam depth h, which will be used to interpret the results in the next sections. For each beam type, three specimens were prepared, in order to have some statistics of the results.

Table 1:-Specimen series

	b (mm)	h (mm)	s (mm)	A_S
Effect of steel reinforcement ratio (Series 1)	200	200	1200	1Φ8
	200	200	1200	2Φ8
	200	200	1200	4Φ8
	200	200	1200	4Φ12
	200	200	1200	8Φ12
Effect of beam Slenderness (Series 2)	200	200	600	4Φ12
	200	200	1200	4Φ12
	200	200	2400	4Φ12
	200	200	4800	4Φ12
Size effect with constant reinforcement area (Series 3)	200	200	1200	2Φ12
	200	400	2400	2Φ12
	200	800	4800	2Φ12
	400	1600	9600	2Φ12
Size effect with constant reinf. Ratio (Series 4)	200	200	1200	4Φ12
	200	400	2400	3Φ20
	200	800	4800	6Φ20

2.2 Material properties and specimen preparation

Preparation of concrete, reinforcement arrangement and casting were accomplished in a concrete beam preparation plant in Shantou, Guangdong Province (see Fig. 2).

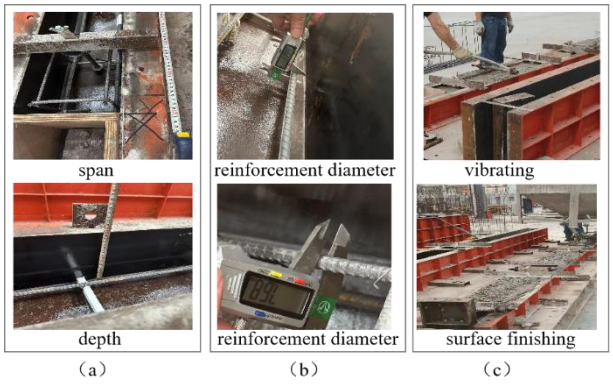

Figure 2: Specimen preparation: (a) formworks; (b)reinforcement arrangement ; (c) concrete casting.

The main prescriptions for concrete are: strength class, as defined by GB/T 50107—2010 [19], equal to C30, maximum aggregate size equal to 20 mm (defined through the slump test according to JGJ 2011 [20]). The actual mechanical properties of concrete were assessed with specific tests carried out 28 days after casting. The compression strength, f_c, was measured on 150 x 150 x 150 mm cubic specimens; the secant value of the elastic modulus, E_c, was assessed on cylindrical specimens 100 mm in diameter and 300 mm in depth, following the Standards GB/T 50081-2019 [21]; the fracture energy, G_F, was evaluated by means of three-point-bending tests carried out on pre-notched beams following the procedure prescribed by RILEM Recommendation TC-89 [22].

Table 2:Mechanical properties of concrete

f_{ck} (Mpa)	f_c (Mpa)	Ec (Mpa)	G_F (N/mm)
30	21.43	32836	0.14

As regards the reinforcement bars, only three diameters have been used, 8 ,12 and 20 mm, in order to limit the effect of the steel–concrete bond properties (note that 8 mm bars were used to provide the correct amount of reinforcement only in the case of very low percentages). The HRB335 steel was used, characterized by a yielding strength standard value of 335 MPa and an ultimate standard value strength of 490 MPa. The design yield strength is 300 MPa and the design ultimate value strength is 445 MPa.

Table 3:Mechanical properties of steel

Φ (mm)	f_y (Mpa)	f_t (Mpa)
8	300	455
12	300	455
20	300	455

3 SIMULATIONS BY THE COHESIVE/OVERLAPPING CRACK MODEL (COCM)

This section presents and discusses the results of the simulations tests. The numerical simulations is based on the data obtained from the Cohesive/Overlapping Crack Model [17]. This section focuses on the load vs. deflection curves to predict the failure modes of the beams.

3.1 Effect of the reinforcement amount

The first series of specimens was designed to analyze the effect of the reinforcement amount on the load-carrying capacity and the crack pattern. It consists of 5 types of beams, having the same size and different reinforcement percentages, ρ, varying in the range 0.13% to 2.26% (see Fig. 3).

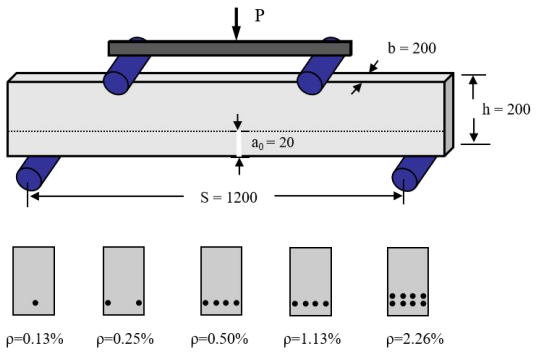

Figure 3: Set of beams used to analyze the effect of the reinforcement amount, ρ.

With numerical simulations, it is possible to obtain the load-deflection curves (Fig. 4). From the figure, the beams with reinforcement ratio 0.13% to 1.13% are ductile and the beams with reinforcement ratio 2.26% show brittle crushing failure.

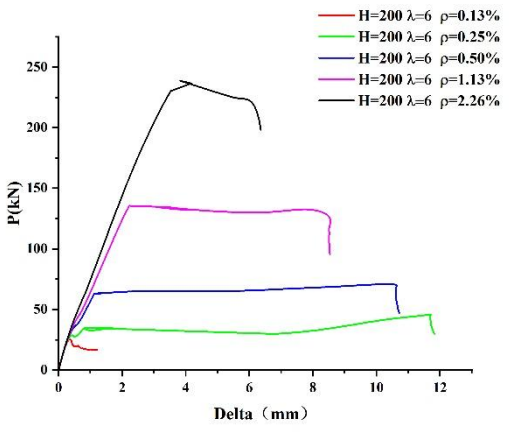

Figure 4: Load vs. mid-span deflection curves for the series of beams designed to analyze the effect of the reinforcement amount.

3.2 Slenderness effect

The slenderness effect was analyzed by considering beams with a constant reinforcement ratio ρ = 1.13%, a constant depth h = 200 mm, and different beam spans varying from 600 to 4800 mm (Fig. 5).

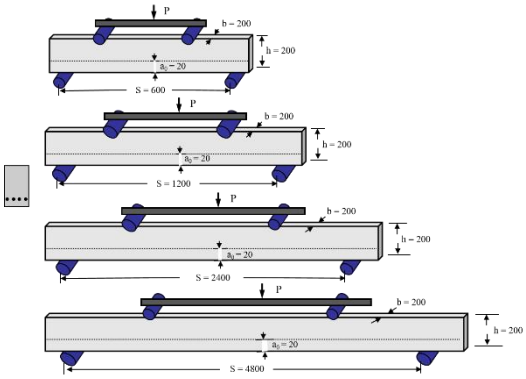

Figure 5: Set of beams used to analyze the slenderness effect with constant reinforcement ratio, ρ= 1.13 %.

With numerical simulations, it is possible to obtain the load-deflection curves (Fig. 6). The beams with slenderness 6 to 24 are ductile, whereas for slenderness 3 a brittle crushing is detected.

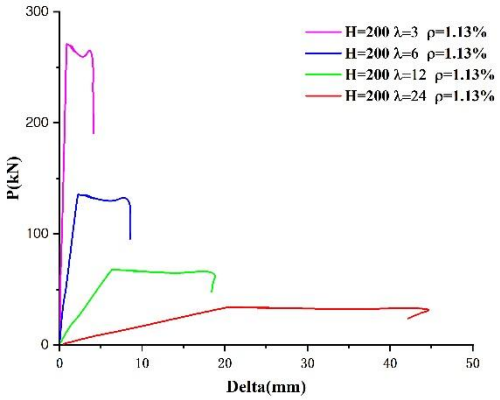

Figure 6: Load vs. mid-span deflection curves for the series of beams designed to analyze the slenderness effect with constant reinforcement ratio.

3.3 Size-scale effect

The size-scale effect is analyzed by increasing the beam depth from 200 to 1600 mm (Fig. 7). Then, a set of beams will be tested to analyze the size-scale effect with a constant reinforcement percentage, ρ= 1.13% (Fig. 9).

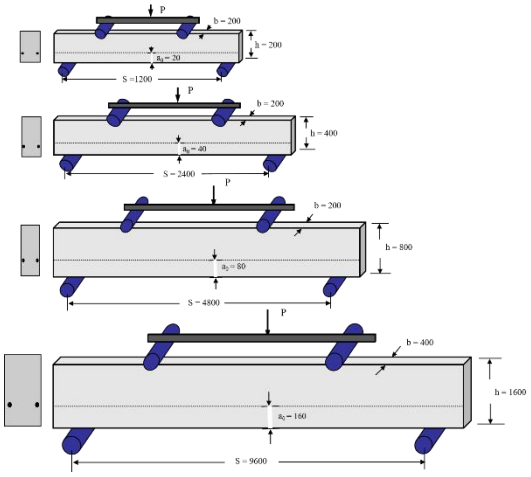

Figure 7: Set of beams with different size-scales and constant reinforcement area, As = 226 mm^2.

With numerical simulations, it is possible to obtain the load-deflection curves (Fig. 8). The beams with constant reinforcement area are ductile except for h=1600 mm that is brittle.

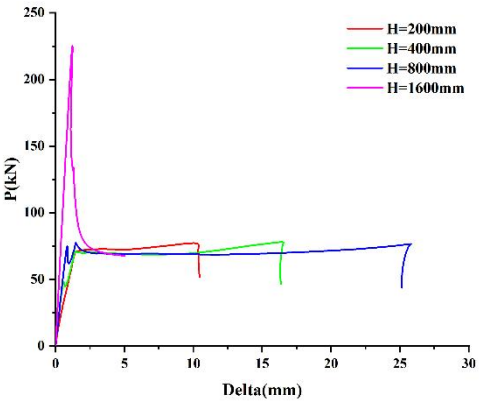

Figure 8: Load vs. mid-span deflection curves for the series of beams designed to analyze the size-scale effect with constant reinforcement area.

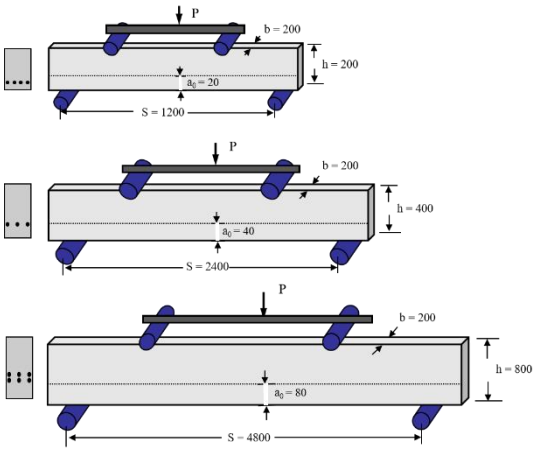

Figure 9: Set of beams used to analyze the size-scale effect with constant reinforcement percentage ρ= 1.13 %.

With numerical simulations, it is possible to obtain the load-deflection curves (Fig. 10). The beams with constant reinforcement percentage are ductile.

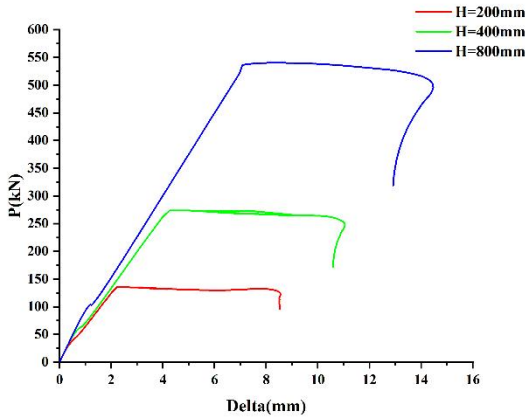

Figure 10: Load vs. mid-span deflection curves for the series of beams designed to analyze the size-scale effect with constant reinforcement percentage.

4 DIMENSIONAL ANALYSIS

Recent studies by the present authors [13,15,18] have shown that the failure mode transitions in RC beams can be conveniently analyzed on the basis of three dimensionless parameters, namely the shear slenderness, λ_l, equal to S/(2h), and two brittleness numbers, N_P and N_C, obtained by applying the Buckingham's Π-theorem of dimensional analysis to the functional relationship of the geometrical and mechanical parameters involved in the problem. The procedure yields to reduce the number of dimensionless variables [2,15,23]:

$$N_p = \rho \frac{f_y\, h^{0.5}}{K_{IC}} \tag{1}$$

introduced by Carpinteri since 1981 [1], and

$$N_c = \rho \frac{f_C\, h^{0.5}}{K_{IC}} \tag{2}$$

introduced by Carpinteri et al [16,17]. The dimensional analysis procedure applied to derive N_P and N_C provides the expression for the dimensionless load:

$$\tilde{P} = \frac{P}{K_{IC} h^{0.5} W} \tag{3}$$

In Eqs. (1) to (3), K_{IC} is the concrete fracture toughness, which has been herein computed by means of the fundamental Irwin's relation, holding in the framework of linear elastic fracture mechanics:

$$K_{IC} = \sqrt{E_c G_F} \tag{4}$$

N_P includes the effects of the reinforcement amount and strength, the concrete toughness, and the structural size-scale, whereas N_C introduces the effect of the concrete compression strength. In dimensionless terms, the structural response is expected to be only a function of the three dimensionless numbers. In particular, physical similarity is predicted when the three dimensionless parameters are constant, although the single mechanical and geometrical properties vary [15,23,24]. The values of the three dimensionless numbers are reported in Table 4 for all the beam types. It is worth noting that, in the first three series of beams, both NP and N_C vary.

Table 4:Dimensionless numbers for each type of beam.

	N_p	N_C	P_{COCM} (kN)	$\tilde{P}$
Effect of steel reinforcement ratio (Series 1)	0.09	5.00	28	0.15
	0.17	5.00	45	0.23
	0.35	5.00	70	0.36
	0.79	5.00	130	0.68
	1.58	5.00	240	1.25

Effect of	0.79	5.00	270	1.41
beam	0.79	5.00	135	0.70
Slenderness	0.79	5.00	78	0.41
(Series 2)	0.79	5.00	34	0.18
Size-scale	0.39	5.00	78	0.41
effect with	0.28	7.07	78	0.29
constant	0.20	9.99	78	0.20
reinforcement	0.08	14.13	230	0.21
area(Series 3)				
Size-scale	0.79	5.00	137	0.71
effect with	1.11	7.07	270	0.99
constant	1.58	9.99	570	1.48
reinforcement				
ratio(Series 4)				

5 CONCLUSIONS

This paper makes a multifaceted contribution to the study of the shear strength of RC members without transverse reinforcement. According to the Global Failure Mode Transition Scheme [25], a double transition from flexural to shear failure and from shear to crushing failure is obtained: by increasing the reinforcement area, by decreasing the scale with constant reinforcement area, by increasing the scale with constant reinforcement percentage, and by decreasing the beam slenderness. We use numerical simulations to predict in advance the relevant data for the beams to be tested, providing a more convenient solution for the experiments.

Moreover, we are about to carry out the experimental program presented in this paper, which is unique in the recent literature in terms of the number of specimens tested and the type of parameters studied, and the results will be able to constitute a comprehensive database that will be very useful for future research work.

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