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FE MODELLING OF CRACK WIDTH IN REINFORCED CONCRETE BEAMS SUPPORTED BY ARTIFICIAL NEURAL NETWORK SURROGATE MODELS

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Abstract: The concrete cracking is simulated by the finite element method combined with the constitutive model based on the nonlinear fracture mechanics using finite element simulation software. It is known that numerical simulations of reinforced concrete using the finite element method can be strongly influenced by the assumptions of crack spacing or crack band size, especially when large finite element sizes are used. The proposed approach attempts to address this issue by using machine learning and artificial neural network surrogate models to estimate crack spacing in reinforced concrete structures. The model uncertainties for mean and maximum crack widths are evaluated using the database of laboratory results. The reinforcement arrangement, dimensional simplification, and numerical discretization effects on the model uncertainty are investigated. The numerical model offers an adequate prediction of crack widths for the beams with a single-layer reinforcement and exhibits less accuracy for the multilayer bar arrangement. The presented numerical model represents an advanced tool for the crack width assessment in the design of reinforced concrete structures in serviceability limit states.

1 INTRODUCTION

The nonlinear finite element analysis is starting to be embraced in practice for the analysis of complex and safety-critical reinforced concrete structures not only for extreme or ultimate loading scenarios but also for checking deflections or crack width at serviceability limit states.

The development of nonlinear FEM for analyzing reinforced concrete structures began in the 1970s with groundbreaking research by Ngo and Scordelis [1], Rashid [2], and Červenka and Gerstle [2]. Over time, many researchers contributed to the advancement of material models for reinforced concrete, including Suidan and Schnobrich [3], Lin and Scordelis [4], De Borst [6], Rots and Blaauwendraad [7], Pramono and Willam [8], Etse [9], Lee and Fenves [10], and Červenka [11],[12]. These models are typically implemented in FE software, where each integration point is assigned a material model to evaluate internal forces. To overcome issues related to mesh size and to ensure realistic energy dissipation, Bažant and Oh [13] introduced the crack band method, which remains widely used in such analyses. For structures discretized with large finite elements or with a lot of reinforcement, the crack band theory used by default does not give very reliable results because it assumes that the crack spacing is greater than the element size (resulting in a maximum of one crack per element). This element is then "damaged" as a whole, resulting in conservative but very brittle results. This is usually not a problem when simulating experiments, where specimens are relatively small and can be meshed with fine elements. However, when modeling a real structure, such as a whole multi-story building or a bridge with dimensions of tens or hundreds of meters, it is inconvenient to set a fine mesh, so elements likely to be larger than one or two meters are used in such analyses.

The limitations of the crack band method, particularly in cases involving very small or very large finite elements, were explored in greater depth by Červenka et al. [14], who introduced minimal and maximal constraints on the crack band size. The minimal limit is generally governed by the smallest feasible crack spacing, which depends on the aggregate size, while the maximal limit corresponds to the expected crack spacing under typical conditions. In nonlinear finite element analysis, these limits can be specified as user-defined parameters, providing flexibility in the model setup.

However, selecting an appropriate value for this parameter in practice is challenging due to its dependence on numerous factors. These include the material properties of the concrete, the size, quantity, and arrangement of reinforcement bars, the concrete cover thickness, and other structural characteristics. This parameter significantly influences the accuracy of the analysis, impacting not only the computed load-bearing capacity but also deflection predictions and crack width estimations, which are critical for assessing serviceability limit states.

Furthermore, inappropriate parameter selection can lead to inaccuracies in the simulation, such as overestimating or underestimating crack development, which directly affects the reliability of the model's predictions. Given its importance, researchers and engineers often rely on experimental calibration or sensitivity analyses to determine an optimal value. Understanding the crack band size and its practical limitations is essential for developing robust nonlinear finite element models that closely align with real-world behavior.

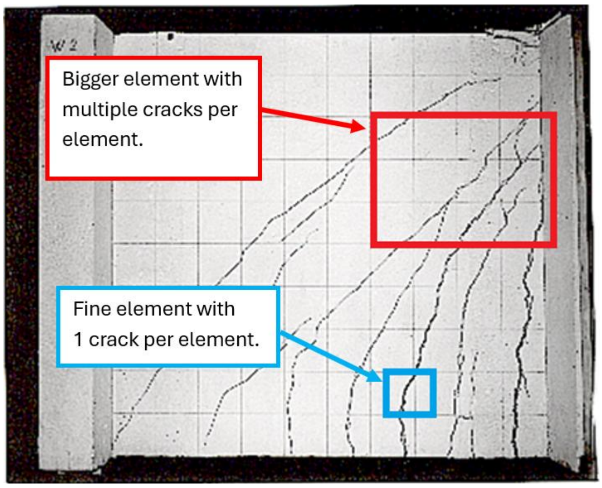

Figure 1: The illustration of the dependence of the element size on the number of possible cracks contained.

2 SIMULATIONS OF EXPERIMENTS INVESTIGATING THE EFFECTS OF MESH SIZE AND CRACK SPACING

This behavior will be demonstrated in the simulation of three experiments simulations performed in ATENA software [15]. In these calculations, the mesh size effect and its correction by crack spacing were investigated.

It is important to acknowledge that the coarse mesh discretization employed in these simulations is not ideal for models of this type. However, given the real experimental data and the dimensions of the specimens, this approach was necessary to effectively analyze the issue and ultimately proved to be a valuable method for addressing the study's objectives.

The first example is a four-point bending test of a deep beam by C.R. Braam was studied. This model was part of a wider investigation described in [16]. The beam selected is 5.5 meters long and 0.8 meters high with a "T" cross-section. The meshes studied were chosen with elements of 30 mm for a fine mesh and cca 300 mm for a coarse mesh.

Figure 3 shows the L-D curves of the experiment and numerical simulations, focusing on the study of the influence of mesh size. The orange curve marked "M30" (i.e. mesh size 30 mm) shows satisfactory agreement with the experimental result curve. The blue curve "M300" shows results for a coarse mesh with no crack spacing.

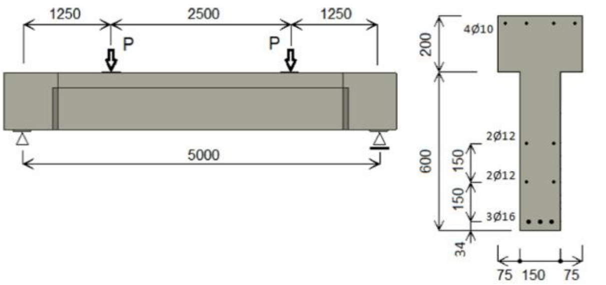

Figure 2: Deep beam B8 tested Braam dimensions

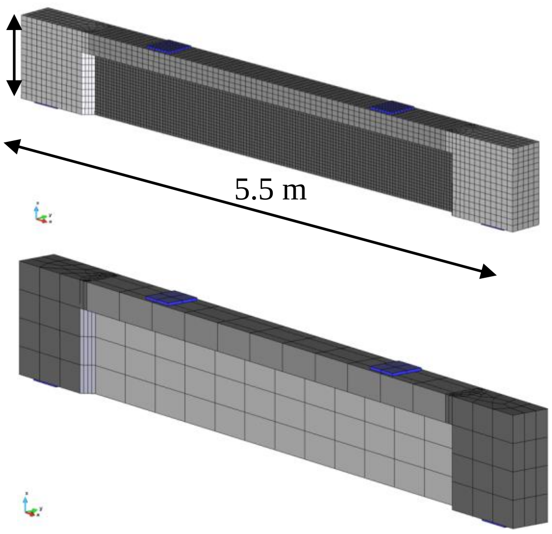

Figure 3: Finite element discretization of the investigated beams

This model clearly demonstrates the deficiencies of the crack band model when large finite elements are used. In this case, the assumption of a single crack occurrence inside the finite element is not valid and crack spacing should be specified. After adjusting the crack spacing value, a closer agreement with the experiment (or the fine mesh model) is obtained. With the crack spacing value of 100 mm a closer match is obtained and with 70 mm, which is the mean crack spacing measured in the experiment, an almost identical curve to that of M30 is obtained. This is another good demonstration of the importance of the crack spacing parameter in the case of large finite elements and the presence of reinforcement.

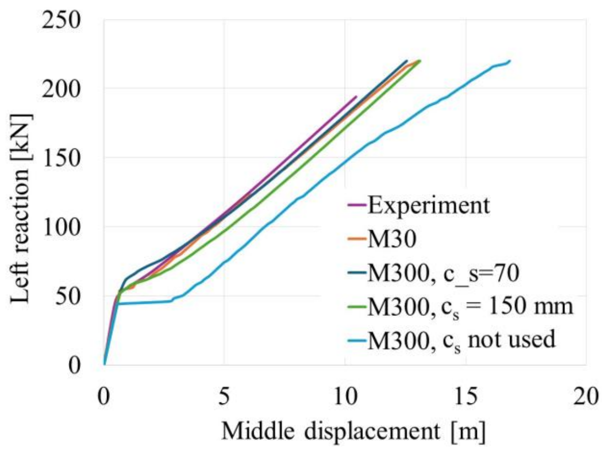

Figure 4: L-D for simulation of Braam's Deep Beam Experiment

The second example comes the experimental database that will be also subsequently used for the development of an artificial neural network (Section) learning (closer introduction in next chapter) dealing also with the four-point bending test of a prefabricated, lightweight concrete beam (described in more detail in [17]. The beam is 4 meters long with a cross-section of 0.4x0.27 m. Two meshes were studied, a fine one with an element of 25 mm and a coarse one with elements of about 300x135 mm. It should be mentioned again that for such an experiment, the mesh is not suitable, but it was used to investigate the effect of the crack spacing parameter in the case of large elements.

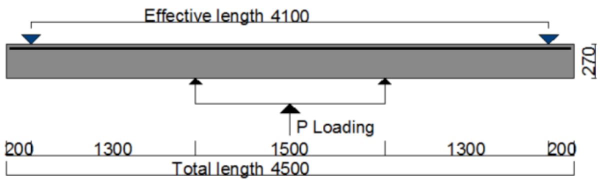

Figure 5: Test setup and beam dimension [mm]

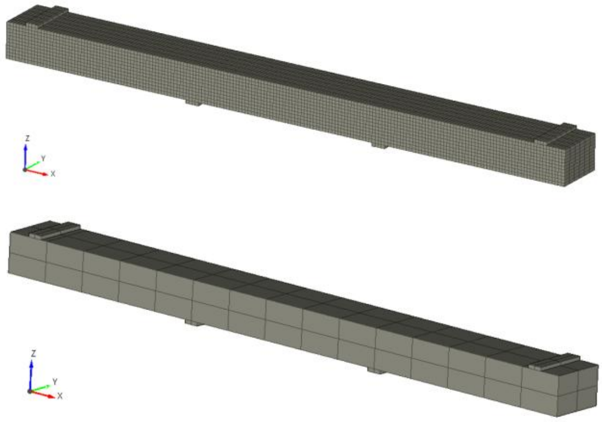

Figure 6: Beam with fine mesh (approx. 25 mm) and coarse mesh (approx. 300x135 mm)

The following Figure 7 shows the load-displacement diagrams for different mesh sizes and crack spacing parameters. As mentioned in the first example, it shows again that the fine mesh follows the experimental curve quite well. The coarse mesh without crack spacing is noticeably softer than the correct results. However, after the correction using a suitable value of the crack spacing parameter, it is possible to obtain results like those of the fine mesh and the experiment.

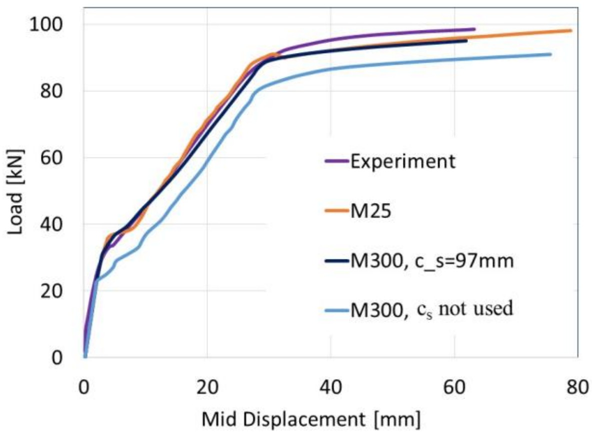

Figure 7: L-D diagram of the light-weight prefabricated beam, the M25 or M300 is the size of the elements described above, c_s is the set crack spacing value.

The next Figure 8 shows the beam at the loading stage, where the final crack spacing and widths were measured. The crack pattern shows that the cracks occur every 3rd to 5th element, given the element size of 25 mm, which corresponds approximately to the experimentally measured mean spacing of 97 mm.

It is also worth comparing the calculated crack width of about 0.22 mm with the experimental one of 0.26 mm at the same loading level.

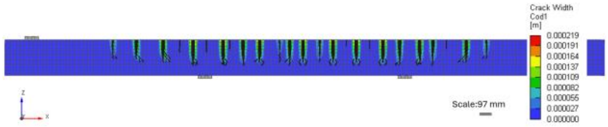

Figure 8: Crack pattern on the beam - the crack spacing of 97 mm was measured at the level of stress in reinforcement 452 MPa, cracks in this figure are shown at the step where the stress in reinforcement reaches 451 MPa.

The last presented example corresponds to a test from the experimental research by Gribniak et al. [18]. This example represents a prismatic beam in direct tension with a cross-section of 150x150 mm, length 1210 mm, from concrete C30 that is reinforced by four ø10 mm bars with a cover 30 mm. Three different meshes with different element sizes were studied (**Figure 10**).

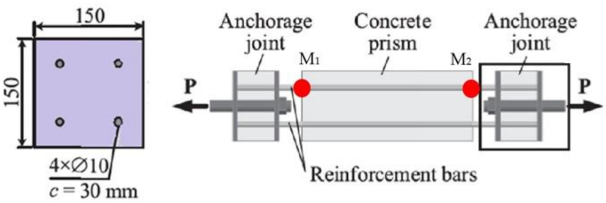

Figure 9: Direct tension test beam dimensions

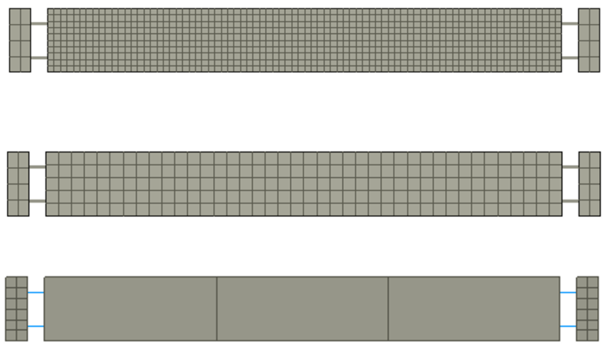

Figure 10: Direct tensile test specimens from [18]

The following diagrams (Figure 11) show the results of the experiment and the simulations. The legend labels M15/M30/M500 denote the size of used the elements 15, 30, and 500 mm respectively. The c_s represents the value of the crack spacing parameter. The curves show that with a fine mesh, reasonable results are obtained. However, with a mesh set to 500 mm, after the cracks occur, the element stiffness is significantly reduced, and the concrete contribution severely decreases. To eliminate this, the crack band approach needs to be corrected with the crack spacing parameter.

The diagrams in Figure 11 show different curves with crack spacing parameters (c_s = 130 mm and c_s = 50 mm). After this correction, they are noticeably closer to the experimental curves.

Another parameter that can be used to improve the response of the nonlinear concrete model in the case of large elements is tension stiffening. This parameter introduces a bottom limit on the softening response. See the green curve in Figure 12 marked as "T-M500, c_s=13, t_s=0.4". Both parameters must be specified manually, which can be a challenge for an analyst.

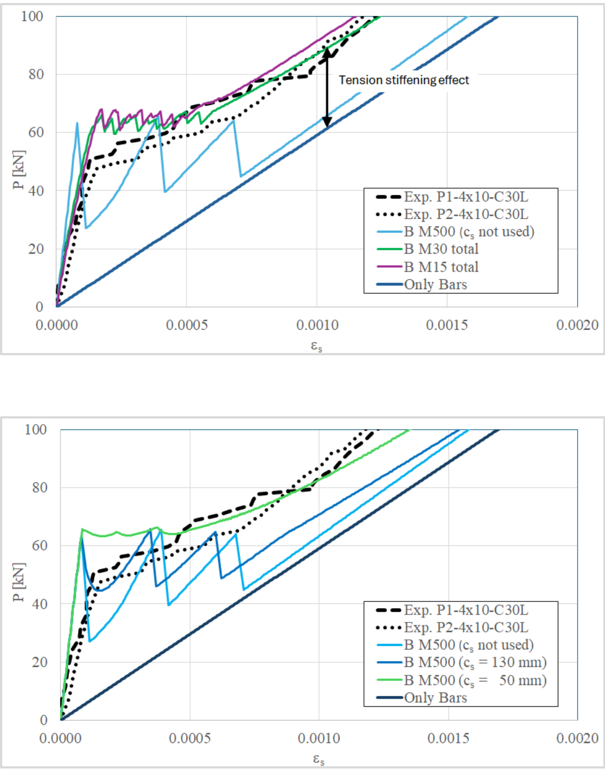

Figure 11: L-D diagrams for direct tension test beam illustrating the influence of mesh size

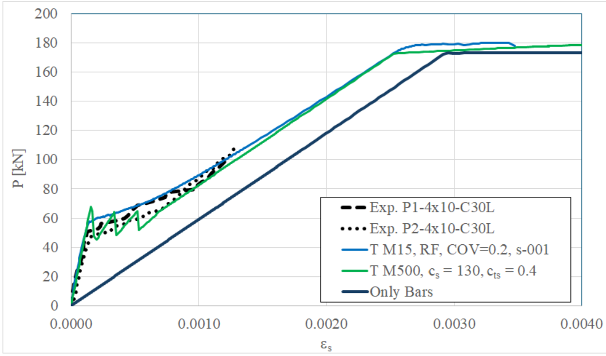

Figure 12: Selected results illustrating the influence of parameters (crack spacing, tension stiffening)

In the simulation, the following material parameters were used:

Table 1: Material parameters used in the simulations

Parameter	Braam	Prefa beam	Direct tension beam
Concrete			
E[MPa]	30 860	19 793	33 061
ν [-]	0.2	0.2	0.2
f_c [MPa]	45.9	43.9	36
f_t [MPa]	4	2.73	2.79
G_F [N/m]	145	140	139
Reinforcement			
E_s [MPa]	200 000	200 000	200 000
f_y [MPa]	550	550	550
f_u [MPa]	620	700	594
$ε_u$ [-]	0.07	0.06	0.05

3 ARTIFICIAL NEURAL NETWORK AND DATABASE OVERVIEW

As previously mentioned, when modeling reinforced concrete structures with large finite elements, additional material parameters such as crack spacing, or tension stiffening should be introduced to the model. It is, however, clear that these parameters are not pure material constants, but can be influenced by numerous factors (e.g. reinforcement bars diameter, arrangement, number of bars, concrete cover, structure geometry, concrete properties etc.). Addressing this problem is well-suited for machine learning, specifically using artificial neural networks with multiple inputs and one or several outputs.

An Artificial Neural Network (ANN) for crack spacing prediction is currently under development. The aim is to be able to eliminate manual guessing of the spacing as an input to the analysis but rather to determine the spacing for each element during the calculation based on the dimensions of the element, embedded reinforcement location, directions, spacing, and cover depth.

The crucial part of creating such a network is having a sufficient amount of data. For the first step, a well-organized and unified database of numerous experiments was used [19]. The database summarizes about 30 experiments, describing the specimen geometry, material parameters, reinforcement and prestressing (if present) and the test setup along with the results. The results focus on crack development and provide values of crack widths and crack spacing at different stages of loading (if measured during the experiment).

The **inputs** to ANN were selected as follows:

- Specimen geometry – cross-section dimensions, length.
- Concrete properties – compressive and tensile strength, modulus of elasticity.
- Reinforcement - number of bars, diameter, concrete cover (single layer only).
- Reinforcement stress level (representing the loading stage).

The desired **outputs** were:

- Mean crack spacing
- Crack width

After data preparation (input preparation, normalization, etc.), different architectures and learning algorithms were tested, and finally, a feed-forward ANN with input/output layers and three hidden layers, each with ten neurons, was trained using the Bayesian regularization training algorithm.

Figure 15 shows that the predictions in the region of 50-150 mm crack spacing are not far from the target values. However, as the targeted value goes in higher numbers, the ANN predictions are not satisfactory. This requires more detailed investigation.

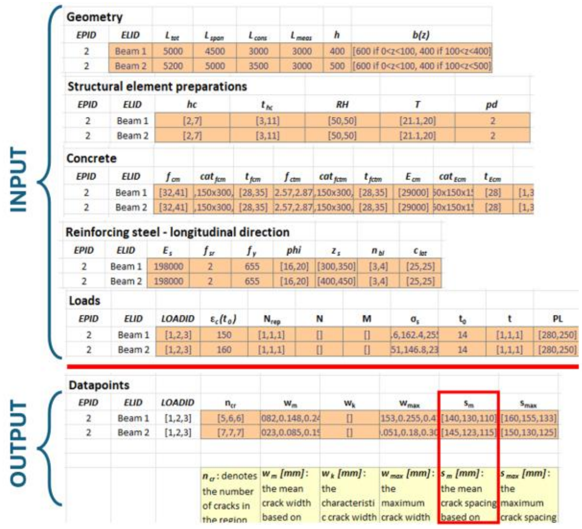

Figure 13: Sample of the database created by van der Esch et al. [19] showing various experiment data.

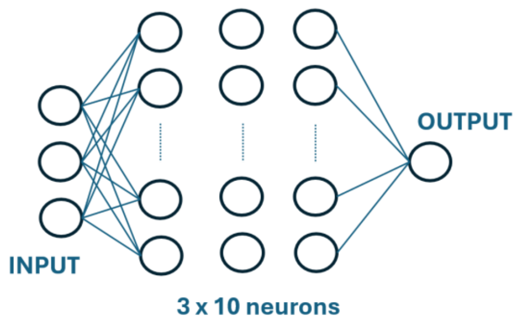

Figure 14: The illustration of the feed-forward ANN architecture.

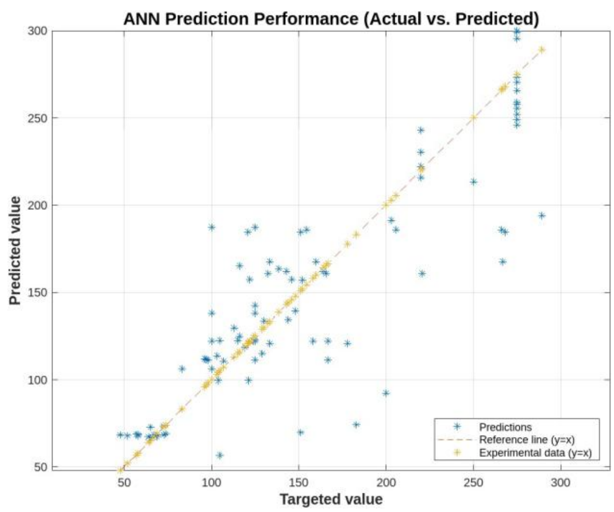

Figure 15: Predictions of the trained ANN. The blue line represents the target value, the blue points represent the predicted values. The distance from the line demonstrates how close the prediction is to the target value.

It may be attributed to the evaluation of crack spacing in the experiments, which was usually done by visual inspection with a lot of inaccuracies and subjective approach of the researchers involved. This might introduce inaccuracies and sudden discontinuities in the training data. Another reason can be the relatively small training data set, which could be addressed by future work or the training data could be enhanced by numerical simulation.

Further development will therefore focus on the following key steps:

1. Separation of four-point bending and direct tension experiments, as these loading modes have significantly different crack evolution. Either take the loading mode as another input with significant weight or train different ANNs for both modes separately.
2. Consider only the final crack spacing value. The crack spacing in the early stages of loading (e.g. 1 m) proves to be misleading when training the ANN.
3. Only considering crack spacing values less than 350 mm.
4. Optimize the training parameters such as learning rate, impulse, and number of epochs as the ANN shows a tendency to overtrain.

After data processing based on the above steps, the training data is significantly reduced from thousands to tens to lower hundreds. This suggests a further step for improvement:

5. Expand the database using ATENA software, simulating more documented experiments with a focus on crack spacing investigation and determination. This can later provide more training data for the ANN.

4 CONCLUSIONS

This paper demonstrates the importance of additional modeling parameters for accurate simulation of reinforced concrete material for large scale nonlinear finite element analyses. Three simulations in ATENA software were introduced, which illustrated that fine meshes closely match experimental results, while coarse meshes tend to be more conservative and require additional user input parameters.

In cases when finite element sizes above 100-200 mm are used, which is quite common in the nonlinear analysis of real engineering structures such as bridges, tunnels, nuclear containments or high-rise buildings, it is important to introduce additional modeling parameters. This paper demonstrates that two additional modeling parameters representing crack spacing and tension stiffening have the potential to increase the accuracy of crack width and stiffness predictions when large finite elements need to be used. These parameters are not material constants and depend on the reinforcement arrangement.

Therefore, the artificial neural network (ANN) algorithm is presented and investigated for this purpose. This is ongoing research, which aims at applying the ANN to determine the suitable values of these additional modeling parameters based on the actual reinforcement arrangement occurring at each finite element.

An artificial neural network (ANN) was developed to predict crack spacing based on an existing database of experiments, showing the potential to replace manual parameter estimation. While initial results are promising, further refinements, including better data categorization and expanded databases by additional ATENA simulations, are needed. The study demonstrates how combining simulation and machine learning can enhance structural analysis accuracy and efficiency in reinforced concrete design.

5 ACKNOWLEDGEMENTS

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