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STRUT EFFICIENCY ON SHEAR STRENGTH, AND DUCTILITY OF RC DEEP BEAMS WITH LARGE RECTANGULAR OPENINGS IN SHEAR SPAN

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Abstract: This study reports experimental investigations on reinforced concrete (RC) deep beams with web opening in the shear span to evaluate the effect of large rectangular openings on strut efficiency, shear strength, and ductility. The tested beams had dimensions of 1300 x 200 x 500 mm, with rectangular openings varying in size from 200 × 100 mm to 300 × 150 mm. The shear span-to-depth ratio (*a*/*d*) was maintained at 0.75. The beams were reinforced with 0.30% web reinforcement in each direction and 1.80% flexural tension reinforcement. The ultimate load capacities of beams with openings of 200 x 100 mm and 300 x 150 mm were 61 and 76% lower, respectively, than that of the reference beam. The ultimate load of the beam with the larger opening was 38% lower than that of the beam with the smaller opening. Horizontal web reinforcement, detailed at the upper and lower corners of the large openings, effectively mitigated the reduction in initial stiffness and ductility. The inclination of the critical strut varied with the opening size, affecting the ultimate load capacity and failure mode. An accurate prediction of the shear capacity is essential to prevent sudden failures. The strut-and-tie model for deep beams with openings located at the centre of the shear span has been simplified. Additionally, a strut efficiency factor was formulated as a function of concrete strength, considering the influence of the critical strut angle.

1 INTRODUCTION

Web openings in RC deep beams are essential for accommodating electrical and mechanical utility services in the building. While extensive research has been conducted on the behavior of deep beams without openings, limited efforts have been reported on deep beams with rectangular openings in the shear span, particularly those with web reinforcement. Existing studies have not examined the post-peak characteristics of deep beams with openings. It is essential for developing reliable and efficient analytical models for design. Kong and Sharp [1] tested deep beams with web opening in the shear span and different web reinforcement patterns.

Yang et al. [2] conducted experimental investigations on deep beams with various sizes of rectangular openings and percentage of inclined web reinforcement. The size, shape, and location of web openings were identified as major factors influencing the behavior of RC deep beams [3]. In general, the web opening size adversely affects the shear strength of RC deep beams. However, there is a lack of test data on variables such as concrete strength, percentage of web and flexural reinforcement, and the size, shape, and location of openings. Kong and Sharp [4] developed an empirical equation for shear strength based on the idealization of a weaker load transfer path around a rectangular opening. Limited research has been reported on developing strut-and-tie models (STM) for the design of deep beams with web openings. Hwang and Lee [5] proposed a softened strut-and-tie model incorporating a simplified concrete softening coefficient introduced by Zhang and Hsu [6]. Tan et al. [7] proposed a strut-and-tie model that satisfies equilibrium to predict the shear strength of deep beams with web openings and performed an analysis using the limited available test data.

Tseng et al. [8] idealized struts around the opening in the STM as spring elements that satisfy equilibrium and displacement compatibility conditions. Kondalraj and Appa Rao [9,10] performed experiments on deep beams with and without web reinforcement. The ACI318-19 [11] strut efficiency factors for deep beams with and without web reinforcement were refined as a function of the compressive strength of concrete to predict shear capacity. The compressive strength of concrete and strut angle were the major influencing parameters on the shear strength of deep beams with web openings [12]. The inclination of the critical strut connecting the support point to the bottom opening corner is influenced by the size, shape, and location of the opening, as well as *a*/*d* ratio, which governs the ultimate shear strength [12]. However, there are no established guidelines for the design of deep beams with web openings.

2 RESEARCH SIGNIFICANCE

The design of RC deep beams with web openings is complex due to a lack of understanding of the effect of size, shape, and location of openings on the shear strength. The STM is effectively utilized in ACI318-19 code for the design of deep beams, owing to its conservative approach and reliable lower bound predictions. The main diagonal strut governs the shear capacity in most solid deep beam designs. However, in deep beams with openings, the lower inclined struts above and below the opening control the ultimate load capacity. The STM was refined for deep beams with rectangular openings, and the accuracy of the proposed strut efficiency factor for the critical strut is validated with the present study and experimental database.

3 EXPERIMENTAL PROGRAM

3.1 Test specimens

Two RC deep beams with rectangular opening in the shear span and a solid beam were tested to failure. The opening sizes ranged from 200 x 100 mm to 300 x 150 mm, located at the centre of the shear span. The beams were designed with 0.30% horizontal and 0.30% vertical web reinforcement to meet the minimum requirements of the ACI 318-19 code. All beams had dimensions of 1300 x 200 x 500 mm, *a*/*d* 0.75, and 1.80% bottom tension reinforcement, with adequate anchorage at the ends. The geometric details of beams are given in Table 1. In beam details, DB refers to deep beam; the first numerical value (x10) indicates overall depth; N and S denote no opening and opening in the shear span, respectively; the second numerical value indicates percentage web reinforcement ratio; the final numerical values, 2010 and 3015, indicate opening sizes i.e. 200 x 100 mm and 300 x 150 mm, respectively. Where b and d denote breadth and effective depth, ρ_v and ρ_h indicate the percentage vertical and horizontal web reinforcement ratio, respectively. S_h and S_v indicate the spacing of web reinforcement in each direction, ρ_t represents percentage flexural reinforcement ratio, f_{ck} denotes the mean compressive strength of 150 mm standard cubes, and x and y indicate width and depth of opening, respectively. A typical reinforcement layout of the beam is shown in Figure 1.

3.2 Materials

Ordinary 53-grade Portland cement, river sand, 20 mm coarse aggregate, and potable water were used to produce concrete to prepare specimens. All beams were cast with a minimum of three 150 mm standard cubes, which were water-cured for 28 days. The deep beams and cubes were cured under the same exposure conditions and tested on the same day. The characterization of Fe550 high-yield strength deformed (HYSD) rebar is summarized in Table 2.

Table 1: Test specimen details

Beam	b	d	a/d	ρ_t	ρ_v	S_v	ρ_h	S_h	f_{ck}	x	y
	mm	mm	-	%	%	mm	%	mm	MPa	mm	mm
DB50-N-0.30									38.4	-	-
DB50-S-0.30-2010	200	436	0.75	1.8	0.30	165	0.30	165	38.9	200	100
DB50-S-0.30-3015									38.9	300	150

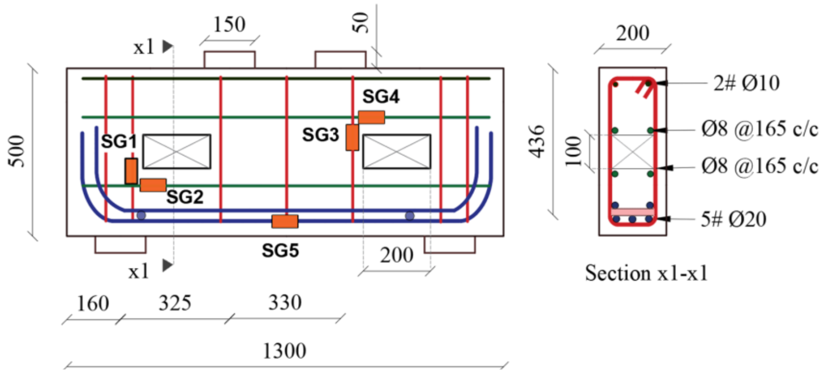


Figure 1: Reinforcement layout and instrumentation of deep beam DB50-S-0.30-2010.

Table 2: Mechanical properties of reinforcement

Properties	Diameter of rebar (mm)	
	8	20
E (MPa)	201	212
f_y (MPa)	624	582
$\mathcal{E}_u$	0.059	0.013

Note: E = Young's modulus, f_y = Yield strength, and $\mathcal{E}_u$ = Ultimate strain

3.3 Test set-up and instrumentation

All beams were tested to failure under the simply supported four-point loading. The beams with web openings were tested under displacement-control at a rate of 0.30 mm/minute using a 1000 kN capacity actuator, while the reference beam was tested under load-control at a rate of 0.50 kN/s using a hydraulic jack capacity of 6000 kN. Different testing methods were selected to account for machine capacity limitations. The load at reaction points was recorded using load cells with a capacity of 1000 kN. The test set-up is shown in Figure 2. An electrical resistance strain gauge with a gauge length of 5 mm was bonded to the web reinforcement near opening corners. A linearly variable differential transducer (LVDT) was used to measure beams' vertical deflection. A 50 mm square grid was marked on one face of each beam to identify crack locations.

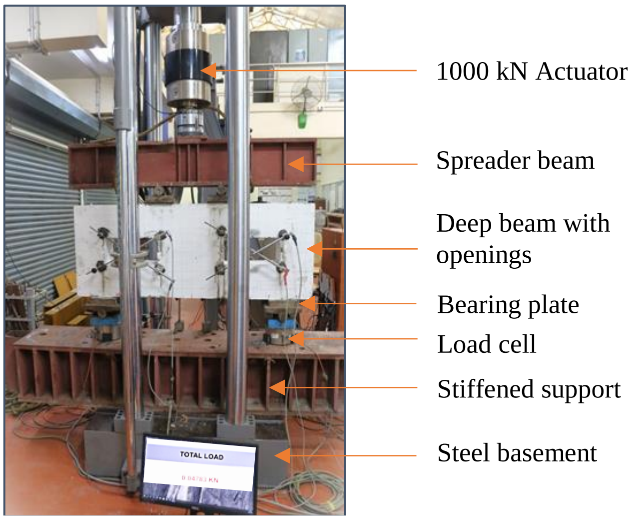
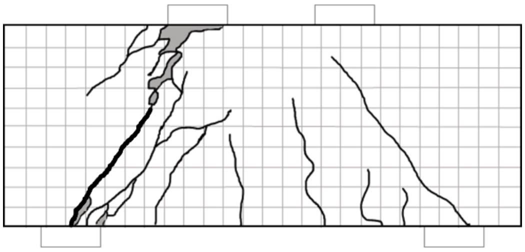


Figure 2: Experimental test set-up of deep beams with web openings.

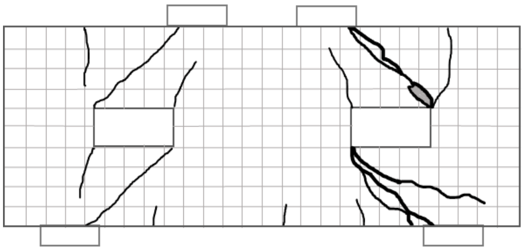
4 EXPERIMENTAL RESULTS AND DISCUSSION

4.1 Crack pattern and failure mode

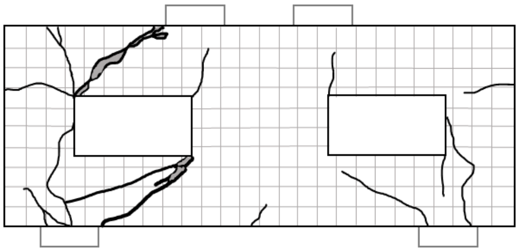
The crack patterns and failure modes of various RC deep beams are shown in Figure 3. The first crack appeared at the bottom and top opening corners, closer to the support and the load points, at 30 and 59% of the ultimate load (V_u) for beams DB50-S-0.30-2010 and DB50-S-0.30-3015, respectively. The flexural cracks in the mid-span initiated at 65 and 19% of the ultimate load in beams with and without openings, respectively. The first diagonal crack in critical strut appeared at 28, 65, and 90% of the ultimate load in beams DB50-N-0.30, DB50-S-0.30-2010, and DB50-S-0.30-3015, respectively. At the failure load, the diagonal crack connected the support with the load points. Shear-compression failure occurred in the beam without opening. The failure mode in beams with web openings was diagonal tension in critical struts, as shown in Figure 3. The experimental results are summarized in Table 3.



a) DB50-N-0.30



b) DB50-S-0.30-2010



c) DB50-S-0.30-3015

Figure 3: Crack pattern and failure mode.

4.2 Load versus deflection response

The load versus deflection response of various beams is shown in Figure 4. The beams with openings exhibited linear behavior until the first crack formed in the upper and lower critical struts. Stiffness degradation was observed in beams with and without openings immediately after reaching the first diagonal crack load (V_{cr}) in the critical strut below the opening. The stiffness degradation after the first crack formation was significantly higher in beams with larger openings compared to those with smaller openings. The curve exhibited a sharp decline at the peak load, accompanied by a rapid load drop.

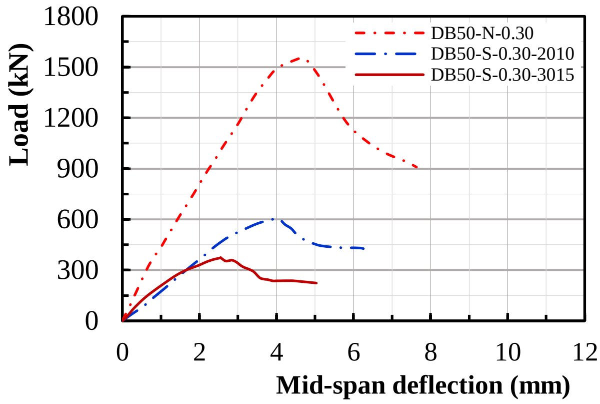


Figure 4: Load versus deflection response.

4.3 Effect of opening in the shear span

The ultimate load capacity of beams with openings, DB50-S-0.30-2010 and DB50-S-0.30-3015, was 61 and 53% lesser than that of beams without openings, DB50-N-0.30, respectively. The secant stiffness of the beams DB50-S-0.30-2010 and DB50-S-0.30-3015 was 56 and 27% lower than that of beam DB50-N-0.30, respectively. The web opening location in the middle of the shear span significantly reduced the shear capacity and secant stiffness due to the obstruction of the force transfer path. The reserve capacity (V_u/V_{cr}) of beams with web openings was significantly lower than that of beams without openings. The ultimate capacity and mid-span deflection of beam with larger openings, DB50-S-0.30-3015, were 38 and 16% less than those of beam with the smaller openings, DB50-S-0.30-2010, respectively. Enlarging the opening significantly reduces the effectiveness of the load transfer paths around the opening.

4.4 Strain in web and flexural tension rebar

The variation of strain in horizontal and vertical web reinforcement with a normalized load of beam DB50-S-0.30-2010 is shown in Figure 5. During the initial loading stage, the strain in web reinforcement was insignificant in beam DB50-N-0.30. After the first crack in the diagonal strut, the rate of strain increment in the web reinforcement became significant with an increase in load. The maximum strain in horizontal and vertical web reinforcement reached 59 and 39% of the yield strain of the rebar at the failure load, respectively. Vertical web reinforcement was effective below mid-depth, while horizontal web reinforcement was effective above mid-depth in the shear span. In beams DB50-S-0.30-2010 and DB50-S-0.30-3015, the horizontal web reinforcement became effective after the initiation of cracks at the top and bottom opening corners closer to load and support points. The strain in vertical web reinforcement was insignificant. The maximum strain in horizontal web reinforcement reached 22 to 54% of the yield strain of the rebar at the failure load in beams with openings. The strain in the horizontal web reinforcement was higher than that in the vertical web reinforcement at the top and bottom opening corners at the failure load. The variation of strain in flexural rebar at the mid-span of various beams is shown in Figure 6. The strain in flexural rebar at the mid-span of beams DB50-N-0.30 and DB50-S-0.30-3015 reached 49 and 19% of the yield strain at failure, respectively.

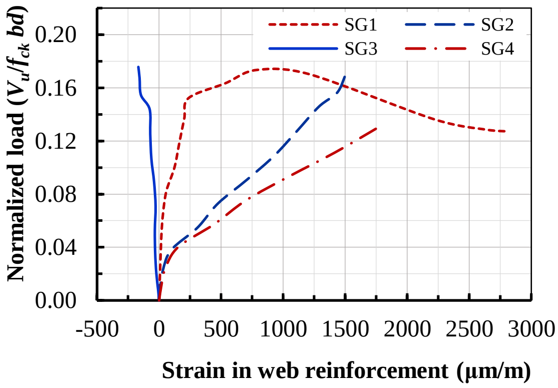


Figure 5: Strain in web reinforcement of beam DB50-S-0.30-2010.

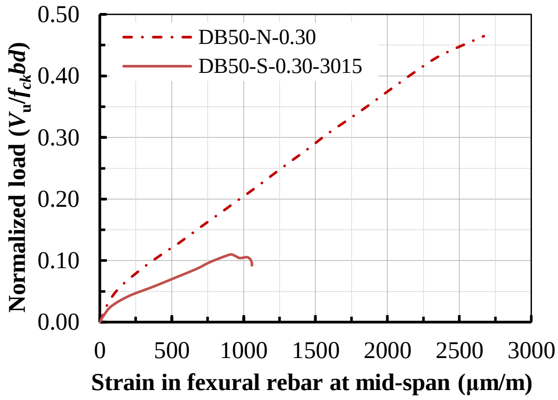


Figure 6: Strain in flexural tension rebar in various beams.

4.5 Ductility ratio

Ahmad et al. [13] found that solid deep beams with web reinforcement exhibit significantly higher ductility than those without web reinforcement, which failed under shear compression or diagonal tension mode. Similarly, Figure 4 shows the post-peak region, indicating that concrete softening occurs in beams with web openings. According to ASTM E2126-11, ductility is defined as the ratio of ultimate displacement-to-yield displacement. The ductility ratios (μ) in beams DB50-S-0.30-2010 and DB50-S-0.30-3015 were found to be 1.70 and 2.67, respectively. The horizontal web reinforcement above and below the opening corners effectively controls the reduction in ductility due to increased opening size. However, larger openings reduce the reserve capacity of the beams, indicating that these beams experience failure in the upper diagonal strut without any warning above the opening.

Table 3: Summary of experimental results

Beam	V_u (kN)	V_{cr}/V_u	μ
DB50-N-0.30	1554	0.28	-
DB50-S-0.30-2010	605	0.65	1.70
DB50-S-0.30-3015	373	0.90	2.67

5 ANALYTICAL INVESTIGATIONS

The STM for deep beams with web openings has been idealized based on the load transfer paths and the position of web reinforcement around the opening. The effectiveness of the load transfer path increases as the strut inclination increases. Struts below and above opening with a lower inclination exhibit weaker arch action. The failure mode is influenced by the inclination of the critical struts, which depends on the a/d ratio and the size, shape, and location of the opening. Therefore, the lower inclined strut below or above the opening is a controlling factor in estimating the shear capacity.

5.1 Simplified strut-and-tie-model for deep beams with web openings

The refined STM for deep beams with web openings is shown in Figure 7. In this model, the forces carried by the struts AD, AB, BC, and DC are designated as F_1, F_2, F_3, and F_4, respectively. The angles of the struts AD and BC are referred to as θ_1 and θ_3, respectively. These critical struts are less inclined than the struts AB and DC, which have inclinations θ_2 and θ_4, respectively. Experimental observations indicate that the strut below the opening, AD, with its angle θ_1, fails before the other force paths around the opening. The total applied load on the beam is denoted as V.

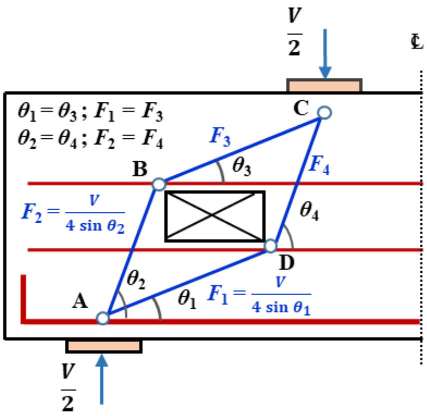


Figure 7: Simplified STM for deep beams with opening.

The vertical equilibrium at Node A is expressed in Equation (1).

$$\frac{V}{2} = F_1 \sin\theta_1 + F_2 \sin\theta_2 \qquad (1)$$

According to the spring model developed by Tseng et al. [8], the force distribution factor can be assumed as 0.5 for the struts AB and AD that branch from Node A when the centroids of the opening and shear span area coincide at the same point. At Node A, the load carried by the lower inclined strut AD is greater than that of the higher inclined strut AB to maintain equilibrium at Node A. Therefore, the shear path AD is the predominant failure path compared to AB. Similarly, at Node C, the force carried by path BC is greater than that of path DC. The force distribution factor for strut AD is defined in Equation (2).

$$\frac{F_1 \sin\theta_1}{F_1 \sin\theta_1 + F_2 \sin\theta_2} = 0.50;\ F_1 \sin\theta_1 = F_2 \sin\theta_2 \qquad (2)$$

From Equations (1) and (2), the capacity of critical strut AD is related to the capacity of the deep beam with web opening V, as described in Equation (3).

$$2F_1 \sin\theta_1 = \frac{V}{2} \qquad (3)$$

5.2 Strut efficiency factor

The effective compressive strength of the strut was estimated by incorporating a multiplication factor into the uniaxial cylindrical compressive strength of concrete (f_c'). This factor, referred to as the strut efficiency factor (β), accounts for various uncertainties and decreases as the concrete strength increases. The force carried by the inclined strut AD in the simplified STM is expressed in Equation (4).

$$F_1 = \beta\,(0.85 f_c')\,A_{str1} \qquad (4)$$

The area of strut AD is defined in Equation (5).

$$A_{str1} = \left(\sqrt{(l_s/2)^2 + w_t^2}\right) \times b \qquad (5)$$

Where l_s is the length of the bearing plate, and w_t is the depth of Node A. From Equations (3) and (4), the efficiency of the lower strut AD is estimated from Equation (6).

$$\beta_{exp} = \frac{V_{exp}}{4\,(0.85 f_c')\,A_{str1} \sin\theta_1} \qquad (6)$$

Rajprabhu and Appa Rao [13] analyzed the influence of various parameters on the shear strength of deep beams with and without openings using a selected database. Test reports of 38 deep beams with rectangular openings in the middle of the shear span and with web reinforcement were filtered from that database. The proposed rational strut efficiency factor, derived from these data points and present studies, is expressed in Equation (7). The strut coefficient is limited to 0.50 and 0.35 for 25 and 55 MPa concrete strengths, respectively. The proposed strut efficiency factor was assumed to decrease linearly with an increase in concrete strength is illustrated in Figure 8.

$$\beta_{proposed} = 0.50 \leq 0.625\left(1 - \frac{f_c'}{125}\right) \leq 0.35 \quad (7)$$

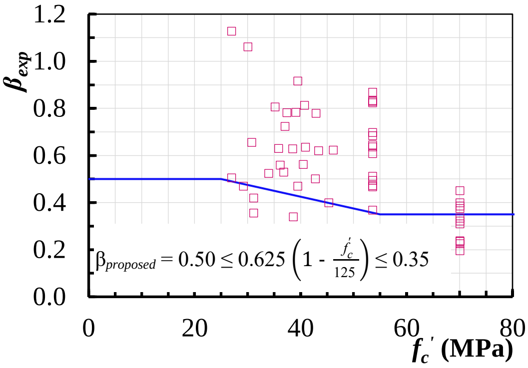
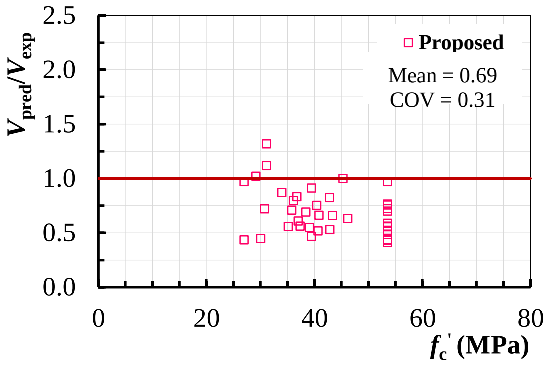


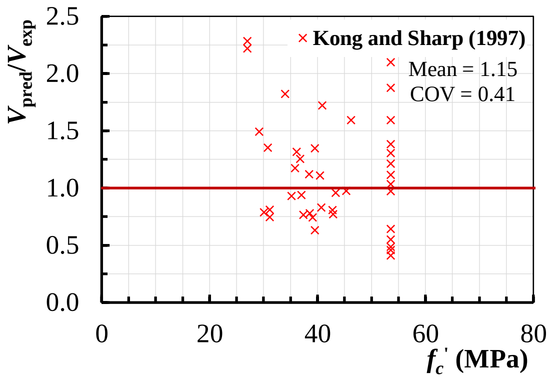
Figure 8: Proposed strut efficiency factor for deep beams with web opening.

5.3 Experimental verification

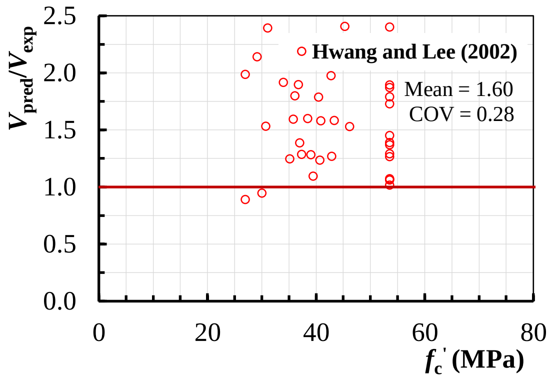
The predicted shear strength (V_{pred}) from the proposed strut efficiency factor and other methods was validated using 40 test results of deep beams with openings, as shown in Figure 9. The mean shear strength ratio (V_{pred}/V_{exp}) obtained from the proposed method was 0.69, with a lower coefficient of variation (COV) of 0.31. The proposed strut efficiency factor results in a lower bound to the experimental shear capacity and 10% of beams being overestimated. Kong and Sharp's [4] equation yielded a mean shear strength ratio of 1.15 and a higher COV of 0.41, with 50% of the beams having overestimated shear capacity due to its empirical form. Hwang and Lee's [5] prediction model results in a mean shear strength ratio of 1.15 and a COV of 0.28, with 95% of the beams overestimated. Tan et al.'s [7] method was overly conservative, underestimating shear strength with a mean of 0.37 and a higher COV of 0.47. This approach only satisfies the equilibrium condition and does not account for the concrete softening. The shear strength of beams was estimated using an assumed force distribution factor of 0.50 for the struts below the opening, branching from Node A. The proposed method provides more conservative predictions compared to other methods and sets a lower bound for the experimental results.



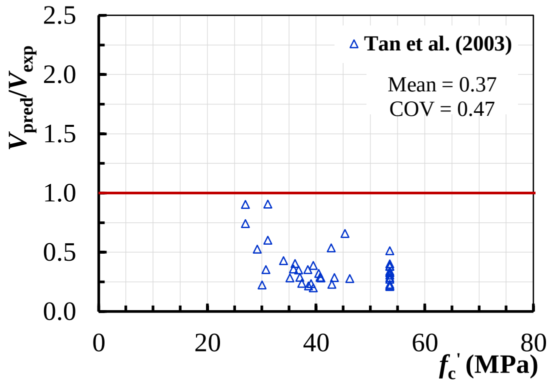
(a) Proposed



(b) Kong and Sharp



(c) Hwang and Lee



(d) Tan et al.

Figure 9: Shear strength ratio versus concrete strength.

6 CONCLUSIONS

The following conclusions have been drawn from the studies on RC deep beams with openings.

Web openings in the middle of the shear span disrupt load transfer paths, splitting them into four smaller struts around the opening. This significantly reduces stiffness, shear capacity, dowel action, and ductility.

The ultimate load capacity, stiffness degradation, and reserve capacity of deep beams are strongly influenced by opening size. An increase in the opening size reduces the angle of the critical struts, which governs the failure mode.

Horizontal web reinforcement, detailed at the upper and lower corners of the opening, effectively mitigates the reduction in initial stiffness and ductility, especially in beams with large-size openings.

The proposed strut efficiency factor, incorporated with the simplified STM, provides conservative shear strength predictions compared to other methods.

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