12^{th} International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-12
B.L.A. Pichler, Ch. Hellmich, P. Preinstorfer (Eds)

https://doi.org/10.21012/FC12.1242
MS05-3:1

OBJECTIVITY ISSUES IN NUMERICAL MODELING OF CRACKING IN RC BEAMS BASED ON ENHANCED SECTION KINEMATICS

L. PARENTE[1,2], D. ADDESSI[3], B. A. IZZUDDIN[4], AND E. SPACONE[1]

[1]Department of Engineering and Geology, University of Chieti-Pescara G. D'Annunzio, Italy
e-mails: *luca.parente@phd.unich.it, enrico.spacone@unich.it*
[2]Department of Civil Engineering and Architecture, University of Catania, Italy
[3]Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Italy
e-mail: *daniela.addessi@uniroma1.it*
[4]Department of Civil and Environmental Engineering, Imperial College London, United Kingdom
e-mail: *b.izzudin@imperial.ac.uk*

Key words: Displacement-based beam finite element, Bond-slip, Warping, Reinforced concrete, Numerical objectivity, Crack localization

Abstract. This paper extends the discussion on critical numerical objectivity issues in finite element analysis of reinforced concrete beam elements, highlighted in previous studies by the authors. We address these challenges using enhanced displacement-based beam finite element models that incorporate bond-slip effects, warping deformation modes, and fracture energy regularization. The introduction of enriched kinematics aims to reduce mesh dependency and more accurately capture localized damage, particularly cracking phenomena. Comparative numerical simulations demonstrate the limitations of classical beam elements and the advantages provided by the enhanced ones, laying the groundwork for future developments in force-based models.

1 INTRODUCTION AND LITERATURE OVERVIEW

Numerical objectivity is an essential requirement for finite element models, ensuring that results are independent of mesh size. In reinforced concrete (RC) beam analysis, classical displacement-based (DB) [1] and force-based (FB) models [2, 3] often struggle with objectivity due to limitations in capturing the complex interactions between concrete and steel reinforcement, particularly bond-slip effects. These deficiencies can lead to unrealistic crack distributions and strain localizations [4, 5].

Fracture energy regularization is commonly used to control tensile concrete softening [6], however, while this technique is widely adopted, it is considered a weak regularization method that ultimately leads to non-objective results and fails to converge in certain scenarios [7, 8, 3]. This is particularly evident when applied to reinforced concrete beam elements, where the regularization does not fully capture the complex material behavior, leading to unrealistic predictions of damage localization [4].

The previous study [5] demonstrated that, even with local regularization techniques, classical DB models exhibit significant numerical issues when analyzing RC beam elements. This advancement explores enhanced DB beam finite element models incorporating bond-slip effects and warping deformations [9, 10, 11, 12]. These modifications aim to address the observed objectivity issues and improve the realism of crack modeling.

Research has sought to enhance DB models to better capture RC behavior. Bond-slip modeling, which incorporates the relative displacement between concrete and steel reinforcement, has shown promise in improving crack representation. In [5], the authors propose bond-slip models that introduce nonlinear springs at the interface, while implicit formulations embed bond-slip behavior directly into the element's kinematics. Integrating bond-slip effects and warping deformation modes significantly enhances DB model performance by localizing cracks within single elements rather than spreading them unrealistically across the mesh [4].

The inclusion of warping, i.e. out-of-plane deformation of beam cross-sections, is another relevant enhancement [12, 13, 14]. Traditional plane sections assumptions can fail in reproducing realistic cracking scenarios, where warping becomes significant. By incorporating warping into beam kinematics, enhanced DB models can represent realistic crack-induced deformations more accurately.

The tests in this study were conducted in a MATLAB environment, specifically using a custom finite element code to simulate the behavior of RC beam elements. This setup enables detailed modeling of a new enhanced 2D DB beam that considers bond-slip effects and warping deformations in the context of RC structures.

2 WARPING AND BOND-SLIP SECTION FORMULATION

The classical element local and basic configuration without rigid body modes for a 2D beam element are reported in Figs. 1 and 2.

The enhanced element used in this paper is formulated according to [4] and its basic configuration without the rigid body modes is shown in Fig. 3. Nodes are assigned for Lagrangian interpolation based on the required polynomial order for each degree of freedom. For example, transverse displacement uses four nodes for cubic interpolation, while other DOFs require fewer nodes for consistency with the beam formulation. It is a 2D DB beam employing 5 nodes with 19 degrees of freedom:

- 3 displacements and rotations in the end nodes u_j, φ_i and φ_j;
- 4 additional displacements and rotations in the internal nodes u_2, φ_2, v_3 and v_4;
- 9 additional DOFs for the warping interpolation;
- 3 for the bond-slip interpolation.

The vector **v** contains the generalized displacements in the basic configuration of the element:

$$\mathbf{v} = \{\, u_j \;\; \varphi_i \; \varphi_j \;\; u_2 \;\; \varphi_2 \;\; v_3 \;\; v_4 \;\; \alpha_i \;\; \alpha_2 \;\; \alpha_j \;\; \beta_i \;\; \beta_2 \;\; \beta_j \;\; \lambda_i \;\; \lambda_2 \;\; \lambda_j \;\; u_{b,i} \;\; u_{b,2} \;\; u_{b,j} \,\}^T \tag{1}$$

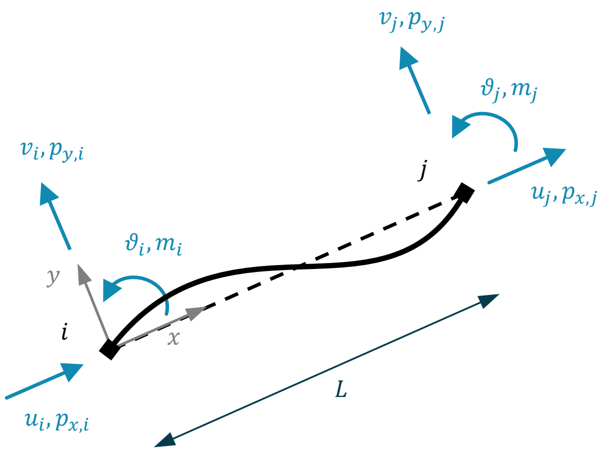

Figure 1: Local configuration for classical beam FE.

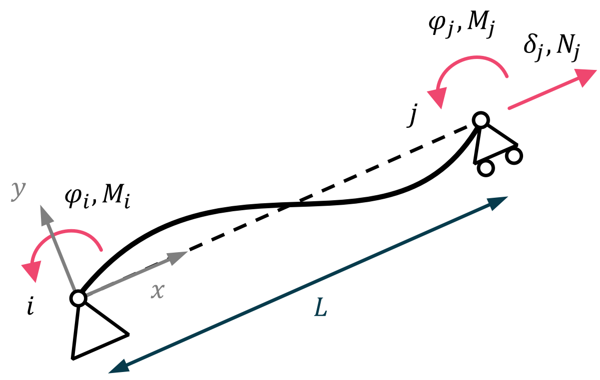

Figure 2: Basic configuration for classical beam FE.

where u and v represent nodal axial and transverse displacements respectively, φ are the nodal rotations, α, β, λ are the nodal warping displacements, and u_b are the nodal slip displacements. By adopting a fiber section discretization, displacements and strains at the

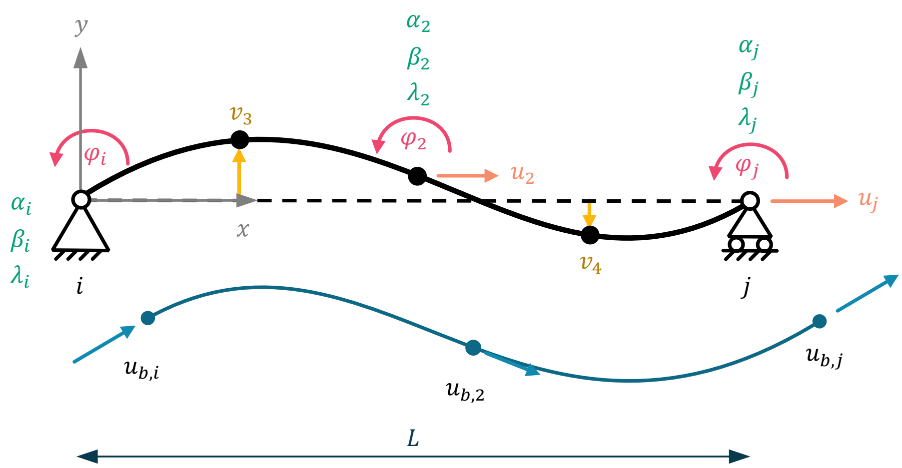

Figure 3: 2D basic configuration of the DB FE with warping and bond-slip [4].

fiber level can be computed considering the axial displacement u_a, the effect of the rigid section rotation $f_1 = -y\vartheta$, selected warping shape functions $f_2,\ f_3, \ldots$ multiplied by the warping DOFs $\alpha, \beta, \ldots$. The bond displacement u_b is included if the fiber lies into the slipping domain, which depends on δ_b (that may be 0 or 1):

$$\begin{aligned} u(x,y) &= u_a(x) - y\vartheta(x) + f_2(y)\alpha(x) + \\ &\quad f_3(y)\beta(x) + f_4(y)\lambda(x) + \delta_b u_b(x) \\ v(x,y) &= v(x) \end{aligned} \tag{2}$$

The displacement vector can be expressed as:

$$\mathbf{u}_m(x,y) = \mathbf{a}_s(y)\mathbf{N}(x)\mathbf{v} \tag{3}$$

where $\mathbf{N}(x)$ are the element shape functions and $\mathbf{a}_s(y)$ contains the section warping shape functions. Compatibility equations are derived in strong form as:

$$\begin{bmatrix} \varepsilon(x,y) \\ \gamma(x,y) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} \tag{4}$$

that, in compact form, read:

$$\boldsymbol{\varepsilon}_m(x,y) = \mathbf{D}(x,y)\mathbf{u}_m(x,y) \tag{5}$$

where $\boldsymbol{\varepsilon}_m(x,y)$ is the section strain. Nonlinear constitutive relationships are used:

$$\mathbf{k}_m = \frac{\partial \boldsymbol{\sigma}_m}{\partial \boldsymbol{\varepsilon}_m} = \begin{bmatrix} \frac{\partial \sigma}{\partial \varepsilon} & \frac{\partial \tau}{\partial \varepsilon} \\ \frac{\partial \sigma}{\partial \gamma} & \frac{\partial \tau}{\partial \gamma} \end{bmatrix} \tag{6}$$

where $\boldsymbol{\sigma}_m = \{\sigma(x,y)\ \tau(x,y)\}^T$ is the section stress response. Using the virtual work principle, the basic forces $\mathbf{q}$ and stiffness matrix $\mathbf{k}$ are computed, resulting as:

$$\mathbf{q} = \int_0^L \int_A \mathbf{a}^T(x,y)\boldsymbol{\sigma}_m(x,y)\, dA\, dx \tag{7}$$

$$\mathbf{k} = \int_0^L \int_A \mathbf{a}^T(x,y)\mathbf{k}_m(x,y)\mathbf{a}(x,y)\, dA\, dx \tag{8}$$

where $\mathbf{a}$ is the compatibility matrix defined by inserting Eq. 3 in Eq. 5:

$$\mathbf{a}(x,y) = \mathbf{D}(x,y)\mathbf{a}_s(y)\mathbf{N}(x) \tag{9}$$

3 NUMERICAL SIMULATIONS

To assess the effectiveness of the enhanced DB beam model, numerical simulations were performed on a RC beam specimen. These analyses were carried out with classical formulations, both including and excluding warping effects.

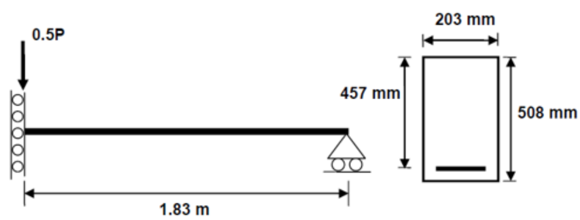

Figure 4: Schematic representation of beam specimen denoted as J4 in [15].

Concrete		Steel	
E_c [MPa]	26200	E_s [MPa]	203395
f_t [MPa]	2.4	σ_y [MPa]	309.65
f_c [MPa]	33.24	b [-]	0.01
G_f [N/mm]	0.0875	A_s [mm^2]	1000

Table 1: Geometric and mechanical parameters of beam J4. b is a Menegotto and Pinto parameter that can be represented as the ratio between the hardening and the elastic tangent.

3.1 Test Setup

The analyses focused on a simply supported RC beam subjected to a midspan concentrated load, with material properties and geometrical dimensions matching those used in the previous study [5]. Simulations were carried out using different meshes to ensure objectivity.

To demonstrate the advantages introduced by the enhanced DB beam model, simulations incorporated bond-slip and warping effects. The goal was to highlight the enhanced numerical objectivity and the more accurate representation of crack behavior.

The numerical tests were conducted on the beam shown schematically in Fig. 4, corresponding to beam J4 in [15]. Geometric and material properties are summarized in Table 1. The same materials as [5] are used for concrete, steel and bond-slip. For simplicity, only half of the beam was modeled, with the left end restrained at the midspan to guarantee symmetry.

Numerical tests were also conducted in [5], where experimental comparisons are provided for validation. In this paper, only good bonding conditions were considered to demonstrate the model's ability to capture flexural cracks, consistent with the expected cracking pattern.

3.2 Non-Objective vs Objective Numerical Results

The results show the performance of the models, highlighting:

- non-objective crack patterns and global response curves in classical beam elements;
- objective, localized crack representation and global response curves in enhanced beam elements with bond-slip and warping.

When a classical DB beam model is used, non-objective results are obtained as detailed in [5] when the number of elements (NE) is progressively increased. In the previous work, not only the crack pattern and the local results show mesh dependency, but also non-objective global response curves are obtained, which is the only case reported here, for the sake of brevity, in Fig. 5. The load and displacement values are the nodal quantities in the midspan. However, when the enhanced DB element with warping and bond-slip is used, both the global and the local strain and crack results improve, as shown in Fig. 6. Even though a relatively fine mesh is used, good results are obtained in this case even with coarser meshes, and the cracks' locations and distribution prediction is in line with the experimental and the analytical outcomes [15].

The slip displacement distributions in Fig. 7 show that cracks occur at the expected locations, where the slip displacement goes from positive to negative. The curvature spikes also indicate where the cracks have formed: even though they are different, as expected as the curvature is supposedly infinite in the discontinuity, they tend to localize in the same area although with a reduced error compared to classical elements.

Finally, by looking at the steel fiber strain, the results converge when the NE, i.e., the number of degrees of freedom, is increased. Fig. 8 shows that classical beam elements fail to capture the correct reinforcement steel strain, and therefore stress, whereas the enhanced beam elements show much improved results when the mesh density is increased. Practically identical results are instead obtained before yielding occurs, while more sparse results were obtained for classical beam elements.

4 CONCLUSIONS AND FUTURE WORK

This study demonstrates the potential of enhanced DB beam elements incorporating bond-

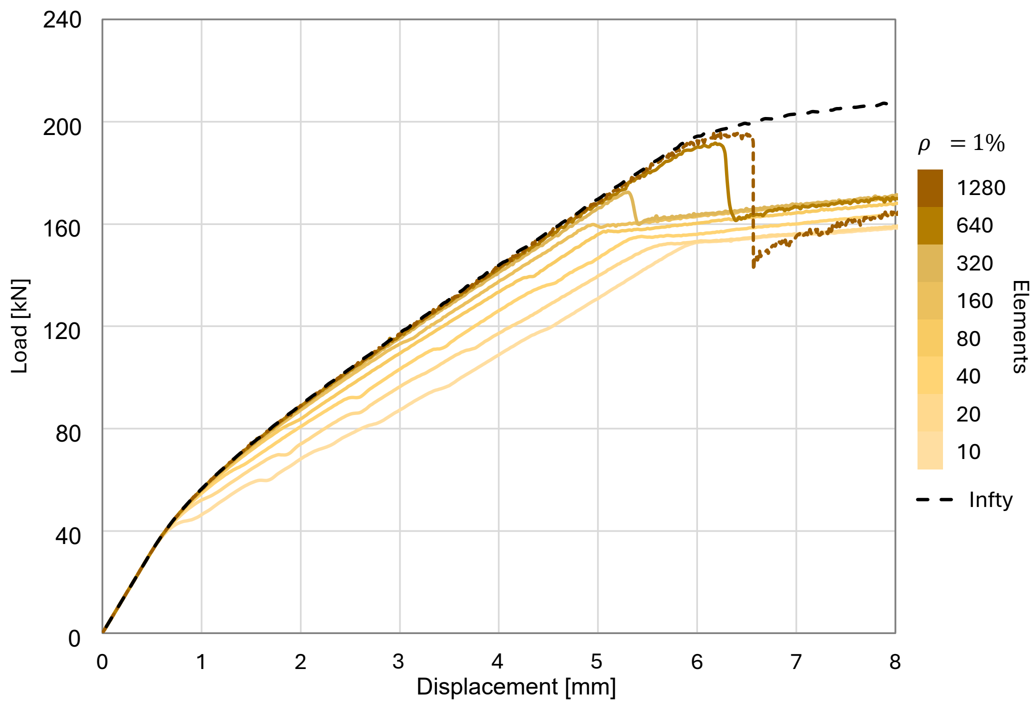

Figure 5: Structural midspan load vs midspan displacement response using classical DB beam elements. ρ is the ratio between the steel area and the concrete area, while *infty* represents the infinite fracture energy limit case.

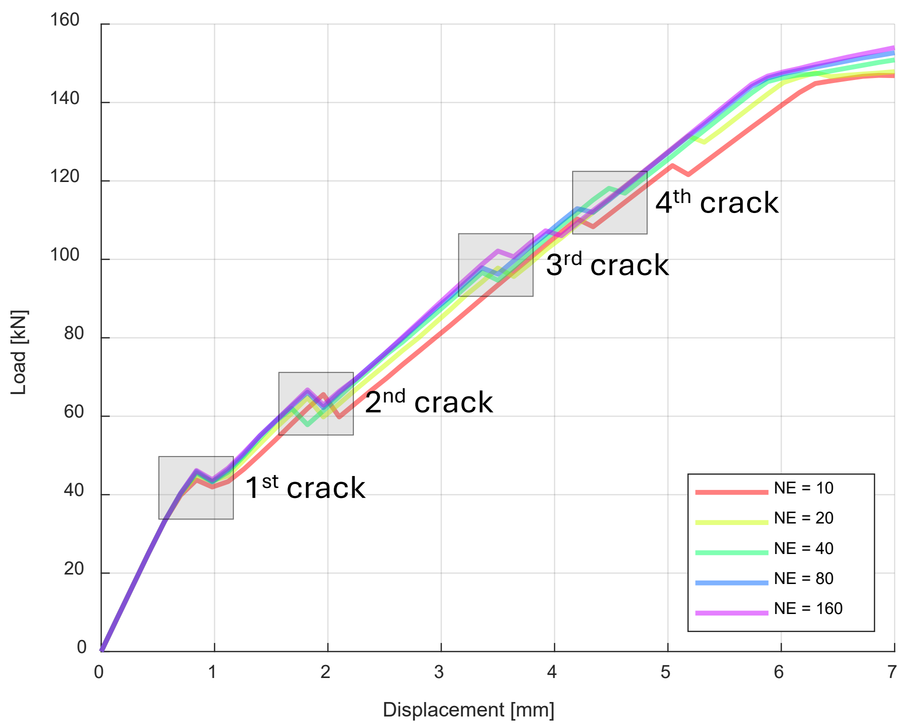

Figure 6: Objective structural midspan load vs midspan displacement response using enhanced DB elements with warping and bond-slip.

slip and warping effects to address numerical objectivity issues in RC beam analysis. Three main advantages are shown when using enhanced elements:

Mesh independency: simulations using classical beam elements show significant mesh de-

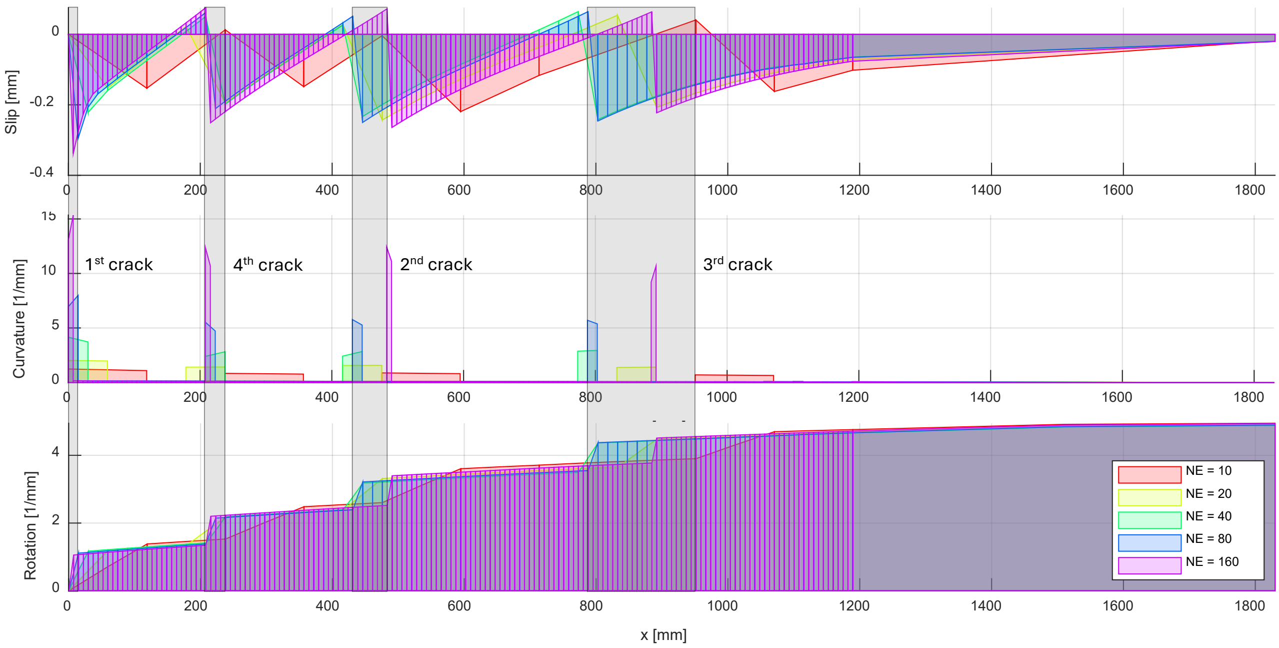

Figure 7: Bond-slip distributions and curvature at the final stage for four different meshes.

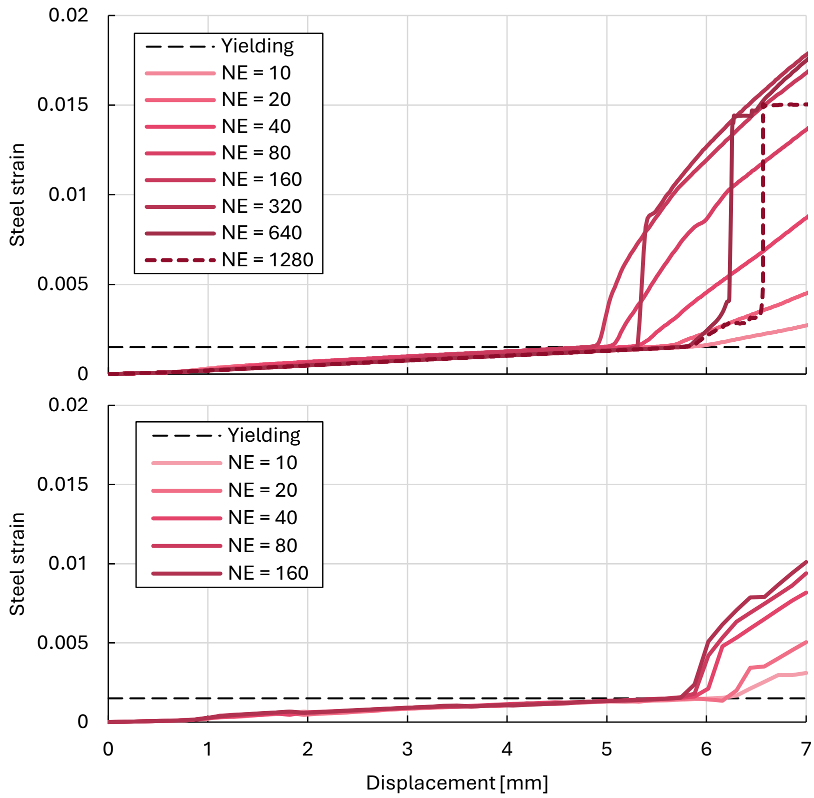

Figure 8: Steel strain in the midspan vs the midspan vertical displacement for classical beam elements (top) and enhanced beam elements (bottom).

pendency, where cracks unrealistically spread across elements with increased mesh density. In contrast, the inclusion of bond-slip localizes the cracks within single elements.

Impact of warping: introducing warping notably improves the section kinematics, allowing the model to capture realistic deformation patterns. Simulations demonstrate convergence and accurate crack localization when warping is included.

Comparative analysis: results comparing classical and enhanced beam elements underline the advancements. The classical model exhibits non-objective behavior, while the enhanced beam shows convergence and reduced strain localization. While these improvements mitigate mesh dependency and enhance crack modeling, further work is needed to extend these advancements to FB models. Future research will explore the integration of higher-order formulations to achieve robust, objective simulations for diverse structural applications.

REFERENCES

[1] E. Åldstedt, P. G. Bergan, Nonlinear Time-Dependent Concrete-Frame Analysis, Journal of the Structural Division 104 (1978) 1077–1092.

[2] E. Spacone, F. C. Filippou, F. F. Taucer, Fibre beam–column element for non-linear analysis of R/C concrete frames: Part I: Formulation, Earthquake Engineering & Structural Dynamics 25 (1996) 711–725.

[3] L. Parente, D. Addessi, E. Spacone, A fiber beam element based on plastic and damage models for prestressed concrete structures, Engineering Structures 292 (2023).

[4] J. Sio, H. Khalid, B. A. Izzuddin, Objective modelling of reinforced concrete planar frame sub-systems under extreme loading, Structures 60 (2024) 105863.

[5] L. Parente, D. Addessi, B. Izzuddin, E. Spacone, Objectivity and Consistency of the Cracking Response of RC Beams with Conventional Models, Proceedings of the Fifteenth International Conference on Computational Structures Technology 9 (2024) 1–16.

[6] M. Petracca, L. Pelà, R. Rossi, S. Oller, G. Camata, E. Spacone, Regularization of first order computational homogenization for multiscale analysis of masonry structures, Computational mechanics 57 (2016) 257–276.

[7] D. Addessi, S. Marfia, E. Sacco, A plastic nonlocal damage model, Computer Methods in Applied Mechanics and Engineering 191 (2002) 1291–1310.

[8] D. Addessi, V. Ciampi, A regularized force-based beam element with a damage–plastic section constitutive law, International Journal for Numerical Methods in Engineering 70 (2007) 610–629.

[9] A. Ayoub, F. C. Filippou, Mixed Formulation of Nonlinear Steel-Concrete Composite Beam Element, Journal of Structural Engineering 126 (2000) 371–381.

[10] S. Limkatanyu, E. Spacone, Reinforced Concrete Frame Element with Bond Interfaces. I: Displacement-Based, Force-Based, and Mixed Formulations, Journal of Structural Engineering 128 (2002) 346–355.

[11] C.-L. Lee, F. C. Filippou, Frame Element with Mixed Formulations for Composite and RC Members with Bond Slip. I: Theory and Fixed-End Rotation, Journal of Structural Engineering 141 (2015) 4015039.

[12] P. Di Re, D. Addessi, F. C. Filippou, Mixed 3D Beam Element with Damage Plasticity for the Analysis of RC Members under Warping Torsion, Journal of Structural Engineering 144 (2018) 1–13.

[13] J. Mazars, P. Kotronis, F. Ragueneau, G. Casaux, Using multifiber beams to account for shear and torsion: Applications to concrete structural elements, Computer Methods in Applied Mechanics and Engineering 195 (2006) 7264–7281.

[14] D. Addessi, P. Di Re, G. Cimarello, Enriched beam finite element models with torsion and shear warping for the analysis of thin-walled structures, Thin-Walled Structures 159 (2021) 107259.

[15] N. H. Burns, Load-Deformation Characteristics of Beam-Column Connections of Reinforced Concrete, Ph.D. thesis, 1962.