12th International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-12
B.L.A. Pichler, Ch. Hellmich, P. Preinstorfer (Eds)

https://doi.org/10.21012/FC12.1258
MS02-1:2

A NEW APPROACH TO CONCRETE FATIGUE S-N CURVES CONSIDERING LOAD APPLICATION RATE

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Key words: Fatigue, S-N curve, load rate, dynamic strength, cycling frequency, compression.

Abstract: Most classical formulations of S-N fatigue curves in concrete assume that, when the maximum cycling is equal to the static strength of concrete, the element fails during the first cycle. However, this assumption ignores that the concrete strength depends on the load application rate. In this work, an S-N curve expression is proposed assuming that failure for N= 1 occurs when the dynamic strength of concrete is reached, which depends on the frequency and the stress range. To validate it, the new equation is fitted to several sets of experimental data. The results reveal that this curve reflects more accurately the test results, with higher R^2 coefficients than the classical S-N curves.

1 INTRODUCTION

Concrete is a quasi-brittle material, which implies that its fatigue strength depends on both the stress range ($\Delta\sigma$) and the maximum stress level (S_{max}) [1]. It is observed that, when $\Delta\sigma$ and S_{max} increase, the fatigue life decreases.

Consequently, to fully characterize the fatigue response of a given concrete, it is necessary to perform many tests by systematically varying the stress levels S_{max} and S_{min}. The results are represented in the so-called S-N curves, which relate the number of resistant cycles (N) for each combination of maximum and minimum stress levels. In a classical S-N curve, S_{max} is plotted on the ordinate axis, N is shown on the abscissa axis and the curves corresponding to certain values of S_{min} are plotted (Figure 1). Therefore, the fatigue behavior of concrete is actually defined by a family of S-N curves, and not by a single one.

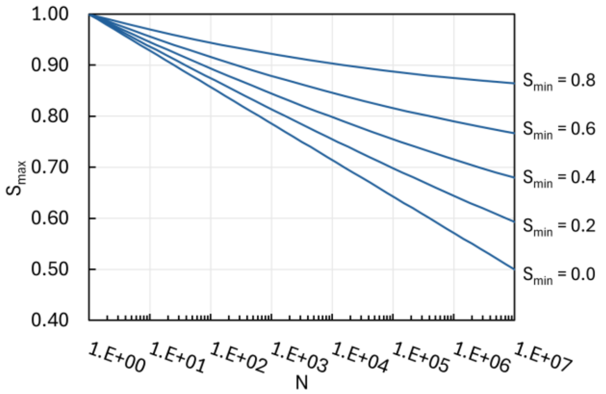


Figure 1: S-N curves of compressive fatigue in concrete according to Eurocode 2 [2].

There are different expressions of S-N curves for concrete fatigue. Due to the ease of execution of the tests, and since they are empirical formulas, most of them have been developed for compressive fatigue of plain concrete. The formulation proposed by Aas-Jackobsen in 1970 [3] was the first to consider the influence of the minimum stress level. It was developed from Whöler curves for metals, and is expressed as (Eq. (1)):

$$S_{max} = 1 - \beta \cdot (1-R) \cdot \log N \quad (1)$$

where β is an experimental fitting parameter and R is the stress ratio, calculated as S_{min}/S_{max}. Aas-Jackobsen set the value of β for compressive fatigue at 0.064. Years later, Tepfers & Kutti [4] set it at 0.0685.

Another important formulation is the one proposed by Petkovic [5], since it was adopted by the 1990 Model Code. It consists of three equations. In Eq. (2) the expression valid for fatigue lives less than 10^6 cycles is presented:

$$S_{max} = 1 - \log N / (12+16 \cdot S_{min}+8 \cdot S_{min}^2) \quad (2)$$

Regarding the S-N curves collected in the structural codes, Eqs. (3.1) and (3.2) show the formulation of the 2010 Model Code [6], valid for N below 10^8 cycles:

$$S_{max} = 1 + \log N \cdot (Y-1)/8 \quad (3.1)$$

$$Y = (0.45+1.8 \cdot S_{min})/(1+1.8 \cdot S_{min}-0.3 \cdot S_{min}^2) \quad (3.2)$$

Finally, the formula contained in Eurocode 2 [2] (Eq. (4)) is presented:

$$S_{max} = 1 - \log N \cdot (1-R)^{0.5}/14 \quad (4)$$

Figure 2 shows the S-N curves of Eqs. (1) to (4) considering a minimum stress level of 0.2. It is observed that the most conservative expressions are those of Petkovic and EC2. Moreover, in all the curves it is satisfied that, when the maximum stress level is 1, the fatigue life is 1 cycle. In other words, it is assumed that, when the fatigue cycles reach a stress equal to the static strength of concrete, the specimen breaks during the first cycle.

However, the physical sense of this hypothesis is debatable, since it ignores the impact of the dynamic component of cycling on concrete strength. It is a well-known fact that the compressive strength of concrete increases with the load application rate. During a fatigue test, loads are applied at a certain velocity, which depends on the cycling frequency, and which is generally much higher than that of static tests. Therefore, in a fatigue test with an S_{max} of 1, the fatigue life of concrete is expected to be greater than 1 cycle.

In fact, the hypothesis that, when S_{max} is 1, N is 1, would only be valid if the application rate of the cyclic loads were equivalent to that of a static test (1 MPa/s). However, for the usual frequencies of fatigue tests in concrete (0.1 to 10 Hz), the velocities are clearly higher.

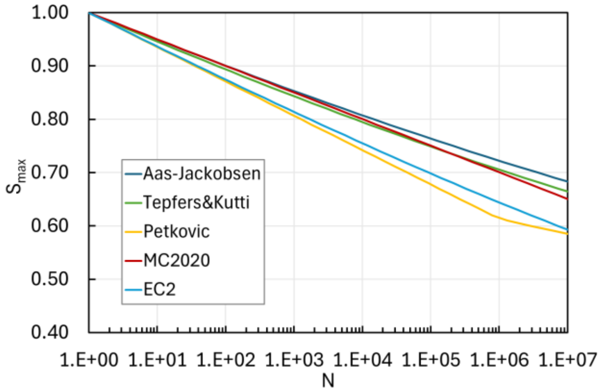


Figure 2: Comparison of S-N curves for an S_{min} of 0.2.

Consequently, it can be stated that the S-N curves in Figure 2 are conservative and unrealistic, at least for reduced fatigue lives.

In this work, an approximation of the S-N curves for compressive fatigue is presented, considering the effect of the loading rate on concrete strength. For this purpose, an expression is proposed in which the intercept is not 1, but an upper value that depends on the dynamic strength of concrete, and consequently on the frequency of the fatigue cycles. In order to validate this formulation, it is fitted to several sets of experimental data and compared with the conventional expression of the S-N curves.

2 METHODOLOGY

The following is assumed as the general expression of the S-N curves (Eq. (5)):

$$S_{max} = a - b \cdot \log N \quad (5)$$

where *a* is the intercept and *b* is the slope of the line. In all the equations listed in the previous section, *a* is 1. On the other hand, *b* is a parameter that depends on the material, on S_{min} and, in some cases, also on S_{max}. In such cases, where the parameter R is involved (Eqs. (1) and (4)), the approximation to Eq. (5) has been found to be reasonable.

This research proposes the premise that, for the fatigue life to be 1 cycle, the maximum fatigue stress must be equal to the dynamic strength of concrete. Therefore, the intercept is calculated as:

$$a = \sigma_{fat,max}/f_c = f_{c,imp}/f_c = (f_c \cdot DIF)/f_c = DIF \quad (6)$$

where $\sigma_{fat,max}$ is the maximum fatigue stress, f_c is the static compressive strength of concrete, $f_{c,imp}$ is the dynamic or impact strength and DIF is the dynamic increase factor of the concrete strength. Consequently, the intercept is equal to the DIF.

In the scientific literature there are different formulations to calculate the dynamic increase factor of the concrete compressive strength [7,8]. In this case, the widely used expression of the Model Code 2010 is applied. Approximately, the DIF is calculated as:

$$DIF = (\dot{\sigma_c}/\dot{\sigma_{c0}})^{0.014} \quad (7)$$

where $\dot{\sigma_c}$ is the load application rate in MPa/s and $\dot{\sigma_{c0}}$ is the reference rate under static conditions, set at 1 MPa/s. This expression is valid for $\dot{\sigma_c}$ less than 10^6 MPa/s, which falls within the usual range in fatigue testing.

The average loading rate in a fatigue test depends on the cycling frequency, and is calculated according to the following expression:

$$\dot{\sigma_c} = 2 \cdot f \cdot (\sigma_{fat,max} - \sigma_{fat,min}) \quad (8)$$

where f is the frequency, and $\sigma_{fat,max}$ and $\sigma_{fat,min}$ are the maximum and minimum cycling stresses, respectively.

Since $\dot{\sigma_c}$ depends on $\sigma_{fat,max}$, and thus on S_{max}, this implies that the parameter a in Eq. (5) would not be a constant. To address this, it is assumed that the loading rate used to calculate the DIF, given certain values of frequency f and minimum stress level S_{min}, is that for which the static strength f_c is reached. Therefore, $\dot{\sigma_c}$ is calculated as:

$$\dot{\sigma_c} = 2 \cdot f \cdot (f_c - \sigma_{fat,min}) = 2 \cdot f \cdot (f_c - S_{min} \cdot f_c) = 2 \cdot f \cdot f_c \cdot (1 - S_{min}) \quad (9)$$

Finally, Eqs. (10.1) and (10.2) present the proposed formulation for the S-N curves in compressive fatigue. These are referred to as load rate-adjusted S-N curves:

$$S_{max} = S_{max,0} - b \cdot \log N \quad (10.1)$$

$$S_{max,0} = [2 \cdot f \cdot f_c \cdot (1 - S_{min})]^{0.014} \quad (10.2)$$

where b is an experimental fitting parameter, the frequency f is expressed in Hz and the static compressive strength f_c is expressed in MPa.

As expected, the constant $S_{max,0}$, which defines the intercept, is always greater than 1 and increases with cycle frequency. Figure 3 plots the value of $S_{max,0}$ as a function of frequency for an f_c of 50 MPa and an S_{min} of 0.2.

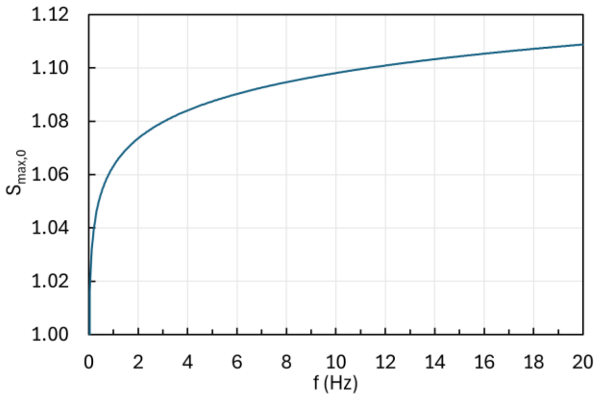


Figure 3: $S_{max,0}$ vs. frequency for an f_c of 50 MPa and an S_{min} of 0.2.

As a result, the S-N curves depend on the cycling frequency, so that the higher the f, the longer the fatigue life. Figure 4 shows the S-N curves for different frequencies, assuming an f_c of 50 MPa, an S_{min} of 0.2 and a fitting parameter b of 0.07.

It should be noted that other authors have already studied the influence of frequency on the fatigue life of concrete, and in fact there are expressions of the S-N curves that introduce frequency, such as those of Hsu [9] and Zhang [10]. The approach of the latter has certain similarities with this work. Zhang et al. [10] propose to apply a load frequency coefficient C_f that multiplies the Aas-Jackobsen S-N curve function (Eq. (1)). This coefficient depends on the frequency and a series of material parameters obtained from numerous impact and cyclic tests. In general, the S-N curves that consider frequency reproduce more faithfully the experimental results [11].

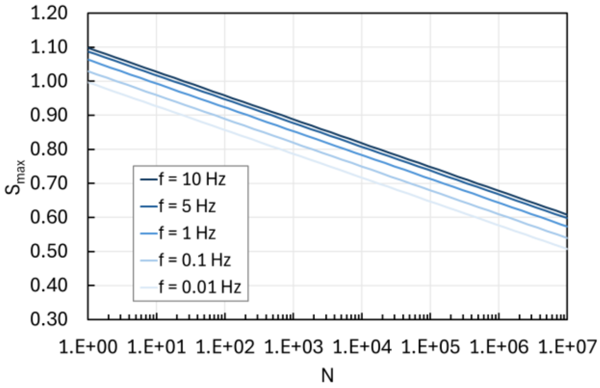


Figure 4: Load rate-adjusted S-N curves as a function of frequency (f_c = 50 MPa, S_{min} = 0.2, *b* = 0.07).

3 LITERATURE REVIEW

In order to validate the proposed S-N curve expression, it has been fitted to several sets of experimental data and then compared with the classical equations. This section describes the test campaigns that were selected. It is worth mentioning that only works on compressive fatigue of plain concrete were considered.

The research by Do et al. [12] is one of the first extensive fatigue campaigns on high-performance concrete. Two types of concrete were tested: one with an average compressive strength f_c of 95.3 MPa and another with 113.0 MPa. The cylindrical specimens (100 × 200 mm) were subjected to fatigue, maintaining a minimum stress level of 0.05 and varying the maximum stress level (0.70, 0.75, 0.85, and 0.95, respectively). The cycle frequency was 1 Hz. The tests were conducted when the concrete was between 2.5 and 3.5 months old.

Another large campaign is that of Kim & Kim [13], who tested concretes with different compressive strength levels, ranging from 26 MPa to 103 MPa. Cylindrical specimens of 100 × 200 mm were used. Fatigue tests were carried out with a constant S_{min} of 0.25 and four S_{max} values (0.75, 0.80, 0.85, and 0.95). The fatigue cycle frequency was set at 1 Hz. The average age of the specimens at the time of testing was 30 days. A drawback of this work is that, although an average of seven specimens were tested per concrete type and stress level pair, only the mean fatigue life of each series is presented in the paper. For the fittings performed, data from concretes with f_c of 52 MPa and 103 MPa were used.

More recently, Oneschkow et al. [14] conducted a very extensive study, examining the behavior of high-strength concrete depending on multiple variables (moisture, frequency, and specimen size). For the fittings performed, data from cylindrical specimens of 60 × 180 mm in a dry environment were used. Three data sets were considered: one from concrete with f_c of 96 MPa and a 1 Hz frequency, another with f_c of 108.8 MPa and a frequency of 10 Hz, and a third with f_c of 115 MPa and a frequency of 0.1 Hz. In all three cases, S_{min} was 0.05, and S_{max} ranged from 0.6 to 0.95.

Finally, in the work by Lv et al. [15], the fatigue response of self-compacting concrete with lightweight aggregates was studied. The specimens subjected to compressive fatigue were prismatic, with dimensions 70.7 × 70.7 × 210 mm. The tests were conducted at a frequency of 10 Hz, with a constant S_{min} of 0.08 and three S_{max} values (0.75, 0.80, and 0.85). For the fittings performed, only data from the series without rubber particles were used, as it is the most similar to conventional concrete. This series has a f_c of 37.3 MPa.

Table 1 identifies the 8 data series used for the validation of the S-N curves, indicating the assigned nomenclature, as well as some main characteristics of the tests. In the nomenclature, the series belonging to the same work are differentiated by the acronym indicating the type of concrete (NSC-normal strength concrete, HSC-high strength concrete, VHSC-very high strength concrete).

Table 1: Data sets used for fitting the S-N curves.

Ref.	ID	f_c (MPa)	f (Hz)
[12]	Do et al.-HSC	95.3	1
[12]	Do et al.-VHSC	113.0	1
[13]	Kim & Kim-NSC	52	1
[13]	Kim & Kim-VHSC	103	1
[14]	Oneschkow et al.-HSC	96.0	1
[14]	Oneschkow et al.-VHSC(1)	108.8	10
[14]	Oneschkow et al.-VHSC(2)	115.0	0.1
[15]	Lv et al.	37.3	10

4 VALIDATION AND DISCUSSION

To validate the formulation proposed in Eqs. (10.1) and (10.2), the fatigue life data from the series in Table 1 have been fitted using least squares to these expressions, obtaining the optimal value of the parameter *b* in each case. In other words, the *n* results with coordinates (log N_i; $S_{max,i}$) from each series were fitted to a straight line, forcing the line to pass through the point (0; $S_{max,0}$).

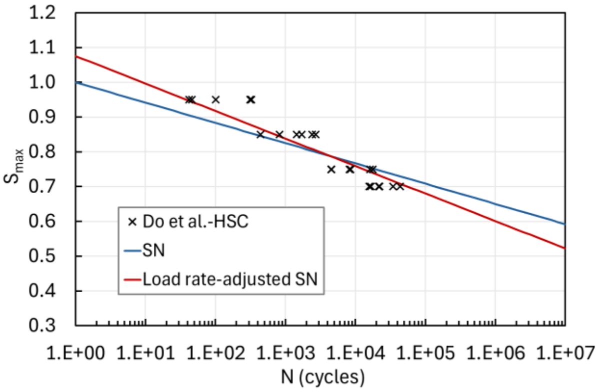
On the other hand, this process has been repeated with the general equation of the S-N curve, expressed as:

$$S_{max} = 1 - b \cdot \log N \tag{11}$$

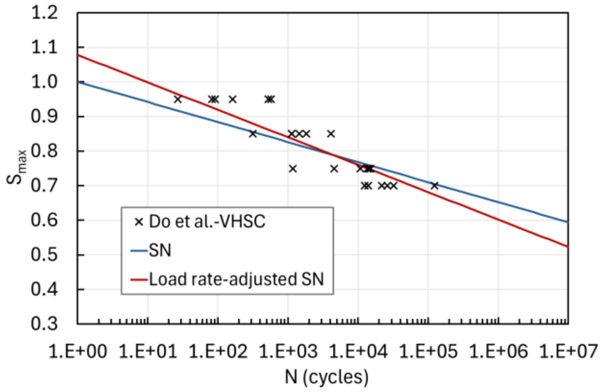
In other words, in this case, the n results with coordinates (log N_i; $S_{max,i}$) from each series were fitted to a straight line, forcing the line to pass through the point (0; 1).

To determine the goodness of fit of the two expressions, the values of the coefficient of determination R^2 have been calculated.

Figure 5 compares the fit of the 8 data sets to the two S-N curves. Table 2 contains the values of the fitting parameter *b* for the classic S-N curves, while Table 3 shows the values of *b* and $S_{max,0}$ for the load rate-adjusted S-N curves. Finally, Table 4 displays the R^2 coefficients.



(a)



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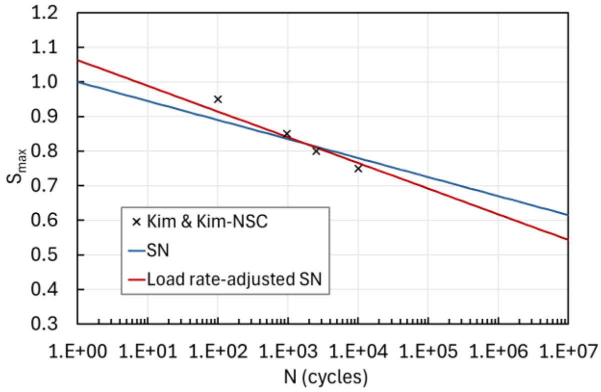

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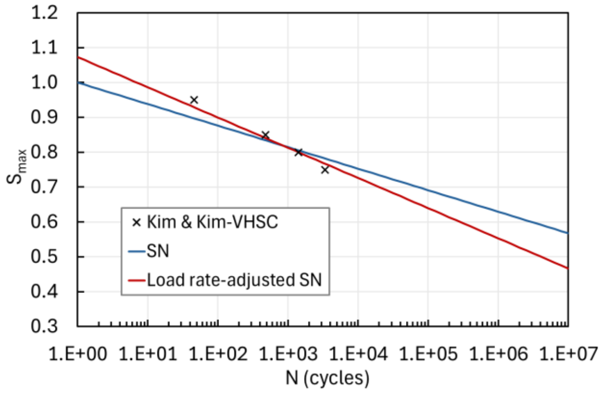

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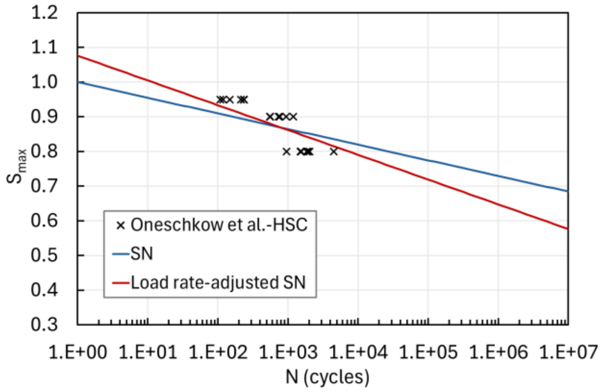

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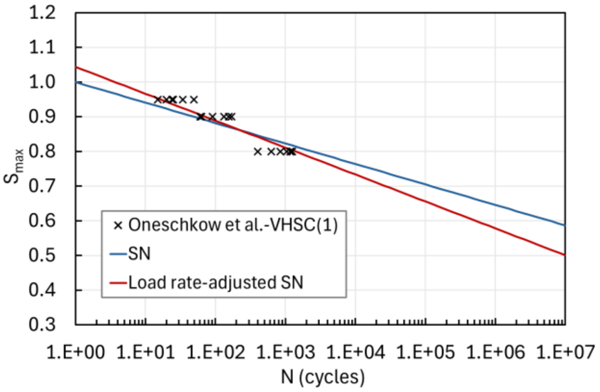

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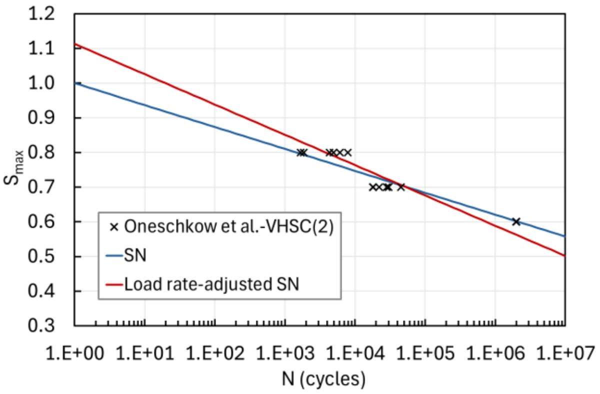

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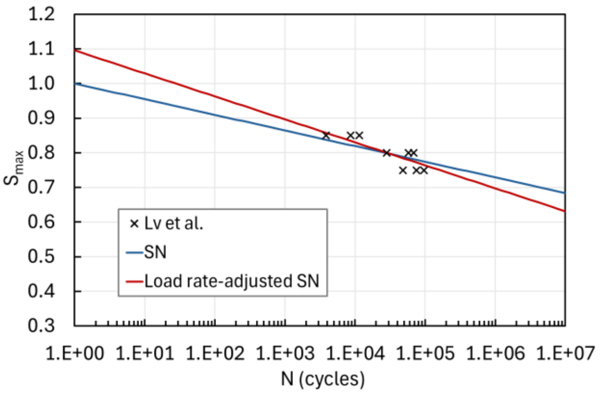

(h)

Figure 5: Comparison of fittings to experimental data for the classic S-N curve and the load rate-adjusted S-N curve.

Table 2: Fitting parameter *b* of the classic S-N curves.

ID	*b*
Do et al.-HSC	0.058
Do et al.-VHSC	0.058
Kim & Kim-NSC	0.055
Kim & Kim-VHSC	0.062
Oneschkow et al.-HSC	0.045
Oneschkow et al.-VHSC(1)	0.059
Oneschkow et al.-VHSC(2)	0.063
Lv et al.	0.045

Table 3: Fitting parameter *b* and y-intercept $S_{max,0}$ of the load rate-adjusted S-N curves.

ID	*b*	$S_{max,0}$
Do et al.-HSC	0.079	1.075
Do et al.-VHSC	0.079	1.078
Kim & Kim-NSC	0.074	1.063
Kim & Kim-VHSC	0.087	1.073
Oneschkow et al.-HSC	0.071	1.076
Oneschkow et al.-VHSC(1)	0.078	1.044
Oneschkow et al.-VHSC(2)	0.087	1.113
Lv et al.	0.066	1.096

Table 4: R^2 coefficients.

ID	SN	Load rate-adjusted SN
Do et al.-HSC	0.73	0.86
Do et al.-VHSC	0.70	0.81
Kim & Kim-NSC	0.78	0.92
Kim & Kim-VHSC	0.81	0.96
Oneschkow et al.-HSC	0.47	0.65
Oneschkow et al.-VHSC(1)	0.80	0.90
Oneschkow et al.-VHSC(2)	0.89	0.85
Lv et al.	0.63	0.74

Figure 5 shows that the load rate-adjusted S-N curve more accurately reflects the experimental results than the classic S-N curve. It is clearly observed that fitting straight lines to the point clouds of each series requires an intersection with the y-axis above 1, which is consistent with the proposal of this work. As shown in Table 3, the intercept values of the load rate-adjusted S-N curves range from 1.04 to 1.11.

Furthermore, in all cases, it is observed that the proposed S-N curves have a greater slope than the classic S-N curves; that is, the value of *b* for the former is higher, as seen when comparing Tables 2 and 3. This implies that the proposed expression is less conservative than the classic formulation for low-cycle fatigue (N $< 10^3$ cycles and $S_{max} < 0.8$, approximately). On the other hand, it is more conservative for high-cycle fatigue.

In any case, the fits are significantly better in the load rate-adjusted S-N curves, as reflected in the higher values of the R^2 coefficient (Table 4). This coefficient is, on average, 14% higher in the proposed S-N curves. Therefore, it is concluded that considering the effect of the loading rate on concrete strength leads to the development of more representative S-N curves.

5 CONCLUSIONS

Most classical expressions of S-N fatigue curves in concrete are based on the premise that when the maximum stress of the cycles equals the static compressive strength (S_{max} = 1), the element fails during the first cycle (N = 1). However, this hypothesis overlooks the fact that the concrete strength depends on the load application rate. In a fatigue test, the loads are applied at a certain rate, which depends on the frequency.

Following this reasoning, this work proposes a formulation for the S-N curves considering the premise that concrete fails at N = 1 when the maximum fatigue stress reaches the dynamic strength of concrete. This formula is a straight line relating S_{max} with log N, where the slope is an experimental fitting coefficient *b*, and the y-intercept $S_{max,0}$ is calculated by adapting the expression for the dynamic compressive strength from MC 2010 to account for the frequency of the fatigue cycles.

To validate this formulation, fits were performed to several series of experimental data, comparing them with the classic S-N curves. The results show that the load rate-adjusted S-N curves more accurately reproduce the test results, with R^2 coefficients 14% higher. Since the slope of these curves is greater, they are more conservative for high-cycle fatigue, and vice versa. In any case, it is concluded that the S-N curves that incorporate the impact of the load rate on concrete strength, and therefore frequency, more faithfully reproduce the experimental results.

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