

STEADY-STATE SUBCRITICAL FRACTURE GROWTH OF PARALLEL NATURAL CRACKS IN SHALE GOVERNED BY OSMOTIC GRADIENT AND FLUID DIFFUSION

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Abstract. Natural cracks in sedimentary rocks such as shale are potential weak paths for hydraulic fracturing to create fracture networks. The mechanism of formation of natural cracks in sedimentary rocks in the geologic past is an important problem to be understood. Why are the natural cracks roughly parallel and equidistant, and why is the crack spacing in the order of 10 cm rather than 1 cm or 100 cm? Here it is proposed that fracture mechanics must be coupled with the diffusion of pore fluid and solute ions to answer these questions. Parallel equidistant natural cracks are considered to develop in a subcritical manner driven by shear deformation and governed by the Charles-Evans law. Shear dilatancy in the fracture process zones (FPZ) induces a drop in the concentration of ions that increases the material fracture energy, and a drop in pore pressure that increases the resistance to frictional sliding. Both processes will lead to a decrease of the fracture propagation rate, and such an impact will be counteracted by the recharge of pore fluid and ions from the rock matrix to the fracture. We study the steady-state propagation and periodic cracks and derive an analytical solution of the crack spacing as a function of the properties of the rock, the solvent and solute, together with the imposed far-field deformation.

1 INTRODUCTION

The shale strata are often intersected by systems of nearly parallel natural cracks filled by mineral deposits. Their spacing is nearly uniform and on the order of 0.1 m, as confirmed by optimal fitting of the gas production rate [1]. These preexisting natural crack clusters likely play a major role in hydraulic fracturing for

gas or oil recovery (aka fracking, fraccing or frac) [2]. Therefore, understanding the mechanism of their formation in the distant geologic past is of great interest for permeability enhancement for energy extraction in shale formation.

What controls the spacing of the nearly parallel cracks in shale? Obviously not fracture

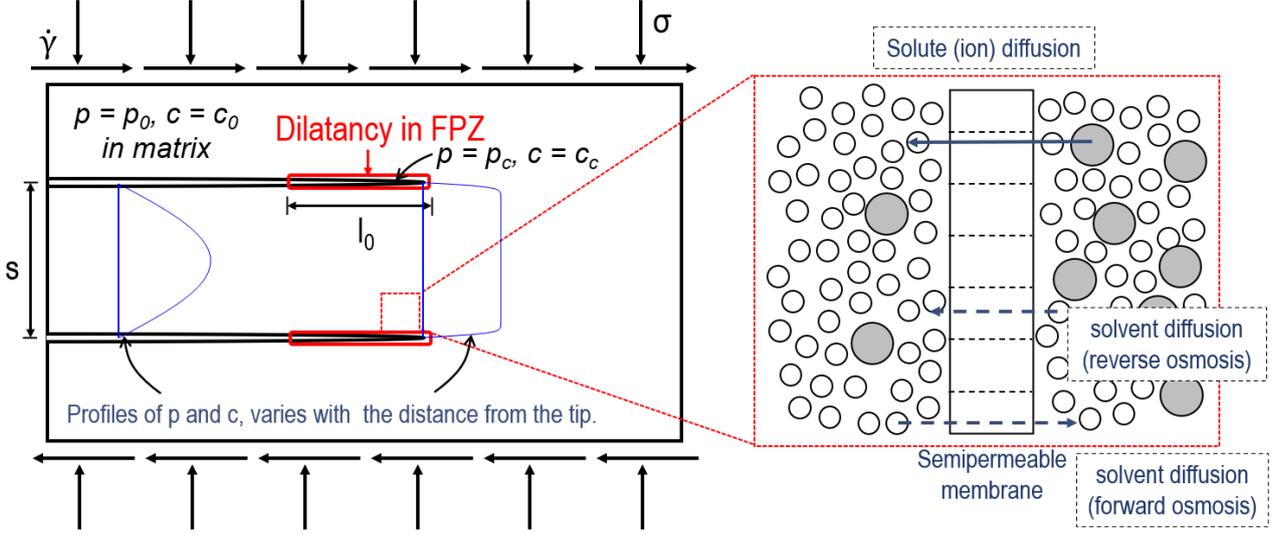


Figure 1: Left: The periodic representative domains of parallel shear cracks growing under the effect of subcritical far-field stresses and strains; Right: Exchange of solvent and solute between the FPZ and the rock matrix.

mechanics alone. The main hypothesis in this study is that the crack spacing is determined by the exchange of fluid and ions between the cracks and the rock matrix [3]. To be specific, it is proposed that the natural crack spacing is dictated by an osmotic pressure gradient which drives diffusion of solvents and solute ions, such as Na^+ and Cl^- , in the transverse direction to the crack. Since the diffusion cannot be instantaneous, one must consider slow subcritical crack propagation.

This study considers mode-II cracks with friction under a far-field confining pressure and shear strain, as the tectonic pressures impede the crack opening at the typical depths of hydraulic fracture (about 3km). When the fracture process zone (FPZ) advances, it will dilute the concentration of ions within the FPZ and reduce the magnitude of the stress-corrosion process, which slows down the crack growth. In return, the drop of ionic concentrations compared to the adjacent rock and the semipermeable nature of the tight gas shale will cause a diffusion of both the solvents and ions into the FPZ that counteracts the effect. At the same time the residual far-field strain resulting from tectonic activities in the past will trigger the crack growth. The balance between these these competing mechanisms enables a stable crack

growth with a certain spacing.

2 SUBCRITICAL GROWTH OF PARALLEL NATURAL CRACKS

We investigate the formation of nearly parallel natural cracks in shale and estimate the crack spacing. The setup of the problem can be found in Fig. 1, where we study the periodic parallel cracks that grow under the effect of subcritical far-field stresses and strains.

The evolution of parallel cracks under subcritical condition is governed by Charles-Evans law [4]

$$\dot{a} = \kappa(c_c) e^{\frac{Q}{R}(\frac{1}{T_0} - \frac{1}{T})} \left(\frac{\mathcal{G}}{\Gamma_{II}} \right)^{n(c_c)/2}, \quad (1)$$

in which the parameters κ and n depend on the ion concentration c_c in the FPZ, which can be expressed using a first-order Taylor expansion as

$$\kappa(c) e^{\frac{Q}{R}(\frac{1}{T_0} - \frac{1}{T})} = \kappa_T + \kappa'_T(c - c_0), \quad (2)$$

where

$$\kappa'_T = e^{\frac{Q}{R}(\frac{1}{T_0} - \frac{1}{T})} [\partial \kappa(c) / \partial c]_{T_0, c_0}. \quad (3)$$

$$n(c) = n_T + n'_T(c - c_0). \quad (4)$$

For most cases, we have $\kappa'_T > 0$ and $n'_T < 0$ to reflect accelerated crack growth due to chemical weathering effects. c_0 stands for the ion concentration in the rock matrix. c_c is determined by

the synergistic effect of fluid transport, ion diffusion, and dilation of the FPZ.

The mode II stress intensity factor for the parallel cracks can be expressed as [5]

$$K_{II} = (\tau_0 - \tau_r) \sqrt{s/(1 - \nu)}, \quad (5)$$

where τ_0 stands for the far-field shear stress, $\tau_r = k\sigma$ is the residual shear stress, and ν is Poisson's ratio. The crack extension force \mathcal{G} for the system can be expressed as

$$\mathcal{G} = \frac{s(\tau_0 - k\sigma)^2}{2G}, \quad (6)$$

where G is the shear modulus of the rock. Combine Eqs. (1-6), the evolution equation for the subcritical growth of the parallel crack system can be expressed as

$$\dot{a} = \kappa_T(c_c) \left(\frac{(\tau_0 - k\sigma)^2}{2G\Gamma_{II}} s \right)^{n(c_c)/2}, \quad (7)$$

and the spacing between the parallel cracks can be estimated as

$$s = \frac{2G\Gamma_{II}}{(\tau_0 - k\sigma)^2} \left(\frac{\dot{a}}{\kappa_T(c_c)} \right)^{2/n(c_c)}. \quad (8)$$

3 COUPLED FLUID FLOW, ION DIFFUSION, AND SHEAR DILANTANCY IN THE FRACTURE PROCESS ZONE

As illustrated in Fig. 1, clay minerals in shale possess negatively charged surfaces and a nano-scale pore structure, which acts as a semipermeable membrane that induces the osmotic effect for fluid and ion exchange between the FPZ and the rock matrix [6]. For fluid flow through the rock matrix, the mass conservation equation reads

$$\phi c_t \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{q}_w = 0, \quad (9)$$

where ϕ is rock porosity, p_f stands for pore fluid pressure, \mathbf{q}_w refers to the Darcy flux given as

$$\mathbf{q}_w = -\frac{\chi}{\mu} [\nabla p_f - \nu_f \nabla (2RTc)] \quad (10)$$

where χ = intrinsic permeability of the rock matrix, and μ = dynamic viscosity of fluid. ν_f = osmotic efficiency, which is about 0.96 for shale.

In terms of ion diffusion, the mass conservation equation reads

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot \mathbf{q}_s = 0, \quad (11)$$

and the diffusion equation is given as

$$\mathbf{q}_s = -(1 - \nu_f) D_e \nabla^2 c, \quad (12)$$

where D_e is the diffusion coefficient of the rock matrix.

To evaluate ion concentration in the FPZ, we solve the steady-state profiles of c and p in the coordinate system that moves with the crack tip and the conservation conditions in the fracture process zone. The governing equations can be expressed as

$$-\phi c_t \frac{\partial p_f}{\partial X} = \frac{\chi}{\mu} (\nabla^2 p_f - 2\nu_f RT \nabla^2 c) \quad (13)$$

$$-\phi \frac{\partial c}{\partial X} = (1 - \nu_f) D_e \nabla^2 c. \quad (14)$$

Mass conservation of fluid and ion in the FPZ yields

$$\delta [c_t(p_c - p_0) + \alpha \Delta \phi] + 2q_w^y t^* = 0, \quad (15)$$

$$\delta [(\phi_0 + \Delta \phi) c_c - \phi_0 c_0] + 2q_s^y t^* = 0. \quad (16)$$

In this expression, δ is the width of the fracture process zone, which is usually several times of the mineral size. $\Delta \phi$ is the increase of porosity due to dilation in the FPZ. q_w^y and q_s^y stand for the flux of solvent and ion into the FPZ. $t_a^* = l_0/\dot{a}$ is the characteristic time scale for crack propagation, and l_0 is the characteristic length scale the FPZ. As shown in [7], the characteristic slip displacements is in the order of several millimeters for shale.

By solving Eqs. (13-16), one can evaluate c_c in the FPZ as a function of the crack spacing s . Such a relationship together with Eq. (8) can determine the crack spacing. Alternatively, one can reduce this formulation to one dimensional fluid transport and ion diffusion problem between the FPZ and the adjacent rock matrix, considering that material transport mainly takes place along the transverse direction to the parallel cracks.

4 ESTIMATION OF CRACK SPACING IN BARNETT SHALE

Barnett Shale is one of the most significant shale formations in the United States, primarily valued for its vast reserves of natural gas and associated hydrocarbons. We use the proposed framework in this study to estimate the spacing of cracks in Barnett shale with a burying depth around 3km. We first estimate in-situ stress conditions. We assume the stratum is in hydrostatic condition and estimate the horizontal stress σ_h using the formula proposed by Engelder and Fischer [8]

$$\sigma_h = \left(\frac{\nu}{1-\nu} \right) \sigma_v + \alpha \left(\frac{1-2\nu}{1-\nu} \right) p_f \quad (17)$$

where α refers to the Biot coefficient, and $\sigma_v = \rho g z$ is the vertical overburn stress. The normal stress component on the cracks can then be calculated as

$$\sigma = \sigma_h - \alpha p_f. \quad (18)$$

The estimated $\sigma = 12.5\text{MPa}$. It is reported that the residual friction coefficient $k = 0.4$ for Barnett shale [9], thus $\tau_r = k\sigma = 5\text{MPa}$.

Next we estimate the far-field shear stress. The earth's crust behaves in a viscous manner when subjected to long-term tectonic shear load, and the viscosity $\eta \sim 10^{21}\text{pa}\cdot\text{s}$ for a stratum at around 3km. The creep shear loading $\dot{\gamma} \sim 10^{-14}\text{s}^{-1}$, which corresponds to 1mm/year given 3km depth. We thus estimate the far-field shear stress $\tau_0 = \eta\dot{\gamma} = 10\text{MPa}$, which is a reasonable estimation, evidenced by findings indicating that the maximum shear stress in soft rocks increases with depth around 3.8MPa/km [10].

In terms of the shear fracture energy Γ_{II} , Choo et al. [7] conducted regression analysis and found the dependence of Γ_{II} on the effective normal stress for clay and shale, given as

$$\Gamma_{II} = 65.66 \left(\frac{\sigma}{p_{\text{atm}}} \right)^{0.72}, \quad (19)$$

where $p_{\text{atm}} = 101.3\text{kPa}$ is the atmospheric pressure. With this formula, we estimate the shear

fracture energy of Barnett shale as 2.1kJ/m^2 . Insert these estimations in Eq. (8), we can express the crack spacing as

$$s = 33.6\text{cm} \left(\frac{\dot{a}}{\kappa_T(c)} \right)^{2/n(c)}. \quad (20)$$

Table 1: Parameters calibrated for Barnett shale

Parameter	Value	Unit
ϕ_0	0.05	-
ρ	2600	kg/m ³
μ	0.8	cP
χ	2.5	μD
c_t	1.25	GPa ⁻¹
ν_f	0.96	-
c_0	20	kg/m ³
D_e	$10^{-12} - 10^{-18}$	m ² /s
δ	0.2	mm
$\Delta\phi$	0.05	-
k	0.4	-
η	10^{21}	Pa·s
G	2	GPa
ν	0.2	-
α	0.9	-
$\dot{\gamma}$	10^{-14}	s ⁻¹
κ_T	1	mm/s
κ'_T	0.095	(mm/s)/(kg/m ³)
n_T	60	-
n'_T	-4	(kg/m ³) ⁻¹
\dot{a}	1	mm/year
l_0	5	mm

The concentration of ions in the FPZ can be calculated based on the steady-state transverse profile of c between adjacent crack tips. The shape of the profile is determined by two characteristic time scales, one for crack propagation, $t_a^* = l_0/\dot{a}$, and the other for ion diffusion between cracks, $t_i^* = \phi\delta^2/((1-\nu_f)D_e)$. Here δ rather than s is set as the characteristic length scale, as it controls the storage of the FPZ and thus the time it takes to recharge the FPZ to the level in the rock matrix. The concentration of ions in the FPZ can be calculated based on the relations of the two time scales:

(I) when $t_a^* \gg t_i^*$, the propagation of the cracks is slow and the decrease of ion con-

centration in FPZ due to dilation can be fully recharged by the diffusion process. In this case, $c_c = c_0$ and we can estimate the crack spacing $s_0 = 18.9\text{cm}$.

(II) when $t_a^* \ll t_i^*$, the cracks propagate fast and the FPZ is barely recharged by any ions. In this case, the ion concentration in the FPZ can be estimated as $c_c = \phi_0 c_0 / (\phi_0 + \Delta\phi)$, which decreases from the concentration of ions in the matrix and will suppress the growth of the cracks. $s_i = 25.3\text{cm}$ for this case.

(III) when t_a^* and t_i^* are comparable, ion concentration in the FPZ should be computed via the coupled fluid flow and ion diffusion processes. Considering that $\delta \ll s$, only the solution before the penetration front of ion diffusion reaches the centerline between two cracks is meaningful, for which c has not been fully recharged to c_0 . Such a scenario corresponds to the Phase I solution in [11, 12], where the problem can be simplified to a one-sided diffusion problem into a semi-infinite domain. For this case, we conducted a series of simulations by varying the ratio t_i^*/t_a^* . The estimated ion concentration in the FPZ as well as crack spacing are demonstrated in Fig. 2. The results converge to the two asymptotic values predicted in case (I) and (II), and we note that with the increase of the ratio t_i^*/t_a^* , the ion concentration in the FPZ decreases while the crack spacing increases.

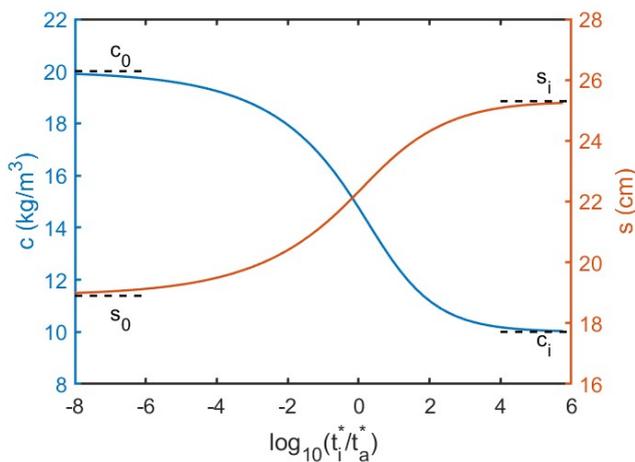


Figure 2: Estimation of the ion concentration c in the FPZ and crack spacing s given different t_i^*/t_a^* ratio.

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