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COMPUTATIONALLY EFFICIENT, DISCRETE MECHANICAL MODELS OF GRID-REINFORCED CEMENT-BASED COMPOSITES

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Abstract. Thin-walled structures made of concrete, or other forms of cement-based composites, are common within the civil infrastructure. In many situations, such structures experience out-of-plane loading, which can lead to various forms of distributed cracking depending on the boundary conditions. Discrete mechanical models are appropriate for simulating such cases of distributed fracture. However, their applications toward modeling the out-of-plane behavior of thin-walled structures are few. One main difficulty involves the large computational expense associated with three-dimensional discretizations of the structure, which are typically needed to capture crack propagation through the wall thickness. In this research, an extension of the Voronoi-cell lattice model (VCLM) is proposed to simulate the behavior of planar structural elements subjected to out-of-plane loading. Based on a two-dimensional network of nodes, a layered assembly of the element cross-sections provides a three-dimensional description of section behavior. With sufficiently fine discretization of the planar structure, this layered VCLM is shown to be elastically uniform for combined membrane and flexural loadings. Compared to corresponding three-dimensional discretizations, computational expense is greatly reduced, thus extending the range of modeling applications. Other capabilities of the layered VCLM, and the consequences of mesh resolution of the lattice structures, are demonstrated through elastic stress analysis and fracture analyses of grid-reinforced cement-based composites.

1 INTRODUCTION

Thin-walled cement-based composites are being developed for a variety of applications, including structural panels, free-form architectural units, and retrofit technologies. The reduction in materials usage associated with thin-walled elements may also have environmental benefits. At another scale of observation, reinforced concrete slab/wall elements can viewed as a form of thin-section construction. For in-plane loading (e.g., in form of uniaxial tension or diagonal tension/compression), such structures can be analyzed using planar models [6]. For cases of out-of-plane loading, however, analyses are complicated by the need to track crack movement (or other forms of nonlinearity) within the thin section. Models based on three dimensional discretizations of the material domain can provide accurate results [23], yet are computationally expensive even for moderately sized structural members.

This research employs layered Voronoi-cell lattice models (L-VCLM) to simulate the behavior of such thin-walled/thin-section members under out-of-plane loading. The VCLM belongs to the family of particle-based lattice models, which are applicable to modeling cracking in concrete materials and structures [4, 5, 8, 11, 14, 17, 25]. As distinguishing feature of the L-VCLM, it is based on a two-dimensional network of nodes that defines the central plane of the thin-section member. Relative to three-dimensional discretizations of the material domain, the planar model reduces computational expense. The VCLM, and its layered counterpart considered herein, accommodates various schemes for representing internal reinforcement, including the grid-type reinforcement.

Basic properties of the L-VCLM are first demonstrated through elastic stress analyses. When considering out-of-plane loading, it is found that model behavior depends on the mesh resolution of the planar network of nodes. Reducing nodal spacing, while keeping the thickness dimension constant, leads to elastically uniform representations of the member. Thereafter, simulations of fracture in ferrocement panels demonstrate an acute sensitivity to reinforcement positioning within the thin section.

2 MODEL FORMULATION

2.1 Domain discretization

The process of constructing the layered elements can be done in different ways. After partitioning the central plane of the structure into Voronoi cells, as shown in Fig. 1(a), options include

- partitioning each Voronoi facet into m layers of uniform height. Each layer then becomes the facet of a conventional rigid-body-spring element connected on the mid-plane nodes. This process is akin to the fiber-based discretization of cross-sections of uni-directional elements [2].

- extrusion of the nodal pattern in the thickness direction. For example, for each node in Fig. 1(a), an additional $m-1$ nodes are placed in the thickness direction, uniformly spaced above and below the central plane as shown in Fig. 1(b). The 3D network of nodes is tessellated, resulting in m identical horizontal layers of elements connecting through the same partitioning of the vertical facets. The connectivities of those horizontal elements are then referenced to the mid-plane nodes, which produces the desired layered elements (Fig. 1(c)). The extraneous nodes used for the 3D tessellation have to be removed or constrained.

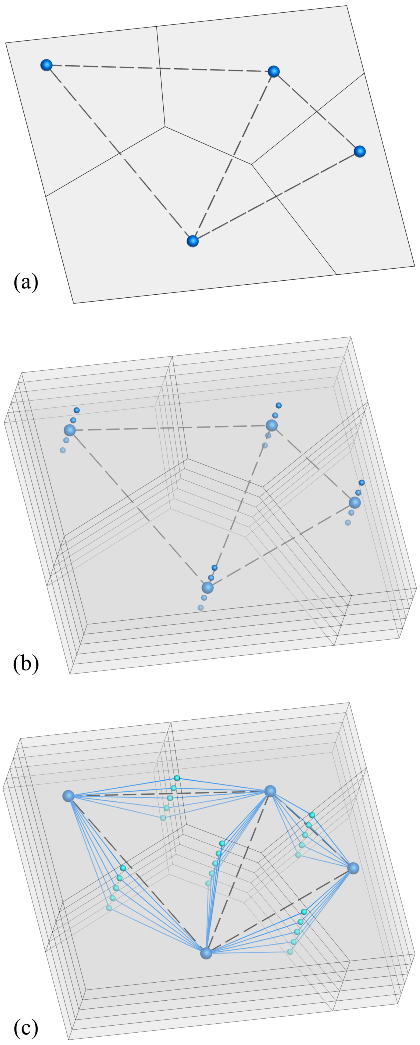

Figure 1: Layered Voronoi-cell lattice model: (a) Voronoi partitioning of mid-plane nodes; (b) 3D tessellation based on an extruded set of nodes (c) layered element construction

The second of these two options is used herein. The 3D discretization produced at the intermediate stage can be used for 3D VCLM analyses of the structure, which serves as a comparator for the L-VCLM analyses. The presence of the 3D discretization also facilitates plotting of the deformed structure.

2.2 Element formulation

The elements within the VCLM utilize the rigid-body-spring concept of Kawai [13]. Each element is composed of a zero-size spring set, positioned at the centroid of the corresponding Voronoi facet, and connected to the Voronoi cell nodes via kinematic constraints. The axial springs are aligned with a coordinate system, defined local to the Voronoi facet, and are assigned the stiffness coefficients:

$$k_s = k_t = \alpha k_n = \alpha E \frac{A_{ij}}{h_{ij}} \tag{1}$$

where E is the elastic modulus of the matrix material; A_{ij} is the area of the Voronoi facet; and h_{ij} is the distance between nodes i and j. Subscripts n, s and t designate the facet-normal and two facet-tangential directions, respectively. The zero-size spring set also includes three rotational springs, one associated with each local coordinate axis. The stiffness coefficients of the rotational springs are given elsewhere [1].

Parameter α is a factor related to macroscopic Poisson ratio ν [9]. For the case of $\alpha = 1$, the lattice model is elastically uniform with respect to in-plane loading, albeit with $\nu = 0$. Through the iterative introduction of auxiliary stresses [1], the stress fields can be accurately represented, in both local and global senses, for arbitrary settings of the elastic constants, E and ν. Heterogeneity can be introduced into such elastically uniform models in a controlled manner [24]. As will be shown herein, however, spurious fluctuations in stress may appear when the models are subjected to flexure loading.

To account for fracture, the spring set stiffness are modified isotropically according to a damage model, in conjunction with a softening relation that preserves fracture energy with respect to changes in element size [3]. A controlled number of fracture events are allowed per computational cycle, as done in other lattice modeling approaches [10, 22]. The strength envelope, which governs tensile softening and nonlinearity in compression, is defined in terms of normal and tangential shear stresses acting on the element facets. The specific form of the envelope is taken from Cusatis et al. [8].

Apart from the layered structure of the L-VCLM, the elements themselves are no different from those used in past studies, e.g., [18]. As such, the L-VCLM can accommodate methods for simulating creep [15] or other phenomena [7] that are compatible with the planar section constraint imposed by the layered structure.

2.3 Modeling of reinforcement

Various forms of reinforcement, such as short or continuous fibers, can be placed within the VCLM [12, 14, 19]. Those forms of reinforcement can be represented

- discretely, such that the reinforcement possesses computational degrees of freedom. The reinforcement nodes are joined to the Voronoi cell nodes, representing the cement-based matrix, via conventional bond link elements; or
- semi-discretely, where the pre- and post-cracking actions of the reinforcement are kinematically constrained to the degrees of freedom of the local Voronoi cells.

The first of these two options is used for modeling the wire-grid reinforcement in ferrocement panels, as presented in Section 4.

3 ELASTIC STRESS ANALYSIS

As a general example, a square panel is subjected to combined uniform loading of in-plane compression and bending moment (Fig. 2). Three different discretizations are considered, as shown in Fig. 3, in terms of average element aspect ratio $\xi = h/w$, where h is the average element size and w is the panel thickness. The number of layers $m = 11$ remains consistent among the three cases.

Loads are imposed via rigid platens located at the slab edges. The vectorial stress components of each spring set, evaluated at the center of each layer, are obtained and visualized as scatters in the $\sigma - \tau$ plane, where each point corresponds to a pair of normalized normal stress σ and shear stress τ.

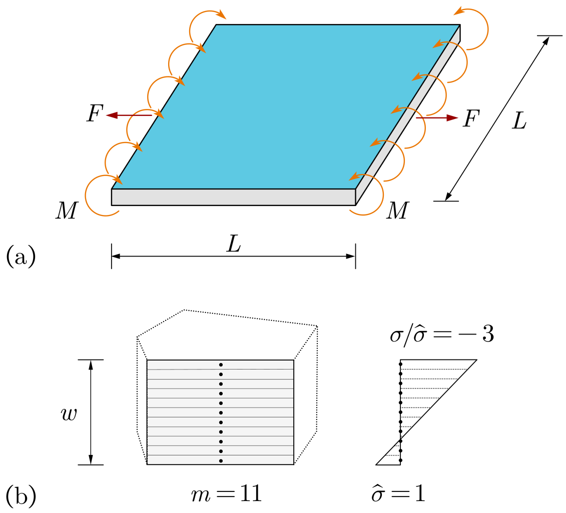

Figure 2: Modeling of planar structure under combined membrane and flexural loading: (a) structural layout and loading configuration; and (b) an element cross section and target stress distribution in the thickness direction.

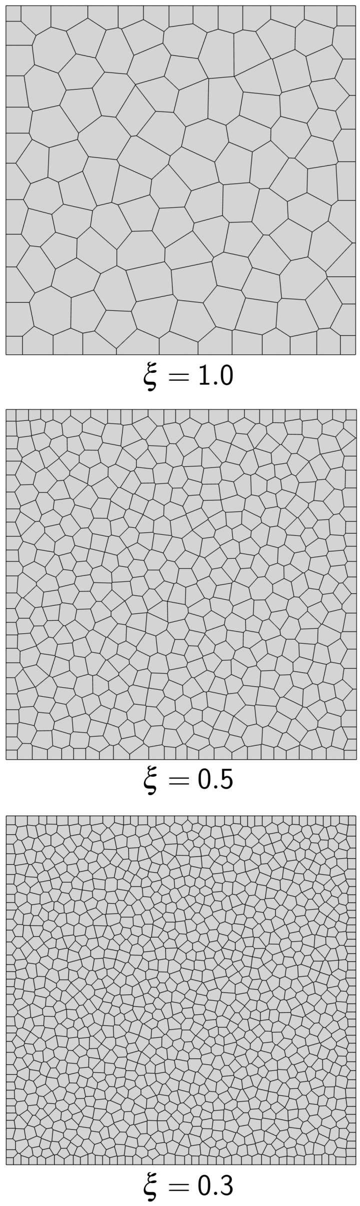

Figure 3: Mid-plane domain discretization based on differing average element aspect ratios ξ

The semi-circular patterns in Fig. 4 generally align with the predictions of Bernoulli beam theory in an averaged sense; however, some degree of randomness in the stress distribution persists. Energy dissipation through shear deformation occurs as shear springs are invoked, triggered by the differences of tangential sliding between the neighboring elements. Therefore, the outer layers undergo larger deflections, causing increased stress fluctuations. The rigid-body nature of lattice elements and the random geometry of Voronoi cells contribute to the stress deviations, which are significantly reduced with mesh refinement, as shown in Fig. 4(c). Therefore, an elastic uniform stress state of a planar structure under bending can be achieved, given the element size to be relatively small.

For elastic deformation and stress analysis of homogeneous plates, the results presented in this section provide deeper insight into the accuracy of planar rigid-body-spring networks under out-of-plane loading. The primary benefits of thickness subdivision become evident when modeling nonlinear material behavior, such as fracture, as detailed in the following discussion.

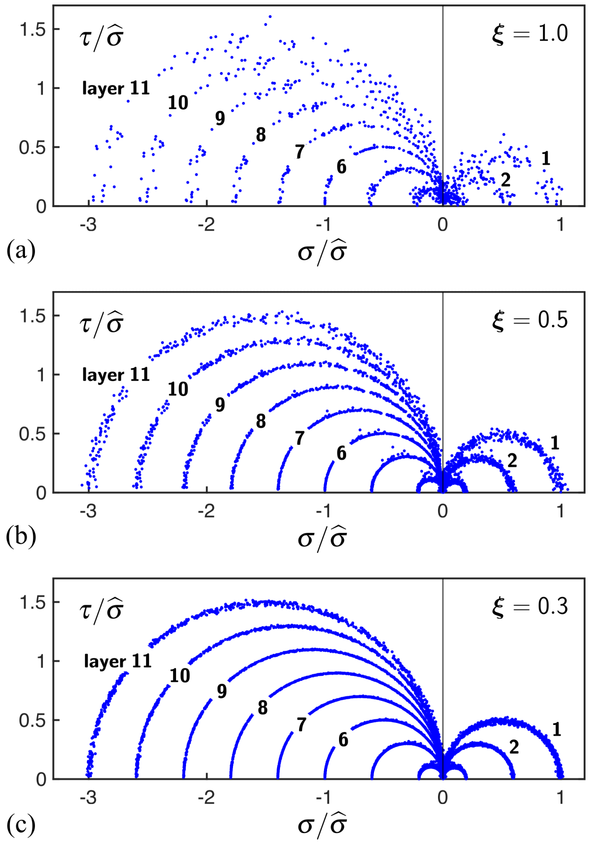

Figure 4: Elastic stresses within the planar model under various mesh resolutions: (a) $\xi = 1.0$; (b) $\xi = 0.5$; and (c) $\xi = 0.3$

4 FRACTURE ANALYSIS OF GRID-REINFORCED COMPOSITES

To further demonstrate capabilities of the L-VCLM, simulations of fracture within ferrocement panels are presented.

4.1 Model configuration

The models are patterned after a series of physical test specimens [20]. In total, 32 panels were tested in third-point loading under displacement control. Each panel had dimensions of $L \times b \times w = 305 \times 76 \times 12.7$ mm^3 and was reinforced with two (staggered) layers of welded wire fabric. The fabric was composed of 19-guage galvanized steel wire spaced at 12.7 mm in each direction.

During the experimental campaign, it was not possible to achieve uniform positioning of the layers of wire fabric within the panel formwork. The wire fabric was supplied in roll form, such that it was difficult to flatten. To mimic such conditions in the modeling framework, the layers of wire fabric were positioned using instances of a spatially correlated random field [16]. In particular, the locations of the wire fabric nodes within the thickness direction were set by controlling the mean value, variance, and correlation length of the random field. One such placement of a fabric layer is illustrated in Fig. 5.

In the experimental program, loading was applied under displacement control to a crossbar that distributed force uniformly to the third-point locations of the panel specimen. This setup was imitated by including an additional, quasi-rigid layer of lattice elements above either the VCLM or L-VCLM representations of the panel specimens.

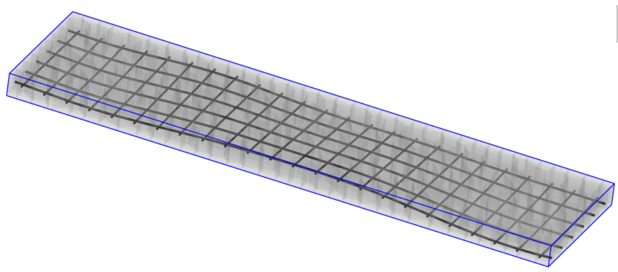

Figure 5: Random positioning of a reinforcement layer within the panel specimen model

4.2 Analysis results

Figure 6 compares load versus mid-span displacement curves provided by the L-VCLM and 3D VCLM simulations for one random positioning of the fabric layers. The elastic loading branches of each curve are practically identical. Within the experimental program, the loading branches exhibited higher initial compliance associated with settling of the specimens within the loading device. Figure 6 shows the elastic stiffness of the test specimens measured after the period of higher compliance. Thereafter, each simulation result exhibits at least two prominent drops in load resistance, associated with the formation of well-defined cracks across the panel width, as shown in Fig. 7. The first two cracks follow the same pathways in both simulations. The third crack differs but follows the same general trajectory. Most of the observed cracking in the experimental program ran perpendicular to the span direction; lightly skew cracking, as seen in Figs. 7(b) and (c), was observed in some cases. For the physical test results shown in Fig. 7(c), the wire-grid reinforcement is distant from the tensile face of the specimen, such that only two cracks formed.

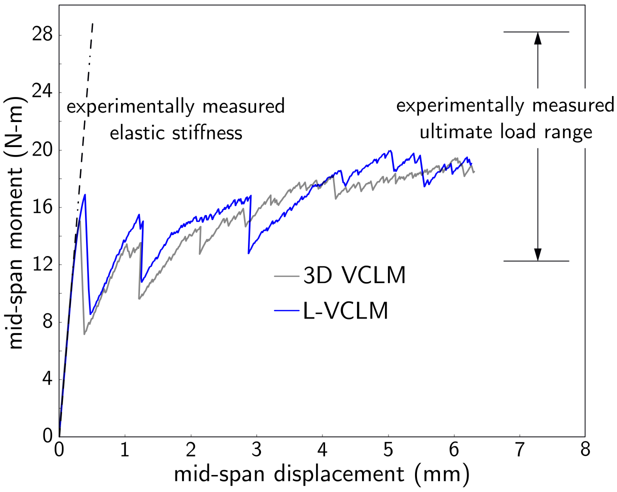

Figure 6: Ferrocement panel response to third-point loading

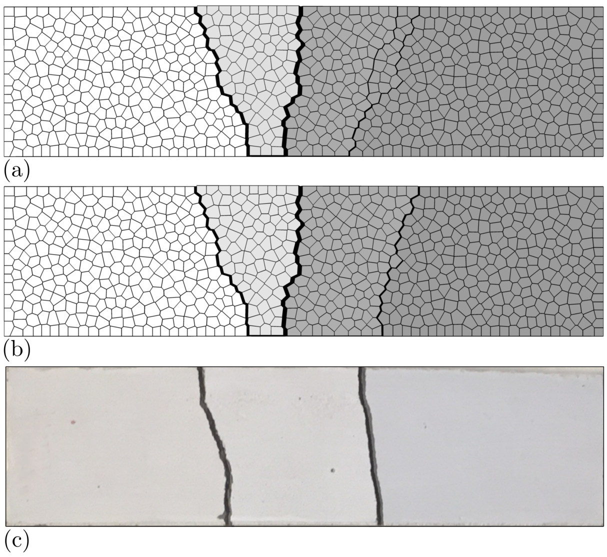

Figure 7: Crack patterns obtained from: (a) 3D VCLM analysis; (b) L-VCLM analysis; and (c) physical test specimen with similar wire grid positioning

The simulated load levels fit within the observed experimental scatter of peak loads, which ranged between 12.2 and 28.2 N·m. Both peak load and the number of cracks that form within the constant moment region depend on the distance of the fabric layers from the tensile face of the panel.

Additional comparisons with the experimental data are forthcoming. In particular, a range of specimen behaviors can be simulated depending on

- the random realizations of wire fabric positioning within the cement-based matrix; and
- the assignments of bond-link properties, accounting for the anchoring actions of the transverse wires of the fabrics.

Preliminary calculations suggest that the simulated range of behaviors is in good general agreement with the experimental results.

When comparing the 3D VCLM and the proposed L-VCLM, the time required to factorize the system of equations is a reasonable measure of computational efficiency. Factorizations were performed using a sparse-matrix solution technique [21]. For the ferrocement panel simulations presented herein, the L-VCLM is faster by a factor of about 25. Even greater improvements in efficiency are expected when modeling larger, more practical structures.

5 CONCLUSIONS

This research involves the development of computationally efficient methods for simulating the behavior of planar structures for out-of-plane loading. Voronoi-cell lattice models (VCLM) are used for this purpose. Efficiency is achieved through a layering of the cross-sections of the lattice elements, such that a two-dimensional lattice can be employed. Some capabilities of the layered Voronoi-cell lattice model (L-VCLM) are demonstrated through elastic stress analysis and fracture analysis of ferrocement panels. The following conclusions can be made:

- The L-VCLM can be rendered elastically uniform under combined membrane and bending actions, provided the length of the elements is small relative to the thickness dimension, i.e., $\xi << 1$. For coarser discretizations, roughly $\xi > 0.5$, the stress field exhibits significant scatter about the theoretical values. This spurious form of heterogeneity reflects local differences in the geometry of the lattice.

- The L-VCLM significantly reduces the computational cost relative to a comparable 3D VCLM. For the ferrocement panel simulations considered herein, the time for each factorization of the system equations was reduced by a factor of about 25. When considering larger and more practical structures, even better improvements in efficiency are anticipated.

- The gains in computational efficiency from the L-VCLM are obtained without appreciable loss in accuracy. Simulations of the third-point bending tests indicate that the L-VCLM produces acceptably accurate results, using 3D VCLM analysis results for comparison. In particular, the initial loading branches and early cracking behavior of the two simulations are practically identical. This correspondence between the analysis results suggests that the grid-type reinforcement is capably represented by the L-VCLM.

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