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A LINEARIZED TOUGHNESS MODEL FOR FIBER REINFORCED CONCRETE USING MULTISCALE APPROACH

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Abstract: Reinforced concrete structures such as offshore supporting structures, bridge decks, road surfaces, machine foundations, etc. are subjected to fatigue loading throughout their service life. Understanding the fatigue behavior of these structures is crucial for ensuring their durability and safety. Fiber-reinforced concrete contains fibrous materials as reinforcement, improving the mechanical properties. Due to the heterogeneity of concrete and variable characteristics of fatigue loading, a multiscale approach is best suited to predict the fatigue life of reinforced concrete. In this study, a linear elastic fracture mechanics (LEFM) based method is attempted to predict the fatigue life of fiber-reinforced concrete using a multiscale approach by modifying the definition of Stress intensity factor (SIF). The nonlinear behavior of the fracture process zone is captured by considering the various toughening mechanisms such as aggregate bridging, fiber bridging and microcracking occuring at meso and micro length scales. The SIF is modified by relating the crack opening displacement at the macroscale and the microscale. This modified SIF based on LEFM approach includes the contributions from bridging stress which occurs due to the bridging of aggregate and fiber at the mesoscale and microcracking occurring at microscale. The modified SIF is validated by computing the fracture energy using available experimental data from literature. Finally, a parametric study is conducted to determine the influence of various parameters on the modified SIF.

1 INTRODUCTION

Fatigue loading or cyclic loading refers to the continuous and repetitive application of load on a material or on a structure that can lead to the degradation of the material and ultimate collapse of the structure even before attaining static load capacity of the structure. Concrete structures like bridges, road surfaces, machine foundations etc. are subjected to fatigue loading throughout their service life. So, understanding fatigue behaviour of structures is of paramount importance to ensure the safety, integrity and durability of the structure. Fiber reinforced concrete is a type of concrete that incorporates fibrous materials as reinforcing elements which can enhance the structural integrity, durability and mitigate detrimental effects like brittleness cracking etc. Fibre reinforced concrete consists of different constituents and their interfaces - cement matrix, aggregates, fibers and weak interfacial zones at aggregate-cement paste interface and fiber-cement paste interface. These heterogenities present in concrete results in its non-linear behaviour whose influences can only be studied by understanding the behaviour of constituents and their interactions at different scales.

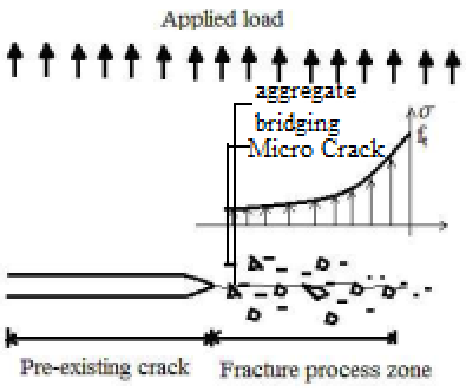

Figure 1: Toughening mechanisms in fracture process zone including microcracking, and aggregate bridging, [14]

Intermediate inelastic zone between cracked portion and uncracked portion of concrete is called fracture process zone which is influenced by various toughening mechanisms like aggregate bridging, microcracking, fiber bridging, crack deflection, crack branching etc, as shown in figure 1, which increase the complexity of the zone. These toughening mechanisms offer resistance to the propagation of cracks. The major toughening mechanisms are aggregate bridging, reinforcement bridging and microcracking in which aggregate bridging and reinforcement bridging falls under macroscopic mechanism where as microcracking falls under microscopic mechanism. Stress intensity factor describes the state of stress in the close vicinity of the crack tip. Material can withstand crack tip stresses up to a critical value of stress intensity factor, beyond which the crack propagates rapidly and leads to failure of the structure. Stress intensity factor plays a crucial role in understanding and predicting the behaviour of materials under fatigue loading. It can be used to estimate the fatigue life of structures and thereby enabling preventive measures to avoid unexpected fatigue failures. Modified stress intensity factor is formulated for fibre reinforced concrete through a multiscale approach. A sensitivity analysis is also conducted to determine the sensitivity of each of the parameters that has been considered in the model.

2 DETERMINATION OF MODIFIED STRESS INTENSITY FACTOR

Stress intensity factor is formulated by considering crack opening displacement at microscale and macroscale.

2.1 Crack opening displacement at macroscale

General equation for crack mouth opening displacement at macroscale of a section determined by conducting three point bending test is given by the equation [16].

$$\delta_{macro} = \frac{4c\sigma}{E} g_2(\frac{c}{D}) g_3(\frac{c}{D}, \frac{x}{c}) \quad (1)$$

σ represents the effective stress on the crack surface at macroscale and is found out by taking the difference between stress due to external applied load and bridging stress that tends to close the crack. For a fibre reinforced concrete, bridging stress is attributed to the bridging action provided by both aggregate and reinforcement.:

$$\sigma = \sigma_a - (\sigma_{ba} + \sigma_{bf}) \quad (2)$$

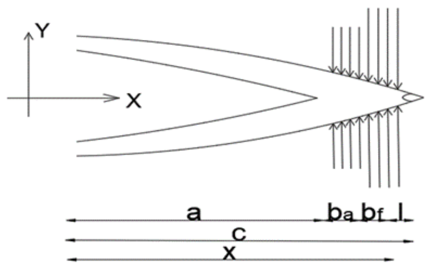

Figure 2: Representation of crack mechanism in fiber reinforced concrete

where σ_a, σ_{ba} and σ_{bf} are stress due to external load, bridging stress due to aggregate bridging and bridging stress due to reinforcement bridging respectively. c is the total crack length which constitutes of macrocrack length (a), microcrack length (l), length of aggregate bridging zone (b_a) and length of fibre bridging zone (b_f). g_2 and g_3 are the geometric factors [15].

$$c = a + b_a + b_f + l \tag{3}$$

2.2 Crack opening displacement at microscale

Concrete structures contain pre-existing internal flaws arising from water filled pores, air voids acquired during casting and shrinkage cracks due to improper curing process. These internal flaws give rise to microcracks at the weak interfaces even prior to the application of mechanical load. Under fatigue loading, these microcracks grow to a certain length and attains a critical value at peak load. Subsequently, it coalesces with the existing macrocracks, contributing to the overall increase in crack length. Crack opening displacement at microscale is derived by Simon and Kishen [12] and is used in this study.

Crack opening displacement at microscale is given by the equation,

$$\delta_{micro} = \frac{Ar^{\lambda 1} K_1^{micro}}{\mu_{micro}(1+\nu_{micro})\sqrt{2\pi}} \tag{4}$$

where K_1^{micro} is the stress intensity factor at microcrack tip and A is the term which depends on parameters like eigenvalue, microcrack angle, Poisson's ratio at microscale, shear modulus at microscale [12].

2.3 Formulation of modified stress intensity factor

Stress intensity factor is formulated by equating the crack opening displacement derived at microscale and macroscale at a section distance x as given in figure 2.

$$\delta_{macro|x=c-\frac{l}{2}} = \delta_{micro|r=\frac{l}{2}} \tag{5}$$

Substituting the equations for crack opening displacement at macroscale and microscale, modified stress intensity factor, $K_{Imacro}{}^{micro}$ can be found out as.

$$K_{Imacro}^{micro} = K_I \frac{2\sqrt{(2c)}(1-\frac{\sigma_{ba}+\sigma bf}{\sigma_a})g_3(\frac{c}{D},\frac{c-\frac{l}{2}}{c})\mu_{micro}(1+\nu_{micro})}{Ar^{\lambda 1}E_{macro}} \tag{6}$$

$$= \alpha\ K_I$$

where K_I is the mode 1 stress intensity factor and is given by the equation [12]

$$K_I = \sigma_a \sqrt{(\pi c)} g_2 \frac{c}{D} \tag{7}$$

By assuming a sharp crack, the microcrack angle and eigenvalue in equation 6 attains a value of π and 0.5 respectively, for ensuring crack tip singularity. The modified stress intensity factor can be simplified as

$$K_{Imacro}^{micro} = K_I 2\sqrt{\frac{c}{l}}(1-\frac{\sigma_{ba}+\sigma_{bf}}{\sigma_a})g_3(\frac{c}{D},\frac{c-\frac{l}{2}}{c})\frac{E_{micro}}{E_{macro}} \tag{8}$$

Fracture energy can be computed from stress intensity factor and is given by,

$$G_f = \frac{(K_{Imacro}^{micro})^2}{E} \tag{9}$$

$$G_f = \frac{(K_I 2\sqrt{\frac{c}{l}}(1-\frac{\sigma_{ba}+\sigma_{bf}}{\sigma_a})g_3(\frac{c}{D},\frac{c-\frac{l}{2}}{c})\frac{E_{micro}}{E_{macro}})^2}{E} \tag{10}$$

where, E is the modulus of elasticity of concrete.

3 VALIDATION OF THE MODIFIED FRACTURE ENERGY

The model proposed to determine stress intensity factor of fiber reinforced concrete as given in equation (8) is validated using experimental data from literatures. The experimental study conducted by Bhosale et al. [2] and Bencardinao et al. [3] is used in the present study for validation. The experimental program was carried out on fiber reinforced concrete beams under three point bending. Bhosale et al. [2] had conducted studies on fiber reinforced concrete with three different fiber types – steel fiber, synthetic polyolefin and the combination of steel and polyolefin at different fiber percentage of 0.5%, 0.75% and 1%.

Table 1: Material and geometric properties of specimen

Specimen designation	Peak load (kN)	Fiber aspect ratio	Max. aggregate size(mm)	Comp. Strength (N/mm2)	Fracture energy (N/mm)
SF1%-1	18.45	50	20	65.36	2.38
SF0.75%	16.61	50	20	63.09	1.85
SF0.5%	16.51	50	20	65.66	1.49
PF1%	13.24	100	20	51.77	1.34
PF0.75%	14.73	100	20	49.67	0.84
PF0.5%	15.89	100	20	58.79	0.60
SF1%-2	30.80	80	15	78.20	3.90

SF-Steel Fiber, PF-Polyolefin Fiber

Bencardino et al.[3] conducted studies on fiber reinforced concrete with two different fiber types- steel fiber and polypropylene fiber at different fiber percentage of 1% and 2%. The validation of stress intensity factor is carried out by validating the fracture energy which is determined from stress intensity factor. The interfacial properties of concrete, microcrack length, bridging zone length and bridging zone stress is determined for the computation of fracture energy. The material and geometrical properties of specimens are depicted in Table 1.

The empirical equation for the determination of modulus of elasticity of fiber reinforced concrete (E) with fiber content up to 1.5% is given by Mansur et al. [2].The equation for tensile strength of fiber reinforced concrete, σ_t is given by Chiranjeevi et al. [4] and is used in the study. Fiber reinforced concrete constitutes of both aggregate-mortar interface and fiber-mortar interface.

Relation between modulus of elasticity of concrete and mortar is found out by the equation

$$\frac{E}{E_m} = \frac{V_f(m)E_m + (1+V_f(ca))E_{ca}}{(1+V_f(ca))E_m + V_f(m)E_{ca}} \quad (11)$$

Interfacial elastic modulus at the interface between cement mortar and fiber can be written by replacing modulus of elasticity and volume of coarse aggregate by the properties of fiber, and is obtained as

$$\frac{E}{E_m} = \frac{V_f(m)E_m + (1+V_f(f))E_f}{(1+V_f(f))E_m + V_f(m)E_f} \quad (12)$$

From the modulus of elasticity of mortar obtained from the above equation, modulus of elasticity of cement paste, E_{cp} can be found out by using the equation,

$$V_f(cp)E_(cp)^2+(1+V_f(fa))E_{cp}[E_{fa}-E_m]-V_f(cp)E_{fa}E = 0 \quad (13)$$

Volume fraction of each constituents in the above equation can be determined from the mix proportion and density of each constituents present in concrete and is given by the equation,

$$V_{(fi)} = \frac{m_i}{\rho_i[\Sigma_{i=1}^{n}]\frac{m_i}{\rho_i}} \quad (14)$$

where m and ρ are the mass and density of constituents. Fracture process zone of fiber reinforced concrete is influenced by the toughening mechanism provided by both aggregates and fibers. The bridging zone length comprises of aggregate bridging zone length and fiber bridging zone length. The length of aggregate zone length can be computed by knowing the maximum size of aggregate and depth of the specimen which is determined through a calibration process by Simon and Kishen [14] and is used in the study . The length of fiber bridging zone is the only unknown parameter for the determination of fracture energy from the proposed analytical model. Fiber bridging zone of fiber reinforced concrete is influenced by the properties of fibers within the concrete, these fibers play a crucial role in transferring stress across the cracked surface after concrete cracking occurs [2]. By considering this, fiber bridging zone length is correlated with the aspect ratio of fibers through a calibration process using available experimental data from literatures. The experimental studies used for the analysis are Bhosale et al. [2] and Bencardinao et al. [3]. The properties of fiber reinforced concrete specimens (SF1%-1 [2], PF0.50% [2] and SF1%-2 [3]) are given in Table 2. The depth of beam considered was 150 mm. By substituting all known parameters into equation 10, unknown value of fiber bridging length for the three samples are determined as given in Table 2.

Table 2: Fiber bridging zone length of specimens

Specimen designation	Fiber aspect ratio	Fiber bridging zone length (b_f) (mm)
SF1%-1	50	64.67
PF0.5%	100	29.83
SF1%-2%	80	45.96

The value of fiber bridging zone length is plotted against aspect ratio of fiber as given in figure 4.

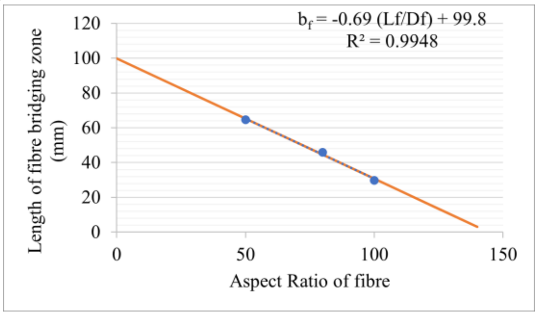

Figure 3: Variation of fiber bridging zone length with aspect ratio of fibers

By assuming a linear fit, fiber bridging zone length is found out as,

$$b_f = -0.69(\frac{L_f}{D_f}) + 99.8 \qquad (15)$$

where L_f and D_f are the length and diameter of the fiber. Fracture energy is then computed from the derived analytical model and is compared with the experimentally determined fracture energy from literatures and is given in Table 3.

Table 3: Comparison of fracture energy

Specimen designation	Fracture energy (N/mm)		Percentage error (%)
	(Model)	(literature[2,3])	
SF1%-1	2.38	2.38	0
SF0.75%	1.91	1.85	3.31
SF0.5%	1.40	1.49	5.08
PF1%	1.28	1.34	4.08
PF0.75%	0.80	0.84	5.38
PF0.5%	0.60	0.60	0
SF1%-2	3.90	3.90	0

The fracture energy obtained from the proposed analytical model shows good agreement with the experimental results found in literature. Percentage error between fracture energy obtained from the proposed analytical model and the fracture energy from experimental data given in literature is calculated and is found to be less, thus validating the same.

4. SENSITIVITY ANALYSIS

The mathematical equation derived for stress intensity factor of fiber-reinforced concrete is complex in nature, involving multiple parameters. Due to the complexity of the mathematical expression, there exists a limited understanding of the relationship between input parameters and the resulting output in the model. The modified stress intensity factor derived to determine stress intensity factor of fiber reinforced concrete contains different parameters such as macrocrack length, length of bridging zone length, ratio between modulus of elasticity at microscale and macroscale, eigenvalue, geometric factor, applied stress, bridging stress and depth of the specimen. Among these parameters, geometric factor can be neglected from the analysis as it does not vary much with the specimen. Since all the random variables are positive, their distributions are taken to be lognormal.

Using statistical parameters such as mean and standard deviation, ensembles of 50,000 values for each random variable are generated using MATLAB R2023a and values for stress intensity factor is simulated keeping one of the seven independent variables as random at a time while maintaining others at their mean value. The cumulative probability distribution is plotted for the modified stress intensity factor as shown in figure 4.

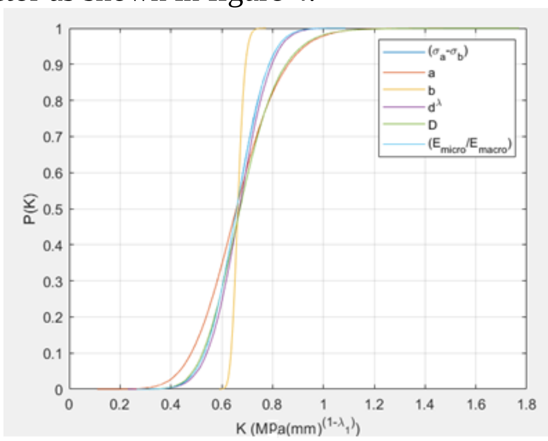

Figure 4: Cumulative probability distribution curves of stress intensity factor for different random variables

From the cumulative probability distribution curve, it is observed that macrocrack length exhibits least slope, indicating it is the most sensitive parameter followed by the depth of the beam. It can also be inferred that bridging zone length is the least sensitive parameter.

Sensitivity of each parameter is also studied by computing the coefficient of sensitivity (*Cs*) and is reported in Table 4. Coefficient of sensitivity (*Cs*) is given by the equation,

$$C_{si} = \frac{C_{vi}^2}{C_v} 100 \tag{16}$$

C_{vi} is the coefficient of variation of the dependent variable when i^{th} independent variable is considered as random with all others at their mean value. C_v is the coefficient of variation when all independent variables are taken as random.

Table 4: Coefficient of sensitivity

Parameters	Unit	Coefficient of sensitivity (Cs), (%)
a	mm	26.613
D	mm	21.243
l^{λ}	mm	10.827
E_{micro}/E_{macro}	-	10.782
$\sigma_a - \sigma_b$	N/mm^2	10.753
b	mm	0.547

From the results, it can be seen that the most sensitive parameter in the modified stress intensity factor is macrocrack length (retaining a coefficient of sensitivity of 26.61%) followed by depth of the specimen (retaining a coefficient of sensitivity of 21.24%). The least sensitive parameter is bridging zone length. The same indications are inferred from the cumulative probability distribution plot.

5. CONCLUSIONS

A modified stress intensity for fiber reinforced concrete is derived using multiscale approach which can capture complex toughening mechanisms at different scales. Further, a sensitivity analysis was also conducted to assess the influence of each parameter on the stress intensity factor. Major conclusions drawn from the study are as follows:

- Modified stress intensity factor derived for the determination of stress intensity factor of fiber reinforced concrete considers material behaviour at both macroscale and microscale.
- An empirical relationship between fiber bridging zone length and aspect ratio of fiber is determined through a calibration process.
- The fracture energy obtained through multiscale approach is compared with the available experimental data in literature and is found to match well with those values reported in literature.
- The proposed model is found to be valid for fiber reinforced concrete with different percentage of fiber and fiber types.
- A sensitivity analysis is carried out to study the influence of different parameters on the stress intensity factor in which macrocrack length is found to be most sensitive parameter followed by depth of the beam.

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