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HIGH-PERFORMANCE REINFORCED CONCRETE STRUCTURES: BRITTLENESS SIZE-SCALE EFFECTS

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Abstract: The present paper deals with the brittle behaviours of high-performance reinforced concrete beams for rather low or high reinforcement percentages. In the former case, the loading drop is due to tensile concrete cracking, whereas in the latter it is due to compression concrete crushing at the opposite beam edge. For the former case, an analytical model is introduced (the Bridged Crack Model) that is able, through a peculiar rotational compatibility condition, to deduce the redundant closing forces applied by the longitudinal reinforcement to the crack faces. This model is conceptually relevant, since it permits to find the minimum reinforcement condition. The linear elasticity of the matrix and the LEFM stress-singularity at the crack tip provide a power-law for the reinforcement percentage as a function of the beam depth raised to –1/2. On the other hand, introducing a numerical model where concrete is considered as a cohesive softening material both in tension and compression, we can obtain a double size-scale brittle-to-ductile-to-brittle transition. By applying Dimensional Analysis and a best-fitting procedure, both in tension and compression, it is possible to find the scaling laws for minimum and maximum reinforcement percentages, respectively. The two exponents become equal to –0.15 and –0.25, respectively. The absolute values of both these exponents are lower than the absolute value of the reference LEFM exponent 0.50 (scaling of extreme severity) and agree with the available experimental results very well. The first has recently been assumed as the reference value in the AASHTO Guidelines for the minimum flexural reinforcement. Unfortunately, we can not affirm the same for the most well-known National and International Standard Codes.

1 INTRODUCTION: BRIDGED CRACK MODEL AND THE PROBLEM OF MINIMUM REINFORCEMENT

The lecture deals with the brittle behaviours of high-performance reinforced concrete beams occurring for particularly low or high reinforcement percentages. In the former case, the loading drop is due to tensile cracking, whereas in the latter it is due to compression crushing of concrete at the opposite beam edge. For the former case, a purely analytical model is introduced (Bridged Crack Model) that is able, through an original rotational compatibility condition, to deduce the redundant closing forces applied by the longitudinal reinforcement to the crack faces [1,2]. This elementary model is conceptually relevant, since it permits to define the minimum reinforcement condition (Figure 1) [3-7].

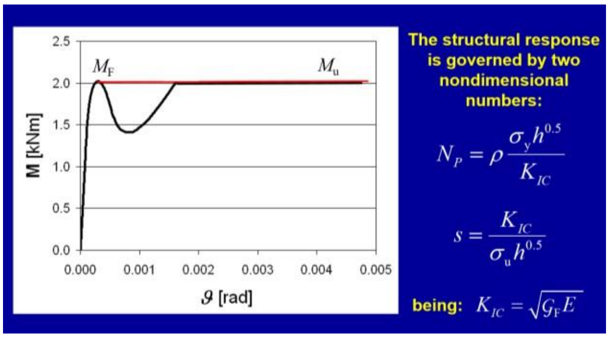

Figure 1. Definition of minimum reinforcement

The linear elasticity of the matrix and the LEFM stress-singularity at the crack tip provide a power-law for the minimum reinforcement percentage as a function of the beam depth raised to the exponent –1/2.

2 QUASI-BRITTLE MATRIX: THE COHESIVE/OVERLAPPING CRACK MODEL

On the other hand, introducing a numerical model where concrete is considered as a cohesive softening material (quasi-brittle) both in tension and compression (Cohesive/Overlapping Crack Model), we can obtain a double size-scale brittle-to-ductile-to-brittle transition [8-14]. When the steel percentage is too low (or the beam depth too small), the peak load is higher than the ultimate perfectly-plastic plateau (Figure 1), and a condition of vertical loading-drop prevails (hyper-strength). On the other hand, when the steel percentage is too high (or the beam depth too large), the ultimate perfectly-plastic plateau reduces its extension to zero (Figure 2) and a condition of vertical loading-drop prevails again, in this case due to crushing at the beam extrados (over-reinforcement) [15-18].

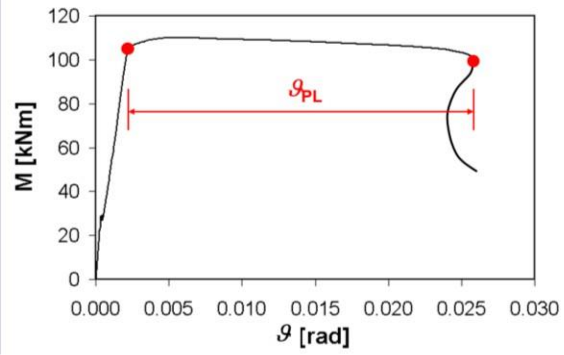

Figure 2. Definition of plastic rotation capacity and maximum reinforcement (ϑ_{PL}=0)

2 MINIMUM AND MAXIMUM REINFORCEMENT CONDITIONS

On these geometrical bases, by equating peak load and ultimate plastic load in the former case, and tending the plastic plateau extension to zero in the latter, it is possible to establish very robust criteria to determine minimum and maximum reinforcement percentages. By applying Dimensional Analysis and a best-fitting procedure, both in tension and compression, it is possible to find the scaling laws for minimum and maximum reinforcement percentages, respectively. The two exponents become equal to –0.15 (Figures 3,4) and –0.25 (Figures 5,6), respectively.

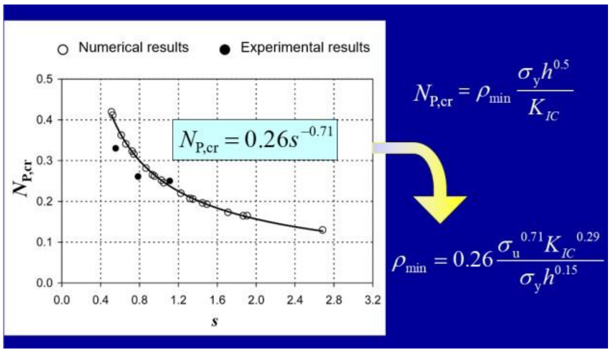

Figure 3. Scale-dependent minimum reinforcement percentage condition

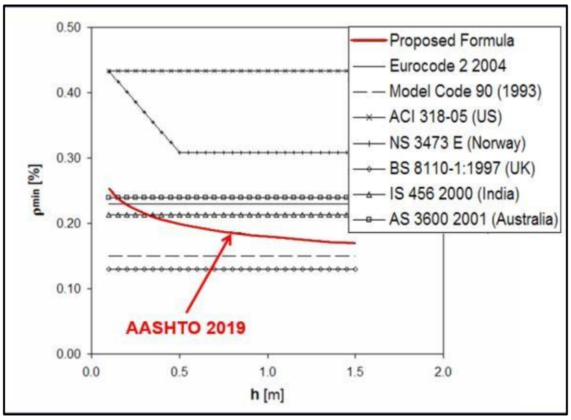

Figure 4. Design code provisions for the minimum reinforcement percentage condition

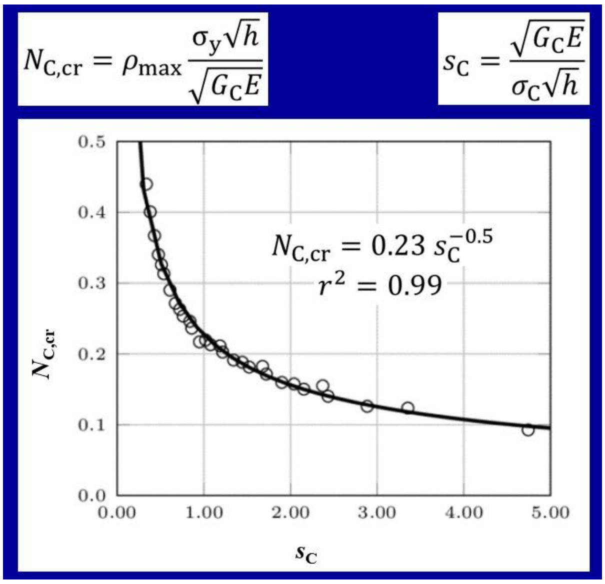

Figure 5. Scale-dependent maximum reinforcement percentage condition

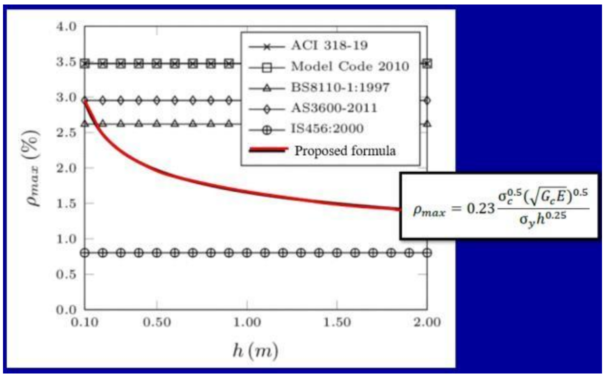

Figure 6. Design code provisions for the maximum reinforcement percentage condition

The absolute values of both exponents are lower than the absolute value of the reference LEFM exponent 0.50 (scaling of extreme severity) and agree with the available experimental results very satisfactorily. The former has recently been assumed as the reference value in the AASHTO Guidelines [19] for the minimum flexural reinforcement (Figure 4). Unfortunately, we can not affirm the same yet for the most well-known National and International Standard Codes.

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