

DUCTILE-TO-BRITTLE TRANSITION IN HIGH-PERFORMANCE PRESTRESSED CONCRETE BEAMS: NEW STANDARDS FOR A SAFE AND EFFECTIVE DESIGN

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Key words: High-performance prestressed concrete, Strain softening, Strain localization, Size-scale effects, Cohesive/Overlapping Crack Model (COCM)

Abstract: The structural behaviour of prestressed concrete beams is considerably affected by different nonlinear phenomena occurring in the post-cracking and crushing regimes, such as snap-back or snap-through instabilities. Design procedures included in current technical Standards are not able to take into account the actual flexural crushing regime, since the adopted constitutive laws overlook the strain-softening and strain-localization behaviour of the concrete matrix. Moreover, design provisions are usually based on Plasticity Theory, leading to completely disregard size-scale effects and ductile-to-brittle transitions as functions of the beam depth. The present work intends to outline a comprehensive theoretical framework for prestressed concrete structural behavior by means of a Fracture Mechanics approach. The Cohesive/Overlapping Crack Model (COCM) is able to simulate the strain-softening and strain-localization behaviour of concrete both in tension and compression, predicting the nonlinear crushing behaviour of prestressed concrete beams. As a matter of fact, the correct estimation of scale effects on maximum reinforcement percentage requires a thorough knowledge of the complex phenomena characterizing the compression crushing failure, leading to define the field in which prestressed concrete structures can develop a safe ductile behaviour. New standard requirements for an effective structural design are formulated.

1. INTRODUCTION

In understanding the flexural behaviour of reinforced concrete (RC) structural elements, several crucial factors must be considered. These include the strain-softening and strain localization in the concrete matrix, along with its different behaviours in tension and compression. Additionally, mechanical instabilities arise during loading, such as cracking in tension, reinforcement yielding, and concrete crushing in compression. Typically, the load-deflection diagram of an RC element begins with an elastic phase up to the first cracking load, P_{cr} , but soon an unstable branch follows where both the

external load, P , and deflection, δ , decrease. This unstable region is often disregarded in experimental testing because it leads to phenomena like snap-through instability, where deflection increases suddenly, or snap-back instability, where bearing capacity drastically drops.

These instabilities are not only observed in RC but also in thin cylindrical shells under axial compression [1,2], and in composite materials like fibre-reinforced concrete, where reinforcements help to arrest cracks and lead to a more ductile response [3]. To better understand these behaviours, the Bridged Crack Model introduces the reinforcement brittleness number, N_P , a nondimensional

number that governs the shift from ductile to brittle failure [4-6]. On the other hand, the Cohesive Crack Model introduces the matrix brittleness number, S_E , which rules the stability of specimen fracture behaviour, depending on specimen size and material properties [7-8].

Modern design practices for RC structures emphasize the need for sufficient deformation capacity, so the structure can provide warning signs before failure. Moreover, it is also important that these structures can redistribute moments in statically indeterminate configurations, withstand accidental loads, and dissipate energy during events like earthquakes and impacts. These requirements highlight the complex, nonlinear behaviour of RC beams, which is influenced by mechanical, geometrical, and loading characteristics [9-10]. This complexity suggests that simplified models cannot fully predict the plastic capacity of real structures. Moreover, experiments and simulations have shown that increasing reinforcement ratio, ρ , or beam depth, leads to a reduction in plastic rotation capacity, further complicating the ability to predict real-world performance accurately [11-12].

The present study uses the Cohesive/Overlapping Crack Model (COCM) [5-6] to examine the ductile-to-brittle transitions in High-performance Prestressed Concrete (HPPC) T-beams. For HPPC beams, the model identifies the scale-dependent reinforcement limits, ρ_{min} and ρ_{max} , within which structures remain stable without catastrophic loss in bearing capacity due to cracking or crushing. In the case of HPPC T-beams, the model emphasizes their brittleness at the Ultimate Limit State (ULS) due to low prestressing forces and high-strength concrete, highlighting the need for scale-dependent minimum and maximum reinforcement ratios as a strict design requirement.

2. COHESIVE/OVERLAPPING CRACK MODEL (COCM)

The Cohesive Crack Model, developed by Carpinteri et al. [8], represents the damage evolution in a concrete cross-section under bending. This model assumes a linear elastic behaviour up to the first peak load and incorporates a stress vs. crack width relationship. More specifically, it introduces a fictitious crack that extends beyond the real crack length, thereby creating a process zone in the tensile region. Along this zone, the material can still transmit tensile forces, although it is partially damaged. This model simulates the residual load-carrying capacity of the structural element by applying closing forces along the crack faces in accordance to a cohesive softening constitutive law. Furthermore, the above model has been improved in the Cohesive/Overlapping Crack Model (COCM) by considering concrete crushing damage through a fictitious interpenetration zone that forms in the region where compressive strain localization occurs [13-14]. The parameters governing this model address the above mentioned crushing behaviour and are analogous to those of the Cohesive Crack Model. The primary variables in the COCM include: the concrete compression strength (σ_c), the threshold value of fictitious interpenetration (denoted as $w_{c,cr}$) beyond which any interaction vanishes, and the crushing energy (G_c) of concrete.

3. RESULTS

The above discussed COCM model is applied to determine the load-deflection behaviour of high-performance prestressed T-beams (Figure 1) in three-point bending. The specimens present different beam depths h , constant slenderness $\lambda = 6$, concrete strength $f_{ck} = 100$ MPa, reinforcement strength $f_y = 1800$ MPa, and initial tendon prestress $f_p = 1400$ MPa. The data presented in Figure 2 indicate that, as a consequence of the model, larger specimens are more susceptible to brittle fracture behaviour [15-17]. Additionally, a minimum amount of

reinforcement is required to achieve ductile failure, and this requirement is influenced by the specimen size and the reinforcement content. For specimens with lower reinforcement, the post-peak behaviour is more ductile, resulting in stable failure. On the other hand, specimens with higher reinforcement exhibit a more brittle post-peak response. The fracture mechanism follows a clear progression: initially, both concrete and steel bar deform elastically, when sufficient damage occurs in concrete at the bottom of beam, the reinforcement bar undertakes the tensile stress until the peak load is reached. Beyond this point, as deflection increases, the steel bar may begin to deform plastically. Moreover, the percentage of steel reinforcement dictates the extension of the plastic rotation plateau. Higher steel percentage may lead to vanishing plastic rotational capacities (over-reinforced beams), potentially resulting in catastrophic failure due to concrete crushing on the compression side. Conversely, lower reinforcement levels produce a loading drop due to tensile cracking.

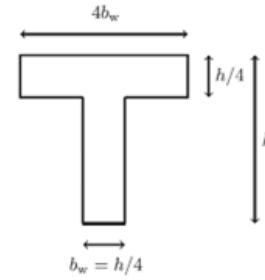
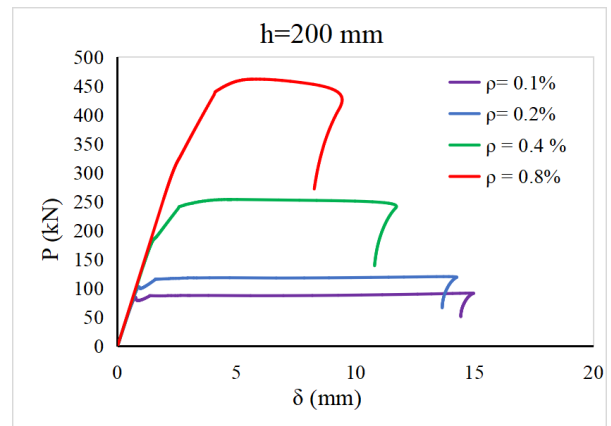


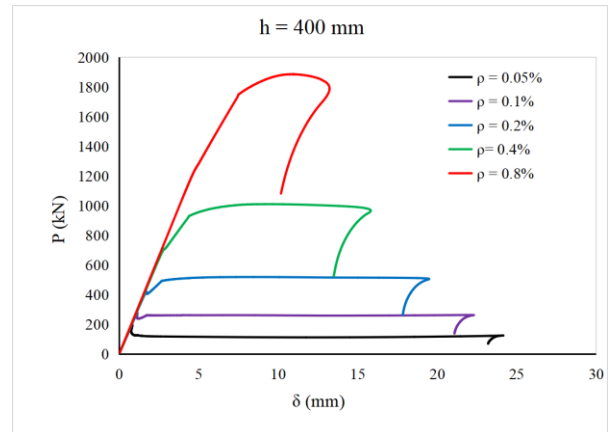
Figure 1: High-performance prestressed T-beam cross-section.

In Figures 2(a)-(d), the load vs deflection diagrams are represented for a different beam depth h in each diagram and parametrically varying the reinforcement percentage ρ . In Figure 2(a), h is equal to 200 mm, whereas ρ ranges between 0.1 and 0.8%. We can observe that, as the loading capacity increases by increasing ρ , so the plastic rotation capacity decreases accordingly. The plastic plateau extension is limited by a snap back instability due to concrete crushing in all the cases. Even for the largest parametric value $\rho = 0.8\%$, we don't reach the maximum reinforcement percentage condition, developing a residual plastic plateau.

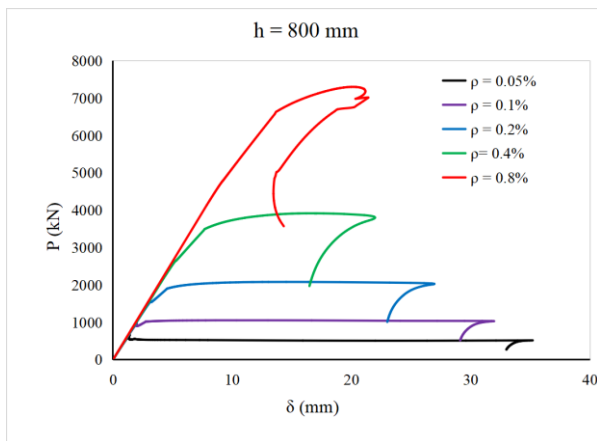
In Figure 2(b), h is equal to 400 mm, whereas ρ ranges between 0.05 and 0.8%. We find the same trends as previously, except for the fact that the plateau is nearly vanishing for the last case. In Figures 2(c),(d), the snap back instabilities due to concrete crushing become more severe.



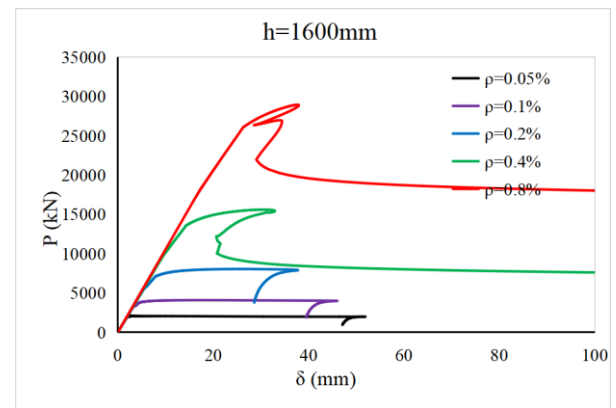
(a)



(b)



(c)



(d)

Figure 2: Load vs deflection for a different beam depth h in each diagram and parametrically varying reinforcement percentage ρ .

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