12th International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-12
B.L.A. Pichler, Ch. Hellmich, P. Preinstorfer (Eds)

https://doi.org/10.21012/FC12.1325
MS04:4

FINITE ELEMENT MODELLING OF CRACKING BEHAVIOUR OF REINFORCED CONCRETE TENSILE MEMBERS USING A COMBINED PHASE FIELD AND COHESIVE ZONE MODELLING APPROACH

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Key words: Cracking behaviour, phase field modelling, finite element modelling, concrete fracture

Abstract: The modelling of cracking behaviour of reinforced concrete (RC) structures represents an intricate challenge because of the heterogeneity of the concrete properties and the complex relationship between concrete and its reinforcement. While experimental methods are limited to inspecting deformations and cracks merely on the concrete's surface, analytical approaches usually assume simplifications as homogeneity of the concrete properties or constant deformation along the concrete section. Concrete's heterogeneous nature, where crack nucleation and propagation are governed by local material properties, introduces uncertainty in mechanical behaviour affecting structural safety, making finite element modelling with varying material properties more suitable for accurately capturing these effects.

The focus of this study is to develop a numerical tool capable of predicting the cracking behaviour of concrete elements reinforced with steel bars subjected to tension. To that aim, finite element models were developed in ABAQUS using a phase field approach to simulate a RC tie element with a single centred steel bar. A stochastic random field was implemented to account for the variability of the concrete's tensile strength. Furthermore, the effectiveness of several techniques to model the concrete-to-steel bond such as a bilinear cohesive interface interaction and a rib-scale model was evaluated. Numerical models with different material field configurations were used to investigate the in-depth variability in cracking patterns and deformations. The numerical results were compared with available experimental results in the literature to validate the proposed model, confirming that the method can effectively model the mechanical behaviour and predict the crack formation stage of RC elements under tensile loading.

1 INTRODUCTION

Concrete is a heterogeneous quasi-brittle material and, in its microstructure, varies spatially across its domain. This variability is due to its composite nature, comprising aggregates, cement, voids, and interfacial zones. As a result, concrete's crack nucleation occurs at random locations and its propagation follows unpredictable paths dictated by local material properties. The heterogeneity of the material causes uncertainty in its mechanical behaviour which affects the safety and reliability of structures. This uncertainty shows up as different mechanical responses and crack patterns under various loading scenarios. In

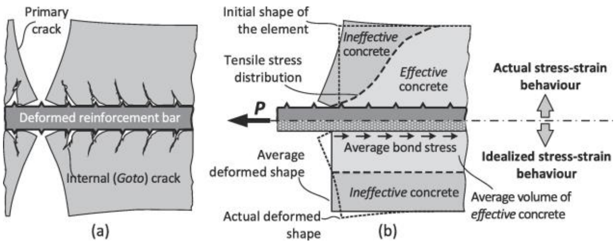

Figure 1: Crack development: (a) primary and internal cracks; (b) effective concrete model[5].

reinforced concrete (RC) elements, the interaction between reinforcement and concrete adds degrees of complexity because the bond mechanism and crack propagation are dependent on the interactions of the materials.

Cracking initiates at the microstructural level influenced by intrinsic defects in the material's composition, often progressing to visible primary macrocracks. After transverse macrocracks form (Fig. 1a), some concrete continues to carry tensile stresses due to the development of internal secondary cracks between the primary ones [1]. The bond mechanism in an RC section makes possible the tensile stress transfer from the reinforcement to the surrounding concrete.

Assumptions such as ignoring the concrete's post-cracking softening, idealizing evenly distributed cracks and assuming that the reinforcement bears all the tension load in a cracked section, are limitations of experimental and analytical approaches that may lead to misrepresent the varying tensile strain gradients and stress distribution across the section. Common techniques for calculating deformations in RC elements include measuring the displacement between two points in the surface of the element to obtain average strains using a linear variable differential transducer (LVDT), internal gauges [2], and using Digital Image Correlation (DIC)[3], but these approaches have limitations because the deformations may vary along the cross-section and the reinforcement. The deformation analysis techniques currently used for RC members usually assume that only a portion of the concrete cross-section, referred to as effective concrete area (Fig. 1b), contributes to carrying tensile loads [4]. The difficulty of this approach relies on the complexity of measuring actual strain distributions within the volume of cracked concrete.

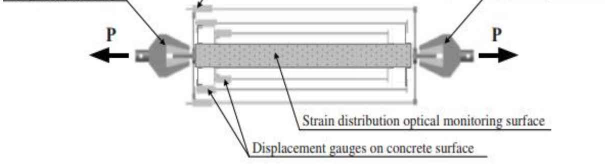

Figure 2: Layout of the test setup and monitoring devices[5].

A standardized test set up for reinforced concrete elements under tension is not yet adopted, however a rectangular concrete prism reinforced with embedded steel bars is a usual test to study the bond behaviour and its influence on the cracking mechanics of RC under tension [5]. The test consists in applying a displacement directly to the steel rebars with the grips of a testing machine, which transfers the load to the surrounding concrete prism (Fig. 2).

A key disadvantage of this configuration is the *end effect*, which results from boundary and geometric characteristics in test specimens. When tensile load is transferred through the bond of the bar reinforcement, the effect introduces localized strain peaks that are not representative of the total structure, making it more difficult to interpret the results. Although attempts have been made to improve experimental configurations with methods such as (DIC) and other sophisticated monitoring methods, these strategies are still limited by loading configurations and specimen geometry. As a result, deformation behaviours seen in lab environments frequently do not match those of real structural components. A viable technique to fill these gaps is finite element (FE) modelling, which may provide an in-depth understanding of crack formation and internal strain distribution. However, the application of conventional FE approaches is limited by issues such as the inability to account for concrete heterogeneity or simulate arbitrary crack nucleation and complex crack propagation patterns.

One of the key components to consider, is how to properly model the bond interaction between concrete and reinforcement. To model the concrete-to-steel bond using FE, several techniques are considered depending on the case scenario and scale of the model [6]. The bond can be modelled at: (1) element-scale, (2) bar-scale, (3) rib-scale, (4) intermediate-scale. Element-scale models use simple line elements that represent a whole structural component. Bar-scale models use truss elements to represent the reinforcement, that is embedded assuming a perfect bond in the concrete, which is modelled with solid elements. In the rib-scale approach, the concrete and reinforcement are modelled with solids and the geometry of the ribs in the reinforcement is explicitly represented as shown in Fig. 3, while the interaction with the concrete has a contact model. A more detailed approach called intermediate-scale defines the concrete-to-steel interaction using a cohesive zone model in the interface between the surfaces of solid elements.

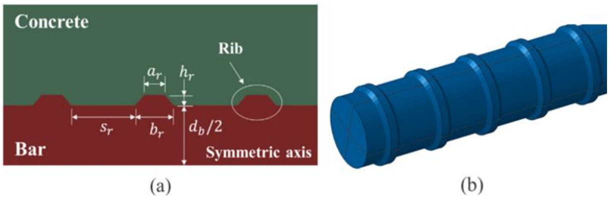

Figure 3: Geometry of the bond interface in a rib-scale model: (a) 2D view; (b) 3D view

In a study [7], a 3D FE model was developed to study strain distributions of RC prisms under tension using a concrete softening law and applying a bond interaction technique that takes into account the effect of the rebar ribs in the cracking behaviour of concrete. The model was able to capture the differences between the concrete and reinforcements strain distributions, the effects of different reinforcement configurations, as well as the resemblance in cracking patterns of the ribbed bar model with experimental results. Additionally, a numerical study on RC prisms [8] examined how different reinforcement configurations influence deformations caused by the heterogeneity of concrete material properties.

In the past decades, the variational phase field approach has gained popularity for modelling fracture and predicting damage in solids [9-11].The phase field fracture method is capable of modelling multiple crack nucleation and propagation, without the need of remeshing techniques. Among the several phase field models published in the literature, the AT2 and AT1 models have demonstrated adequate results when modelling fracture of brittle materials[12-13]. Nevertheless, these models depend only in the fracture energy of the material, and to include the material’s tensile strength, the length scale parameter is taken as a material property, which can be disproportionately large in the case of concrete.

In the regularised phase field cohesive zone model (PF-CZM)[14], the material’s tensile strength is introduced as a material property. This method leaves the length scale parameter as numerical parameter that can be chosen to be adequately small and discards the need of mesh refinement in the expected crack path as commonly done using the earlier PF models. Additionally, strain-softening models can be implemented using the PF-CZM model, such as linear and exponential softening. For normal concrete, a well-accepted softening model based on experimental data [15], can be approximately reproduced by the PF-CZM using an exponential softening curve. In this way, the PF-CZM can include several parameters that define fracture in concrete, the material’s tensile strength that governs the nucleation and location of the crack, the material’s mode-I fracture energy that dictates the crack’s propagation and evolution, the length scale as a physical parameter, and the strain-softening behaviour.

These studies demonstrate the capabilities of the phase field method to satisfactorily model damage in a bulk region, but a more complex scenario arises when there are several components interacting, as in the case of composite laminates and grained heterogeneous materials. The presence of interfaces makes the modelling of damage and crack propagation more challenging. In the case of laminates, coupling approaches were developed [16-17], using a phase field model for damage in the bulk region and a cohesive zone model for damage in the interface, demonstrating capability of the method to predict complex crack patterns. In a heterogeneous granular material such as concrete, accurately representing the mesoscale aggregate structure requires accounting for particles with different material properties that influence the material's overall behaviour. A proposed multi-phase field approach [18] modelled the behaviour of concrete specimens explicitly modelling the aggregates, using two damage variables that separate tension and

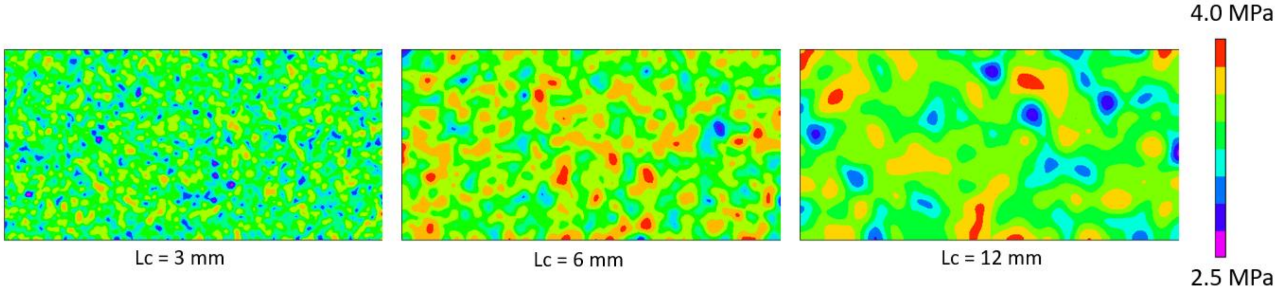

Figure 4: Random field samples (500 x 250 mm) with different correlation lengths (Lc).

compression components.

Some complications arise when modelling the mesostructure of concrete, such as aggregate geometric shape assumptions (polygons, circular, etc), complex meshing techniques, and high computational demands. Statistical tools are required to generate particle fields that properly represent the aggregate size and distribution in concrete mixtures. Stochastic methods can represent the variability of mechanical properties in a material and provide a probabilistic analysis of its behaviour. Using probabilistic methods, it is possible to model the randomness of the system and analyse different possible scenarios. Two approaches have been developed to model the heterogeneity in concrete's material properties using the phase field method, a direct and an indirect method. In the direct method [19], the components of the concrete mixture, namely coarse aggregates, mortar, voids, and the interfacial transition zone (ITZ) are modelled explicitly with their material properties. On the other hand, the indirect method considers that material properties vary spatially throughout the domain and is defined by generating random fields of properties such as fracture energy and tensile strength and mapping them directly to the element mesh of the model. The direct method requires complex fine meshing at a high computational cost, and additional pre-processing steps to generate arbitrary aggregate shapes based on methods such as X-ray computed tomography (CT) [20]. The complexity of this method is further extended with 3D models that require a higher computational cost and sophisticated random aggregate generation algorithms [21-22]. The indirect method arises as a smoothed representation, that reduces the complexity of interactions at the interfaces between material components. This approach is relatively more computationally efficient, because the random field generation is simplified and many samples can be easily obtained for statistical analyses [23-25]. The statistical criteria used to model the spatial variation and distribution of the properties include the spatial correlation length, mean, and variance. The spatial correlation length defines how far apart two points in the field can be while still being correlated. A larger correlation length implies that properties vary more smoothly over the domain, creating a more homogeneous appearance. Variance determines the spread or dispersion of values around the mean in a statistical distribution, with higher variance indicating greater variability and a wider range of possible values. Fig. 4 illustrates three 500 x 250 mm samples of random fields generated with different spatial correlation lengths. Using spatially varying random fields using statistical distributions (Gaussian, Weibull, lognormal, etc) enables the capture of random fluctuations of material properties that cannot be achieved using the direct method that relies only on volume fraction and aggregate size parameters. This stochastic method enables crack nucleation at arbitrary locations and propagation along random pathways, allowing to catch various crack patterns.

Although multiple studies have been reported using the indirect method to model concrete in two dimensions, a 3D model has not yet been developed. Furthermore, the models found in literature represent only plain concrete, a 3D RC model that consists of either direct or indirect method model, to the best of the authors' knowledge, has not yet been developed. While various methods for modelling cracking in reinforced concrete have been explored, few studies integrate phase field methods with stochastic random fields in three-dimensional RC elements to capture realistic crack patterns. Using advanced numerical techniques may improve the accuracy of crack modelling in RC ties and understand more thoroughly the cracking behaviour and mechanisms of RC structures.

Generally, for the implementation of the phase field approach in ABAQUS a User-element (UEL) subroutine is used, which complicates the results visualization process and limits the built-in features of the software. An alternate approach [26] can be done due to the analogy between the phase field evolution equation and the heat transfer equation, where the temperature (T) is analogous to the phase field (ϕ) which is a scalar value that ranges from 0 to 1. In this way the built-in elements in ABAQUS library can be used and take advantage of its post-processing features. The phase field method can be implemented directly in ABAQUS using a user-defined material (UMAT) subroutine and coupled displacement-thermal elements.

In this study, reinforced concrete tie elements are modelled in ABAQUS, integrating a single steel rebar at its centre employing different bond modelling techniques, in combination with spatially varying tensile strength and the phase field approach for quasi-brittle fracture, to evaluate their performance and efficiency.

2 NUMERICAL MODELS

This section reviews the numerical models considered to represent a reinforced concrete tie element, which was experimentally previously tested in [5]. A 60 x 60 mm² rectangular prism, with a 10 mm diameter steel rebar on its centre, and a length of 640 mm subjected to displacement-control loading is modelled using various techniques. The material properties of the concrete are calculated based on a C30/37 mixture, with a compressive strength of 39.5 MPa, resulting in a modulus of elasticity of Ec = 35.5 GPa, and a calculated tensile strength of fct = 3.48 MPa. The assumed values of Poisson's ratio corresponding to ν = 0.2 and the fracture energy of Gc = 100 J/m², were taken from available literature [26]. The modulus of elasticity for the steel rebar is Es = 199.5 GPa, and a Poisson's ratio of ν = 0.3.

While the phase field method is implemented using a UMAT subroutine [26], the concrete-to-steel bond FE is modelled through different techniques to determine their effectiveness in modelling a RC tie. The stochastic random field to model the spatial variability of the tensile strength of the concrete is generated using a Weibull distribution. The phase field parameters used were a length scale lo= 2 mm, an anisotropic formulation

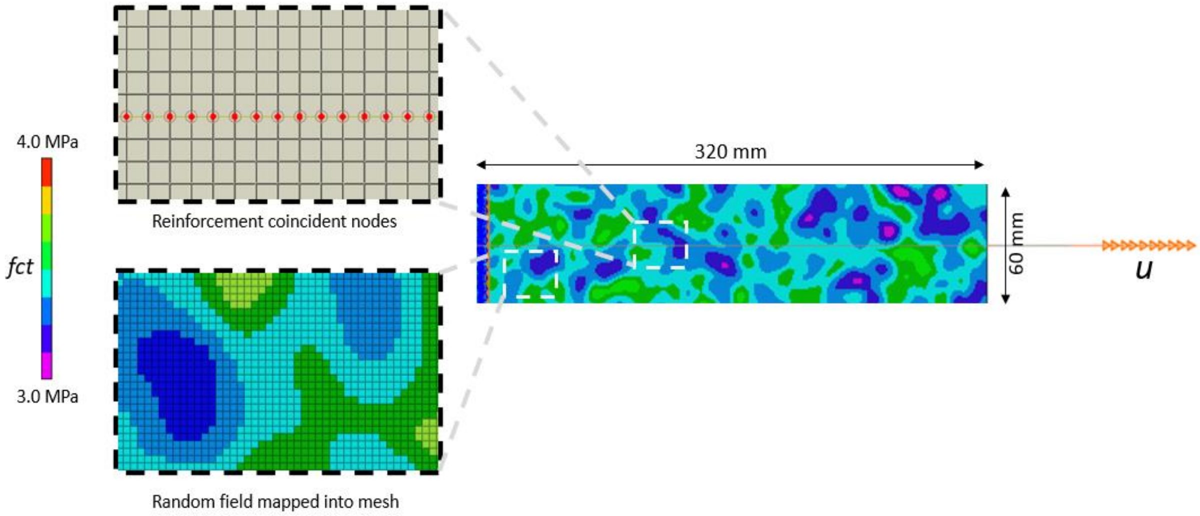

Figure 5: Geometry and mesh of the 2D with bar-scale model.

accounting for the strain energy density split was implemented, the PF-CZM with exponential softening was defined, and a monolithic solution scheme was chosen.

2.1 Model 1: 2D with bar-scale model

Initially, a 2D FE model is simulated to represent ½ of a RC tie element as illustrated in Fig. 5. Quadrilateral C3D4T elements, with displacement-thermal coupling and a maximum size of 1 mm on a structured mesh are selected to map the random stochastic field of material properties to each individual element in the concrete part, and 2-node T2D2 embedded truss elements to model the centred steel rebar. An axis of symmetry is applied to the midsection of the RC tie element with respect to the X axis. The nodes in the rebar are coincident and tied to the corresponding nodes in the structured mesh, and a horizontal displacement is applied to the free end of the rebar. Next, the random field mapping technique is applied properly in this scenario, with the mesh being adequately refined to capture the variability of the material properties.

It is observed in Fig. 6 that, once the displacement is applied, the tensile stresses are not transferred properly from the steel rebar to the concrete using embedded truss elements, and this causes localized damage to develop merely in the adjacent concrete. A stress concentration in the nodes of the rebar generates only localized damage in the concrete elements contiguous to the rebar, and as the displacement increases, the damage is propagated along the concrete surrounding the interface. Furthermore, a 2D model does not represent properly the real geometry of a RC tie element. Several limitations are present, such as the initial assumption that the mechanical properties is uniform along the depth of the prism, which is not realistic because they vary along every direction.

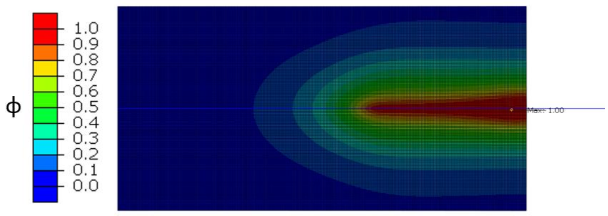

Figure 6: Localized phase field damage in the concrete surrounding the reinforcement.

2.2 Model 2: 3D with intermediate-scale model

To overcome the limitations mentioned in the previous section, a 1/8 3D model was considered, mapping the stochastic random field in hexahedral C3D8T coupled temperature-displacement elements and modelling the steel bar in the centre of the RC tie element using C3D8R and C3D6 elements with elastic steel properties. The random field mapping technique used in the 2D models is extended to a third dimension as shown in Fig. 7, keeping the same random field generation parameters, and a structured mesh with a maximum element size of 2 mm. Multiple random fields are tested to evaluate the different scenarios that can emerge when the material properties are not constant.

In this model, a part with rigid elements was attached to the free end of the RC tie element to avoid the localized damage created in the concrete adjacent to the interface when being pulled. The rigid elements were tied to the coupled temperature-displacement elements to join both parts. The idea of adding this part was to apply the displacement to the whole cross-

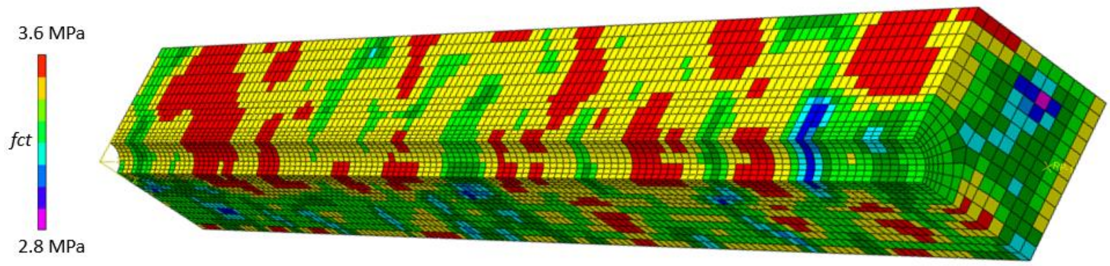

Figure 7: Random field mesh-mapped into a 3D model.

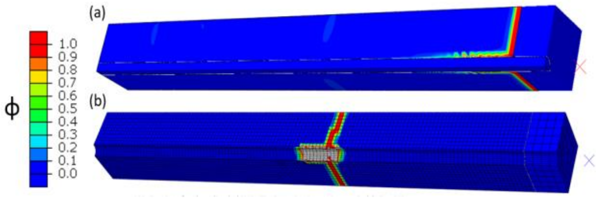

Figure 8: Damage in the interface 3D model. (a) damage initiated in the boundary (b) crack initiated in the middle.

section of the tie, representing a section where the stresses are transferred uniformly and avoid the localized damage.

The bond between the rebar and the concrete was modelled with two different scenarios: a cohesive interface, using a simplified bilinear traction-separation law from the one proposed by the Model Code [27], and a second case with perfect bonding.

From the obtained results, it is observed in Fig. 8, that this model cannot prevent the propagation of the localized damage along the concrete surrounding the interface. For the case where perfect bond was considered, the response was similar to the 2D case, with localized damage. In the case where a cohesive interface was used, several different random fields were generated, and in some samples Fig. 8a, once the tensile strength of the concrete was reached, the concrete elements tied to the elastic part failed and the damage propagated as in previous models. The cohesive interface was undamaged while the concrete adjacent to the rebar failed and prevented further stress transfer from the loaded rebar to the RC tie element. In some other random field samples, there was initial damage in the boundaries, but a crack appeared in the midsection Fig. 8b, propagating towards the middle of the tie, opposite from the loading direction. As the imposed displacement increased, these cracks propagated along the adjacent concrete as in other scenarios.

In summary, with this model, the geometry of the tie can be represented in accordance with the real-life configuration, and the rebar can be modelled according to its geometrical characteristics. In the same way, the spatial variation of material properties is better characterised, and the bond interaction can be defined with more detail compared to the ideally perfect bond used in the 2D model. However the issue with the crack propagation along the interface persists, deterring the stress transfer mechanism and multiple crack formation.

2.3 Model 3: 3D model with intermediate-scale model and phase field coupling

A coupling technique between the phase field model in the concrete and the cohesive zone model in interface is proposed as a solution to the problem of crack propagation. The idea was to assign a coupling function that allows the damage in the bulk to induce damage in the interface. The technique was achieved by using the features available in ABAQUS. In this case due to the analogy between the phase field method and the heat transfer equations, the elements used are coupled temperature-displacement, so the phase field nodal damage can be transferred between surfaces by defining a “gap conductance” definition. Furthermore, the cohesive interface is defined with a UINTER subroutine, that defines the bond behaviour with an imposed condition that uses the transferred nodal variable of the phase field from the UMAT subroutine.

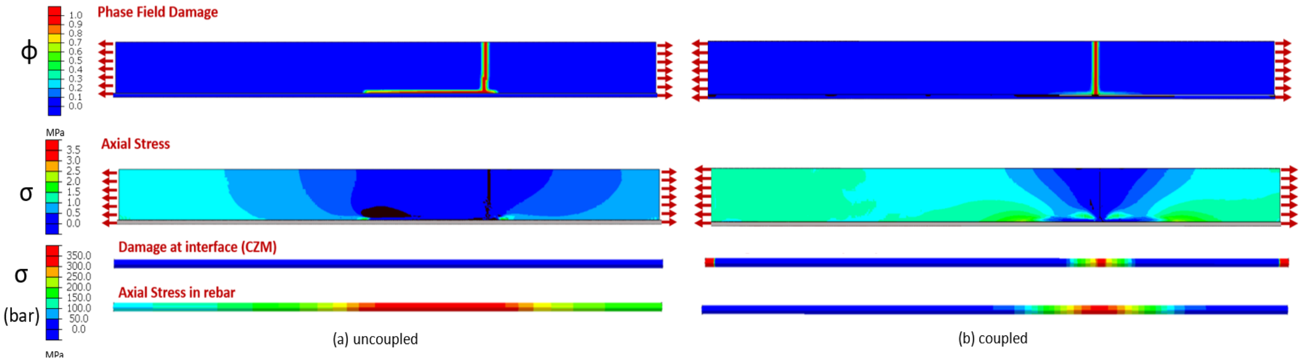

Figure 9: 3D model with intermediate-scale model and phase field coupling.

The coupling function can be defined as a monotonically increasing function dependent on the phase field or assume a brittle behaviour. This implies that it can be defined as progressive damage, where the interface is degraded as the bulk's damage evolves, or to degrade in a brittle way once a desired damage value is achieved. For this model both approaches are tested to evaluate the most adequate to represent this loading scenario.

Fig. 9 compares the behaviour of the tie using the coupling technique with the uncoupled approach used in the previous model. In Fig 9a, once a crack appears and it propagates along the concrete bordering the interface, the concrete section around the crack becomes ineffective and does not carry loading, which is carried solely by the reinforcement. Meanwhile the damage at the interface remains zero and the surfaces remain bonded. On the other hand, in Fig. 9b once a coupling function is introduced, after an initial crack is formed there is no significant damage propagation along the interface and the ineffective concrete area is limited to the concrete surrounding the primary crack. Meanwhile, the axial stress on the rebar has a peak only at the location of the crack and the cohesive interface presents damage.

This approach shows this performance when the displacement is applied to the whole cross-section, but if the displacement is applied directly to the rebar, the same local interface issues are observed as in previous models. Although the coupling prevents the neighbouring concrete from fully damaging, the degradation of the cohesive interface induces loss of stiffness, which deteriorates the stress transfer mechanism and prevents the rest of the concrete away from the crack to continue carrying load until its ultimate strength. This means that the damage in the concrete has no progress, and the reinforcement carries the rest of the loading in the subsequent loading steps.

2.4 Model 4: Rib-scale model

In this section, a ¼ rib-scale model is developed using solid elements, explicitly detailing the steel bar rib geometry and the concrete in contact with it, following the geometrical specifications illustrated in Fig. 3, which are related to the rebar diameter [6].

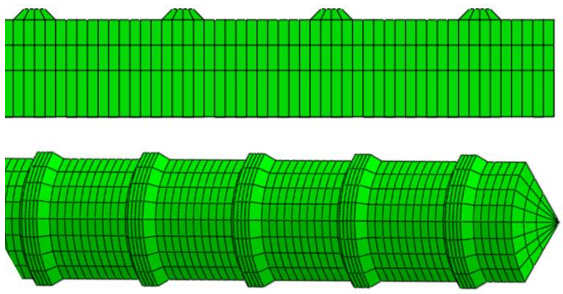

Figure 10: Rib-scale mesh. 2D view (top) 3D view (bottom).

According to this model, shear forces are mechanically transferred between the rebar and the concrete. It denotes the use of mesh refinement to characterise the interface properly and avoid convergence issues. Fig. 10 shows the mesh structure used in this to characterise the ribs in the surface of a steel rebar. The concrete and rebar surfaces are defined with a “contact property”, with a penalty friction of μ = 0.56 in the tangential direction and a ‘hard contact’ in the normal direction to avoid element interpenetration.

The mesh mapping technique used in previous models is used to designate the material random field. A part with mesh

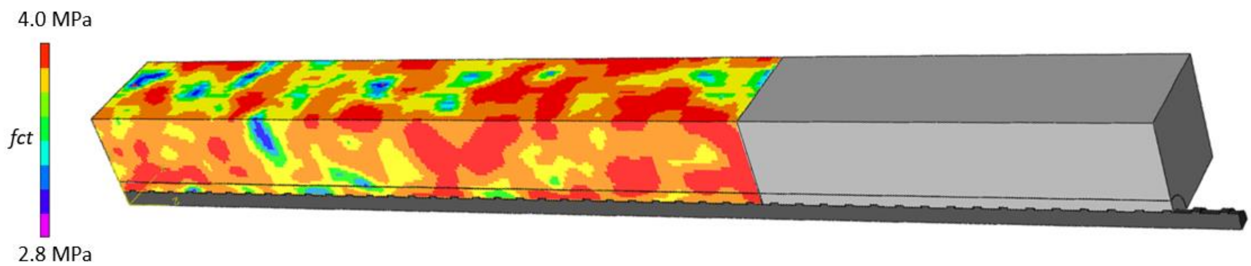

Figure 11: 3D model with rib-scale model and random field.

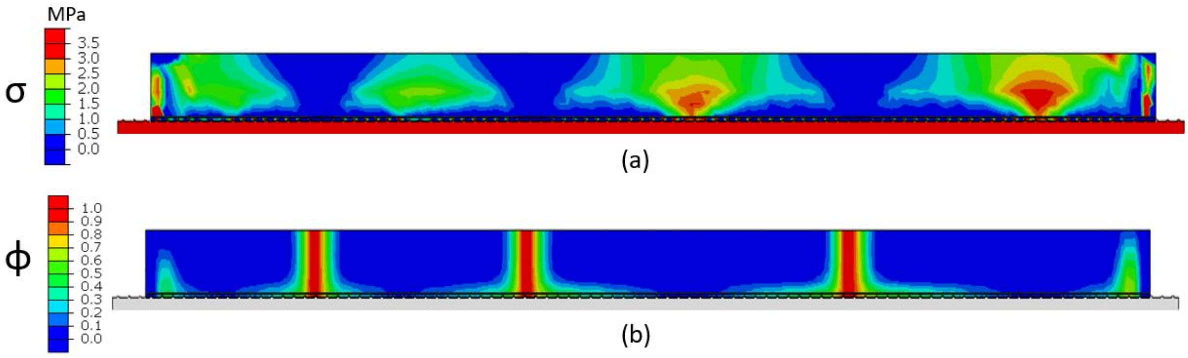

Figure 12: 3D rib-scale model. (a) axial stress contour; (b) crack pattern

refinement is used for the elements in contact with the rebar using 1 mm sized elements to match the geometry of the ribs in the rebar. This part is tied to the rest of the concrete part that is meshed with coarser elements with a maximum size of 5 mm. A method applied to previous models is implemented as well into this approach, which is attaching a part to the free end of the tie as shown in Fig. 11. This part is modelled with 3D solid elements with homogeneous elastic properties of concrete. However, preliminary results show that attaching elastic elements to the free end to prevent the local distortion is not an effective technique. Once the concrete's tensile strength is reached and the initial cracking begins, the end part deforms elastically along with the elastic material of the rebar, preventing the rest of the RC tie element to damage while the load continues increasing.

To avoid this problem, an alternate technique was applied by setting a boundary condition in the nodes at the free end, which sets the nodes to a damage of 0. This allows to have uniform element types throughout the model and preventing the convergence issues caused by the excessive distortion of elements adjacent to the rebar at the loading end. The length of this part assigned with a boundary condition has an impact on the performance of the model. When this length is set as long as the previous elastic part, it behaves in the same way, deforming elastically, so it is set only to the elements in the surface of the free end. This solution also favours convergence, preventing the excessive distortion of the concrete elements surrounding the rebar at the free end.

Furthermore, due to the refined meshing and more complex geometry required for this rib-scale model, the previously adopted random field mesh-mapping technique becomes more troublesome to implement. Instead, a node-based method based on coordinates is enforced, which consists in generating a random field of

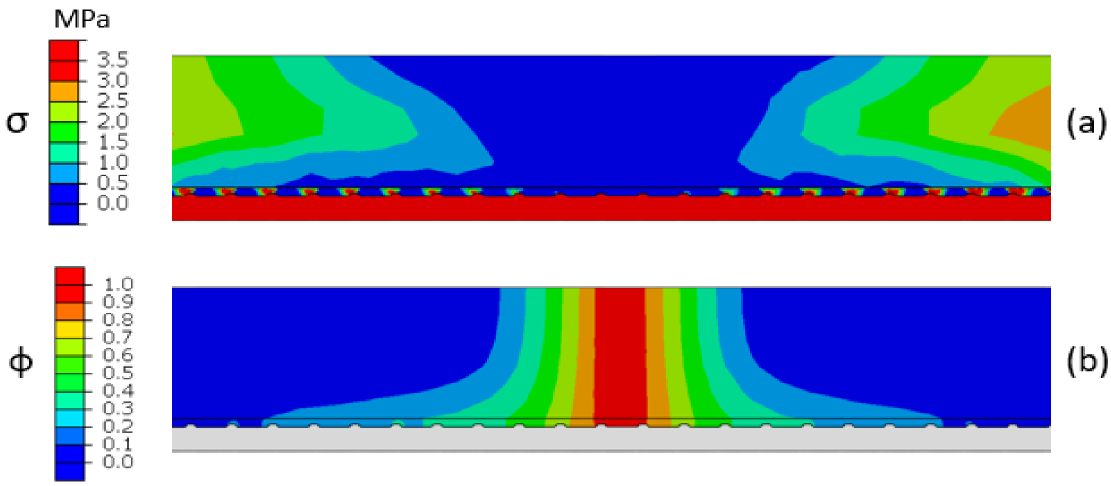

Figure 13: Bond stress transfer mechanism. (a) axial stress contour; (b) primary crack.

values and assigning them directly to the nodes in the model. This makes the mapping procedure simpler, in the presence of curved geometries and the ridges and depressions in the concrete surrounding the rebar. The implementation is straightforward, the UMAT subroutine that calculates the phase field damage can read the values in the random field using a user defined field subroutine (USDFLD) and use them as input material tensile strength in the PF-CZM model. This study evaluates the spatial variation of the tensile strength of concrete only, but the technique can be extended to consider other material properties such as fracture energy, and elastic modulus.

A ¼ model including the node-based random field mapping is used to represent the geometry of a RC tie using a rib-scale model, setting a boundary condition of $\phi=0$ in the surfaces of the free ends and a displacement is applied at the opposite ends of the rebar to induce direct tension loading. The random field is mapped using a node-based approach and a Weibull distribution. The phase field parameters used were a length scale lo=15 mm, an isotropic formulation for the strain energy density, the PF-CZM with exponential softening was defined, and a monolithic solver.

After initial cracking, the “end effect” is observed at the free ends of the RC tie as shown in Fig. 12b, but unlike previous models, this damage does not propagate. Primary cracks are fully formed at arbitrary locations and the effective area of concrete is able to continue carrying some tensile stresses as shown in red contours in Fig.12a. As shown in Fig. 13a, a closer inspection of the interface reveals that the ribs are transferring stresses due to mechanical action, while the crack illustrated in Fig. 13b, does not propagate along the interface.

3 CONCLUSIONS

This study evaluated various numerical approaches for modelling the cracking behaviour of reinforced concrete ties under tensile loading, integrating random fields and phase field methods.

- The 2D model provides a simplified representation but cannot capture the full 3D behaviour of RC ties, particularly the spatial variability of material properties, geometrical features, and detailed bond interactions.
- The 3D model with intermediate-scale bonding offers a more realistic geometry and bond representation, however, challenges remain in preventing localized concrete damage along the interface.
- Coupling the phase field method with cohesive zone models mitigates concrete damage propagation at the interface, enabling a more accurate representation of crack patterns and stress transfer mechanisms.
- The rib-scale model was as the most effective approach, explicitly representing rebar geometry and providing accurate predictions of bond behaviour and crack evolution.

This work demonstrates the potential of advanced numerical techniques in improving the understanding and prediction of RC structural behaviour. Future studies should focus on optimizing computational efficiency; parametric studies including reinforcement configuration, cover length, and external reinforcement; Monte Carlo simulations with large number of random field samples; and validating the models against a wider range of experimental data.

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