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MULTI-SCALE DAMAGE ANALYSIS FOR CONCRETE USING THE SCALED BOUNDARY FINITE ELEMENT METHOD WITH HIGH-PERFORMANCE-COMPUTING

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Abstract: Concrete exhibits complex fracture behaviors, making accurate simulation and analysis essential for understanding material damage mechanisms. This study presents a numerical approach utilizing the novel Scaled Boundary Finite Element Method (SBFEM) to simulate the damage in concrete. The SBFEM is a semi-analytical method capable of using polyhedral elements with arbitrary number of nodes, edges and faces, which is highly complementary with efficient octree mesh generation algorithms. Therefore, octree meshes can be generated directly from the digital scan images of concrete samples, which capture material heterogeneity with high fidelity. Furthermore, the computational demands of damage simulation, especially in multi-scale problems, are addressed by leveraging modern High-Performance-Computing (HPC) techniques. The problem domain is partitioned into a number of parts with similar size which are distributed to multiple computational units. The partition scheme is designed to minimize the data communication between parts, therefore maximizing the computational efficiency. By combining the SBFEM with HPC, this approach is particularly well-suited for simulating concrete's fracture processes, enabling efficient modeling of damage initiation, propagation, and coalescence in complex geometries. Several numerical examples are presented in this work, demonstrating the accuracy and computational efficiency of the proposed approach. This study provides a robust tool for researchers and engineers to address the challenges of damage mechanics in concrete structures.

1 INTRODUCTION

Concrete is one of the most widely used materials in construction due to its durability, high compressive strength, and cost-effectiveness. Its applications range from simple residential buildings [1] to complex infrastructure such as bridges, dams [2], and nuclear power plants [3]. Concrete exhibits a highly heterogeneous internal structure and complex damage mechanisms, including cracking, fracture, and spalling [4]. Accurate simulation and analysis of these damage processes are critical to understanding material behavior and ensuring the reliability and safety of concrete structures.

Finite Element Method (FEM) has been extensively employed to model concrete damage [5]. However, FEM encounters challenges when dealing with the heterogeneous and multi-scale nature of concrete. One major limitation is the difficulty of generating high-quality meshes directly from CT scan images of concrete samples, including aggregates, pores, and cement paste. Additionally, multi-scale problems, which involve regions of widely varying resolution, exacerbate the complexity of mesh transitions and connectivity. This challenge is particularly pronounced in dynamic problems where critical time steps are small, leading to increased computational cost.

The Scaled Boundary Finite Element Method (SBFEM) has emerged as a promising novel numerical method for damage analysis in complex materials like concrete [6, 7]. SBFEM is uniquely suited for handling polyhedral elements with arbitrary shapes, which are naturally compatible with efficient octree mesh generation techniques. SBFEM can directly utilize octree meshes derived from CT scan images [8], preserving the heterogeneity of the material with high fidelity. Moreover, octree meshes inherently support multi-scale modeling due to their hierarchical structure, making them ideal for capturing both fine and coarse features of concrete within a single framework.

Another advantage of octree meshes is their compatibility with modern High-Performance Computing (HPC) environments. The hierarchical structure of octree meshes enables efficient partition for parallel computation, reducing inter-process communication and optimizing load balancing [9]. Recent advancements in asynchronous explicit methods further enhance the computational efficiency by allowing different regions of the domain to operate with different time steps [10]. This feature is particularly advantageous for dynamic problems, where localized regions of fine resolution can adopt smaller time steps without imposing global constraints.

This study explores the application of SBFEM combined with octree-based mesh generation and HPC techniques for multi-scale damage analysis in concrete. The proposed approach addresses the challenges of mesh generation, multi-scale transitions, and computational cost, providing a robust framework for simulating the heterogeneous concrete structures.

2 SCALED BOUNDARY FINITE ELEMENT METHOD

In this section, the construction of polyhedral elements using the SBFEM is briefly discussed. Only the key equations that are necessary for the implementation are presented. Readers interested in a detailed derivation and additional explanations of these equations are referred to the monograph by Ref. [11].

SBFEM is a semi-analytical approach that requires discretization only on the surface of the elements, while the radial direction is solved analytically. The boundary can consist of an arbitrary number of faces, edges, and nodes, provided that it satisfies the visibility criterion. This criterion requires that the entire boundary of the element be directly visible from a defined scaling center "O", located within the element. By connecting the scaling center O to the element boundary, scaling lines are formed, which are represented using the scaled boundary coordinate ξ. The coordinate ξ takes values in the range [0,1] for finite domains and [1,+∞) for infinite domains. The boundary itself is discretized using two-dimensional isoparametric elements in terms of (η,ζ), where the nodal coordinates on the boundary are denoted by $\boldsymbol{x}$. The entire polyhedral element can be represented by continuously scaling its boundary toward the scaling center. The coordinates of any arbitrary point $\hat{\boldsymbol{x}}$ within the polyhedron can then be expressed using the scaled boundary coordinate ξ, the boundary shape functions $\boldsymbol{N}(\eta,\zeta)$, and the nodal coordinates $\boldsymbol{x}$ as

$$\hat{\boldsymbol{x}}(\xi,\eta,\zeta) = \xi\boldsymbol{x}_b = \xi\boldsymbol{N}(\eta,\zeta)\boldsymbol{x} \quad (1)$$

in which $\boldsymbol{x}_b$ represents the coordinates of the boundary. Along the scaling line connecting the scaling center O to a boundary node, an unknown displacement function $\boldsymbol{u}(\xi)$ is introduced. The displacement at any arbitrary point within the element can be obtained through shape function interpolation at the same scaled boundary coordinate ξ, which is

$$\boldsymbol{u}(\xi,\eta,\zeta) = \boldsymbol{N}^u(\eta,\zeta)\boldsymbol{u}(\xi) \tag{2}$$

in which $\boldsymbol{N}^u(\eta,\zeta)$ is the shape function arranged in matrix form. By substituting the above coordinate transformation into the principle of virtual work and performing some mathematical manipulations, the governing partial differential equations of solid mechanics can be transformed into a set of second-order ordinary differential equations with respect to the scaled boundary coordinate ξ. These equations are referred to as the scaled boundary finite element equations, as shown in

$$\boldsymbol{E}_0\xi^2\boldsymbol{u}(\xi)_{,\xi\xi} + \left(2\boldsymbol{E}_0 + \boldsymbol{E}_1{}^{\mathrm{T}} - \boldsymbol{E}_1\right)\xi\boldsymbol{u}(\xi)_{,\xi} + \left(\boldsymbol{E}_1{}^{\mathrm{T}} - \boldsymbol{E}_2\right)\boldsymbol{u}(\xi) = \boldsymbol{0} \tag{3}$$

in which the coefficient matrices $\boldsymbol{E}_0$, $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are determined solely by the geometry and material properties of the discretized boundary. These matrices can be computed using standard numerical integration. The equations can be further transformed into a set of first-order ordinary differential equations, which are solved using eigenvalue decomposition. The eigenvalue decomposition yields the eigenvector matrix, which can be directly utilized to compute the element stiffness matrix

$$\boldsymbol{K} = \boldsymbol{\phi}_q\boldsymbol{\phi}_u{}^{-1} \tag{4}$$

where $\boldsymbol{\phi}_q$ and $\boldsymbol{\phi}_u$ are submatrices of the eigenvector matrix obtained from the eigenvalue decomposition. The computation of the element mass matrix $\boldsymbol{M}$ follows a similar procedure. The assembly of the global stiffness and mass matrices is consistent with the standard FEM.

For nonlinear problems, the SBFEM equations can be used to construct shape functions $\boldsymbol{N}_V$ for polygonal and polyhedral elements,

$$\boldsymbol{N}_V = \boldsymbol{N}^u(\eta,\zeta)\boldsymbol{\phi}_u\xi^{\boldsymbol{\lambda}}\boldsymbol{\phi}_u{}^{-1} \tag{5}$$

in which $\boldsymbol{\lambda}$ is the diagonal matrix containing the eigenvalues. Using this shape function, the displacement at any point within the element can be expressed in terms of the nodal displacements nonlinear computations can be performed in a manner similar to the FEM.

4 OCTREE MESH AND HPC

In this section, the octree-based mesh generation algorithms are briefly introduced, as well as the implementation of parallel computing in an HPC environment.

4.1 Octree mesh generation

Octree-based mesh generation algorithms are robust for complex geometries, as the initial background mesh is generated independent of the model [12]. They are employed to recursively partition a volume in 3D into axis-aligned cubes, as a result, a fast mesh size transition can be achieved. The initial element, which is called root element, is refined into eight smaller elements, which will be further refined until a predefined criteria is met. The information is stored in a hierarchical tree structure, in which the level refers to the number of times an element has been refined. The pointers to the element of the higher and lower levels (called parent and child pointers) are stored for quick element retrieval.

In the current implementation, the so called "1:2 rule" is enforced on the octree mesh, resulting in a maximum size ratio between adjacent elements, which can not exceed 2. This requirement provides a smooth mesh size transition and reduces the number of possible configurations (patterns) of elements. In a balanced octree mesh, the faces of the cubic octree cells have only six unique node arrangements, depending solely on the number of mid-edge nodes and their locations. The faces are treated as polygons in the SBFEM and discretized using rectangular and isosceles triangular elements, which are considered as high-quality elements (no element distortion).

4.2 HPC implementation

In a balanced octree cell, there are 4096 patterns in total considering whether a hanging node is present at the middle of each edge. The 4096 patterns can be further transformed into 144 unique cases using rotation and mirroring operators [9]. These transformation matrices can be arranged and combined to form 48 unique transformations of an octree cell in 3D. Each of the 4096 patterns can be transformed into one of the 144 unique patterns using one of the 48 transformations.

When calculating nodal force vector in explicit dynamics, it can be calculated element-by-element so that the assembly of the global stiffness matrix is not required.

In an octree mesh, elements of the same shape can be grouped together, and therefore, their nodal force vectors can be calculated in one step using a single matrix-multiplication.

Due to the limited surface element shapes, the stiffness and mass matrices for the master element only need to be scaled by suitable factors including information on the element size and material properties. This implementation is highly efficient when dealing with large scale problems as it significantly saves storage requirement and reduces number of floating-point operations.

5 NUMERICAL EXAMPLE

This section presents a numerical example to illustrate the application of SBFEM in multi-scale damage analysis of concrete. The model consists of a mountainous area and several buildings. One building is represented using beams and columns, while a critical joint is modeled at a finer scale based on a concrete specimen obtained from scan images, as shown in Figure 1. The scale of the simulation spans from 100 m to 0.01 m, capturing both large-scale structural dynamics and fine-scale material behavior. Seismic wave propagation is simulated by applying excitation at the base of the model. To enhance computational efficiency, the asynchronous time-stepping technique is employed. This allows the smallest time step, required for the detailed concrete specimen, to be restricted to the localized region, while larger time steps are used in coarser regions.

The concrete material in the specimen is modeled as a two-phase composite comprising mortar (gray) and aggregates (black). The corresponding material properties for these components are specified as follows: the Young's modulus and Poisson's ratio of the aggregate are $E_a = 55\text{GPa}$ and $\nu_a = 0.25$, while the corresponding material parameters of the mortar are $E_m = 10\text{GPa}$ and $\nu_m = 0.2$, respectively. The contour plots of the normal stress in vertical direction in the concrete are shown in Figure 2.

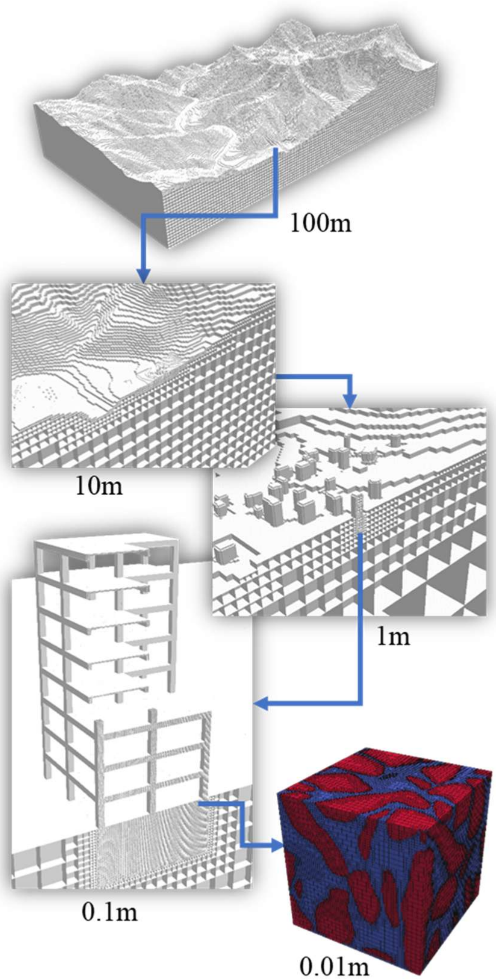

Figure 1: Multi-scale modeling of concrete frame in mountainous region using octree mesh.

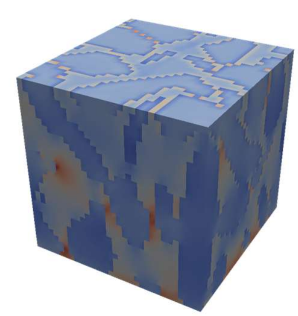

Figure 2: Maximum principal stress contour of the concrete. Unit: MPa

6 SUMMARY

This study develops a multi-scale damage analysis framework for concrete using SBFEM and HPC. Octree meshes generated from CT scans capture concrete heterogeneity, while asynchronous time-stepping improves efficiency in dynamic simulations. Numerical example demonstrates the method's ability to model concrete damage across scales with high accuracy and computational efficiency.

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