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STATISTICAL MODELING OF FATIGUE CRACK GROWTH IN GEOPOLYMER CONCRETE BEAMS

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Abstract. Geopolymer concrete (GPC) is gaining significant attention as an eco-friendly alternative to traditional plain cement concrete (PCC), primarily due to its lower carbon emissions and superior mechanical properties. Recent research highlights its enhanced performance in terms of compressive strength, acid resistance, water permeability, and heat resistance. However, the behavior of GPC under cyclic loading, which is critical for assessing its long-term durability, still needs to be explored. This study aims to provide a better understanding of the fatigue performance of GPC, contributing to the broader assessment of its suitability for long-term use in construction applications. In this work, the fatigue crack growth rate of GPC is investigated using experimental data obtained from three-point bending tests. Beam specimens of GPC, with dimensions of 100 mm x 100 mm x 500 mm, are subjected to cyclic loading at a frequency of 1 Hz. The peak load applied during these cycles is 80% of the material's flexural strength, with a stress ratio of 0.1. A statistical model is employed to fit the experimental data and predict crack propagation trends under repeated loading. The crack length (a) versus number of cycles of loading (N) is recorded for each test, and is used for statistically predicting the fatigue crack growth behavior of GPC. Size adjusted Paris' law parameters are obtained, offering a mathematical representation of how cracks propagate in GPC under repeated loading. The proposed fatigue crack growth equation may be useful for predicting crack behavior in GPC, facilitating more informed decisions regarding its application in infrastructure projects.

1 INTRODUCTION

The increasing emphasis on sustainable construction practices has driven significant research into eco-friendly alternatives to traditional building materials. Geopolymer concrete (GPC) has emerged as a promising substitute for plain cement concrete (PCC), owing to its lower carbon emissions and superior mechanical properties [1]. Unlike PCC, which relies on Portland cement as a binder, GPC utilizes industrial by-products such as fly ash and ground granulated blast-furnace slag, reducing the environmental footprint associated with its production [2]. Furthermore, studies have demonstrated GPC's enhanced performance in terms of compressive strength, acid resistance, water impermeability, and thermal stability, making it an attractive material for a wide range of applications [3].

Despite these advantages, the long-term durability of GPC under cyclic loading conditions remains inadequately explored. Fatigue

Table 1: Composition of GPC mixture per m^3 for dry condition

	Fly ash	**NaOH solution (12M)**	**Na_2SiO_3 solution**	**Fine aggregates**	**Coarse aggregate (10 mm)**	**Coarse aggregate (20 mm)**
kg/m^3	487.10	48.71	121.77	563.84	477.06	715.58

- Composition of Na_2SiO_3 solution: Na_2O - 16.37%, SiO_2 - 34.35%, and water - 49.72% (by mass). Coarse aggregate ratio (by mass), 20 mm:10 mm = 1.5. Na_2SiO_3 to NaOH solution ratio (by mass) = 2.5. Density of 12 M NaOH solution - 1.13 kg/m^3

performance is a critical factor in determining the material's suitability for structural applications, particularly in infrastructure subjected to repeated loading, such as bridges, pavements, and rail tracks. Understanding how GPC behaves under these conditions is essential to assess its potential as a durable construction material.

This study aims to bridge the existing knowledge gap by investigating the fatigue performance of GPC through experimental and statistical analyses. Beam specimens of GPC, with dimensions of 100 mm × 100 mm × 500 mm, were subjected to three-point bending tests under cyclic loading. The tests were conducted at a frequency of 1 Hz, with a peak load set to 80% of the material's flexural strength and a stress ratio of 0.1. Crack length (a) versus the number of cycles (N) data was recorded during the tests to evaluate the material's fatigue crack growth behavior. A statistical model was subsequently employed to analyze the experimental data and predict crack propagation parameters based on size adjusted Paris' law [4].

This work offers insights into the fatigue performance of GPC and contributes to the broader understanding of its application in infrastructure projects, supporting the development of GPC as sustainable and durable construction materials.

2 MATERIALS AND METHODS

2.1 Materials

Geopolymer concrete (GPC) specimens were prepared using fly ash and ground granulated blast-furnace slag as primary binders. These materials were activated with an alkaline solution consisting of sodium hydroxide (NaOH) and sodium silicate (Na_2SiO_3). The mix design was optimized to achieve the desired compressive strength of 25 MPa which suitable for regular structural applications. Aggregates used in the mix were graded to ensure uniformity, and potable water was added to achieve the necessary workability. The composition of the GPC mix is listed in Table 1 and the corresponding measured mechanical properties are listed in Table 2.

2.2 Specimen Preparation

Beam specimens with dimensions of 100 mm $\times$ 100 mm $\times$ 500 mm were cast and cured. The curing process involved heat curing for 24 hours at 60 $^\circ$ Celsius, followed by ambient curing. The initial notch of 10 mm was provided at the mid span of the beams using saw cutter.

2.3 Fracture properties

The flexure strength using three beam specimens under monotonic loading was determined prior to fatigue testing to calculate the peak load for cyclic loading. The representative diagram of the three point bend testing is shown in Figure 1. The obtained static fracture properties are listed in Table 3.

Table 2: Mechanical properties

14 days compressive strength [MPa]	28 days Compressive strength [MPa]	Elastic modulus (E) [GPa]	Flexural strength [MPa]	Density [kg/m^3]
25.79 ± 4.14	31.83 ± 1.04	25.28 ± 0.04	7.50 ± 0.21	1736.29 ± 64.21

Table 3: Fracture properties

	Peak load [kN]	Average peak load [kN]	Fracture energy [N/m]	Average fracture energy [N/m]
Beam -1	11.02		294.46	
Beam -2	9.15	9.43	312.58	298.94
Beam -3	8.13		289.78	

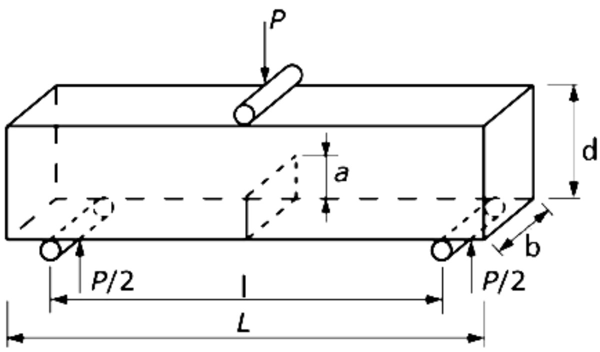

Figure 1: Three point bending setup with initial notch.

2.4 Fatigue testing

The fatigue performance of the GPC specimens was evaluated using three-point bending tests as represented in Figure 1. The experimental setup for this study utilized a universal testing machine equipped with cyclic loading capabilities to investigate the fatigue performance of Geopolymer Concrete (GPC). The loading frequency was maintained at 1 Hz throughout the tests, ensuring uniform cyclic loading conditions. During each cycle, a maximum load corresponding to 80% of the flexural strength of the specimen was applied, simulating realistic service conditions for structural applications. Additionally, the stress ratio, defined as the ratio of the minimum to maximum load in each cycle, was set at 0.1 to replicate typical loading scenarios experienced in infrastructure applications. During the fatigue tests, the crack mouth opening displacement $(CMOD)$ was recorded with the increasing number of cycles (N). The final crack length and corresponding number of cycles at failure were also recorded. The CMOD data is utilised to obtain the crack length (a) versus number of cycles of loading (N) using the compliance approach. Crack length (a) versus number of cycles (N) for GPC specimens is shown in Figure 2. Initial crack growth was slow and stable, but as the number of cycles increased, the rate of crack growth accelerated significantly, consistent with typical fatigue behavior [5]. This $a - N$ data served as the basis for statistical analysis to obtain fatigue crack growth parameters of the size adjusted Paris' equation which is described in the Section 3.

3 STATISTICAL METHOD FOR DERIVING FATIGUE CRACK GROWTH PARAMETERS

The derivation of fatigue crack growth parameters involves processing experimental data of crack length (a) versus the number of cycles (N) to estimate the constants in the size adjusted Paris' law for concrete, which is given by the following equation:

$$\frac{da}{dN} = C\left(\Delta K / K_c\right)^m \tag{1}$$

where:

- $\frac{da}{dN}$ is the crack growth rate (m/cycle),

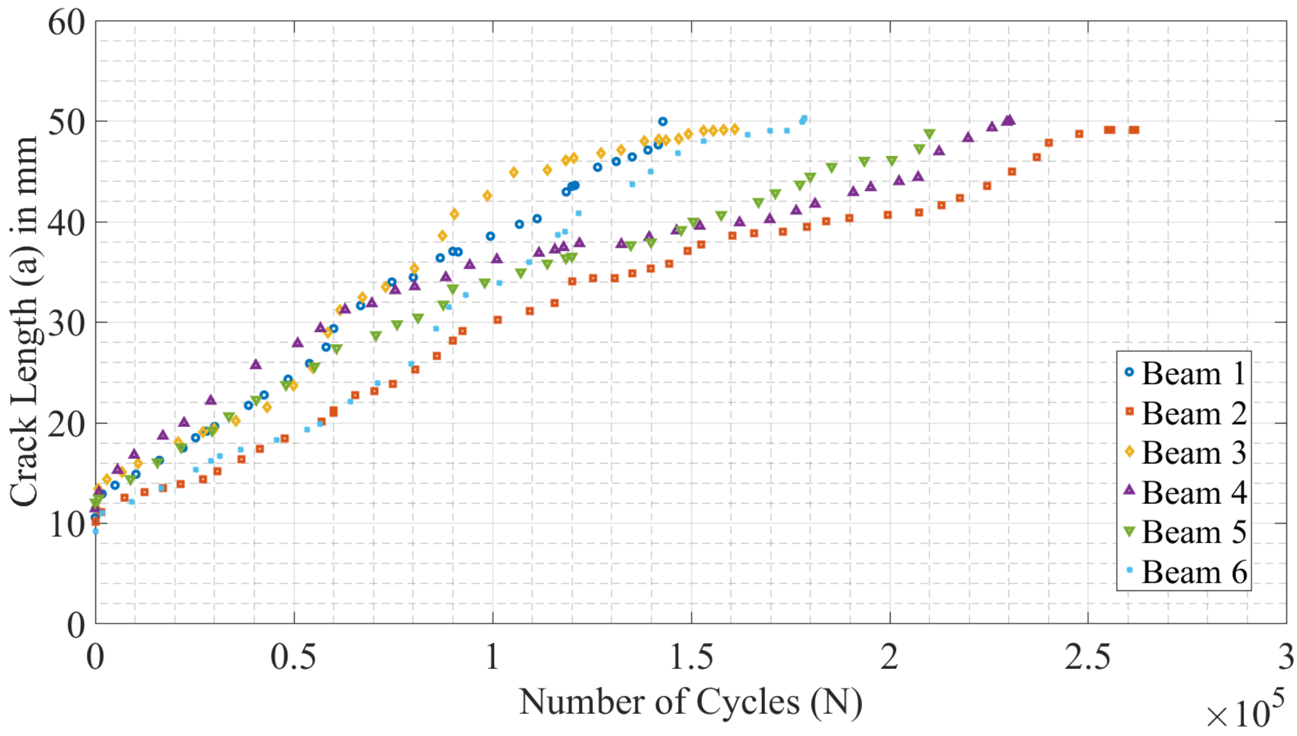

Figure 2: Crack length versus number of cycles for GPC specimens.

- C and m are material-dependent fatigue crack growth parameters,
- ΔK is the stress intensity factor range,
- K_c is fracture toughness.

Equation (1) served as the base equation to obtain the fatigue crack growth parameters using $a - N$ data. The data is transformed and pre-processed as per the following algorithm:

- Compute crack growth rate ($\frac{da}{dN}$) from the discrete a versus N data. The incremental changes in crack length (Δa) and cycles (ΔN) are calculated as:

$$\Delta a = a_{i+1} - a_i, \quad \Delta N = N_{i+1} - N_i \quad (2)$$

  The crack growth rate is then approximated as:

$$\frac{da}{dN} = \frac{\Delta a}{\Delta N} \quad (3)$$

- The midpoint crack length (a_{mid}) and corresponding cycle number (N_{mid}) are:

$$a_{\text{mid}} = \frac{a_{i+1} + a_i}{2}, \quad N_{\text{mid}} = \frac{N_{i+1} + N_i}{2} \quad (4)$$

- Determine Stress Intensity Factor Range (ΔK)
  - Stress Intensity Factor (K) For a three-point bending specimen:

$$K = \Delta\sigma\sqrt{\pi a} \cdot Y(a) \quad (5)$$

    where D is the specimen depth and $Y(a)$ is a geometric correction factor. For span to depth ratio of 4, it is given by:

$$Y(a) = \frac{P(a)}{Q(a)} \quad (6)$$

    where,

$$P(a) = 1.99 - \frac{a}{D} \cdot \left(1 - \frac{a}{D} \cdot (2.15 - 3.93 \cdot \tfrac{a}{D} + 2.7 \cdot \left(\tfrac{a}{D}\right)^2\right)$$

$$Q(a) = \sqrt{\pi} \cdot \left(1 + 2 \cdot \frac{a}{D}\right) \cdot \left(1 - \frac{a}{D}\right)^{1.5}$$

  - The stress range is given by:

$$\Delta\sigma = \sigma_{\text{max}}(1 - R) \quad (7)$$

- Stress Intensity Factor Range (ΔK) At the midpoint crack length (a_{mid}):

$$\Delta K = \Delta\sigma\sqrt{\pi a_{\text{mid}}} \cdot Y(a_{\text{mid}}) \quad (8)$$

- Calculation of fracture toughness, K_{1C} There are several ways to calculate the fracture toughness of beams analytically. In this work, the double K fracture model [6] is adopted, which is an analytical method where inputs are based on the experimental $CMOD$ and load values. It is calculated using the following equations [7]:

$$\alpha_c = \frac{X}{Y} \quad (9)$$

where,

$$X = \gamma^{3/2} + 0.4460\gamma$$

$$Y = \left(\gamma^2 + 2.2538\gamma^{3/2} + 2.9950\gamma + 3.4135\right)^{3/4}$$

$$\alpha_c = \frac{a_c}{D}, and$$

$$\gamma = \frac{CMOD_c \cdot B \cdot E}{6 \cdot P_u}$$

Finally, the fracture toughness, K_{1C}, is calculated as:

$$K_C = \frac{3(P_u + 0.5W) \cdot S \cdot \sqrt{\pi a_c} \cdot Y(a_c)}{2D^2B} \quad (10)$$

The geometric factor at crack length a_c is $Y(a_c)$ which is calculated using Equation (6).

Here:

  - $\alpha_c = \frac{a_c}{D}$,
  - a_c: Equivalent crack size when the load value reaches the peak value P_u,
  - $CMOD_c$: Crack mouth opening displacement corresponding to P_u,
  - S: Span of the beam,
  - B: Width of the beam,
  - D: Depth of the beam,
  - E: Elastic modulus of the beam,
  - W: Self-weight of the specimen between supports.

- Linear Regression for curve fitting
  - Rewrite size adjusted Paris' law in logarithmic linear form

$$\log\left(\frac{da}{dN}\right) = \log(C) + m \cdot \log\left(\frac{\Delta K}{K_c}\right) \quad (11)$$

    where $\log(C)$ is the y-intercept and m is the slope.

  - Fit the linear relationship using least squares regression:

$$m = \frac{\text{Cov}(\log(\Delta K/K_c), \log(da/dN))}{\text{Var}(\log(\Delta K/K_c))}$$

$$\log(C) = \text{Mean}(\log(da/dN)) - m \cdot \text{Mean}(\log(\Delta K/K_c)) \quad (12)$$

  - The parameters are extracted as:

$$C = 10^{\log(C)}, \quad m = \text{slope} \quad (13)$$

- Repeat the process for multiple specimens to derive individual C and m.

- Calculate the average values of the parameters:

$$\text{Average } C = \frac{\sum C_i}{n}, \quad \text{Average } m = \frac{\sum m_i}{n} \quad (14)$$

- Plot $\log(da/dN)$ versus $\log(\Delta K/K_c)$ with the fitted Paris' law curve on logarithmic scale, indicating the averaged parameters.

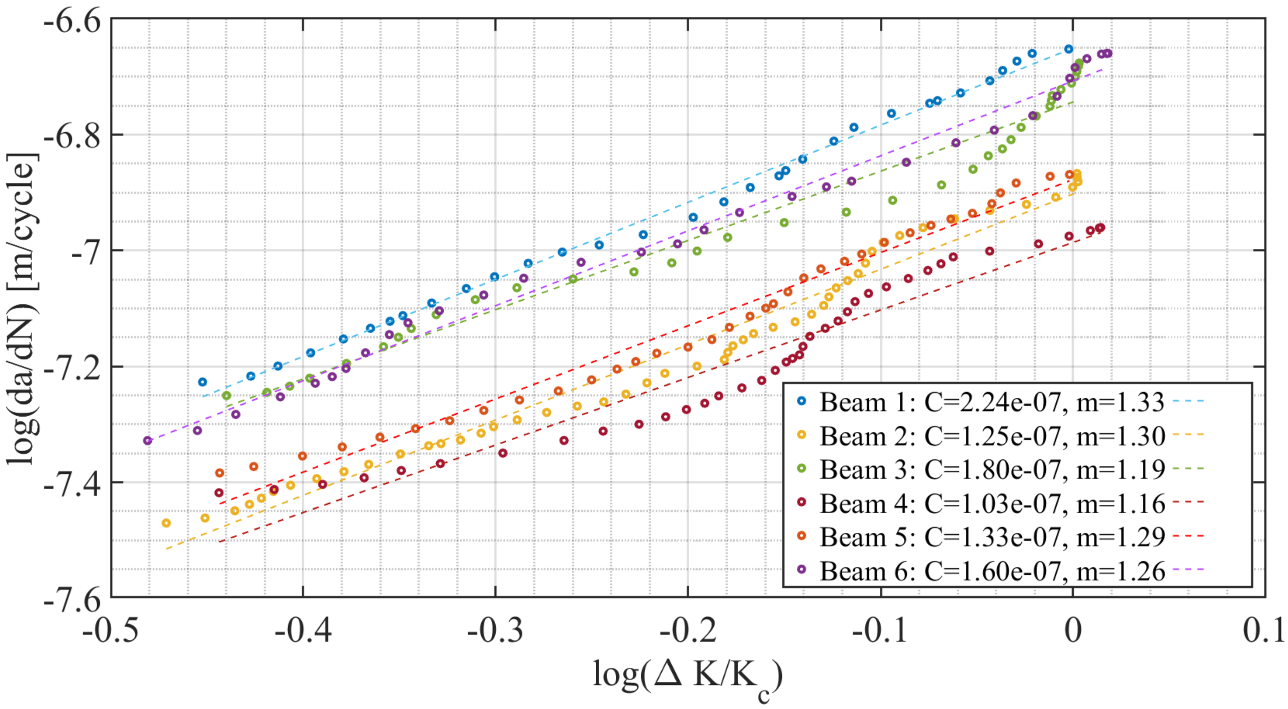

Figure 3: Size adjusted Paris' law curves for the experimental samples.

The above algorithm is executed using MATLAB 2024b version and the obtained fatigue crack growth rate curves are plotted in Figure 3. The plot represents the size adjusted Paris' law relationship for fatigue crack growth rates ($\log(da/dN)$) as a function of the normalized stress intensity factor range ($\log(\Delta K/K_c)$) for individual beams of GPC with a 10 mm notch. Each beam exhibits a linear trend, consistent with the law, where C and m are material-specific parameters. The parameters vary among the beams, as indicated by the different slopes (m) and intercepts (C) of the fitted lines. The average values of the parameters are $C = 1.60 \times 10^{-7}$ and $m = 1.26$. The scatter around the individual trend lines indicates variability in the experimental data, which is consistent for a quasi-brittle material such as concrete [8,9]. This scatter in data is further used in prediction of fatigue life as discussed in Section 4.

4 PROBABILISTIC FATIGUE LIFE PREDICTION

The probabilistic fatigue life of concrete specimens was predicted using a Monte Carlo simulation approach based on the size-adjusted fatigue crack growth rate equation (Equation (1)). To account for variability in C and m, experimental data for these parameters were used to fit statistical distributions. The parameter C was assumed to follow a log-normal distribution, while m followed a normal distribution. The fatigue life (N_f) for each Monte Carlo realization was computed by numerically integrating the crack growth equation over the range of crack lengths:

$$N_f = \int_{a_0}^{a_f} \frac{1}{C\left(\frac{\Delta K(a)}{K_C}\right)^m} da, \tag{15}$$

where a_0 and a_f represent the initial and final crack lengths. Initial crack length is taken as the initial notch length which is 10 mm in this work, while final crack length is crack length corresponding to the failure of the beams obtained through the fatigue experiments. The workflow of the program developed to implement this methodology is as follows:

- Fit log-normal and normal distributions to experimental data for C and m, respectively.

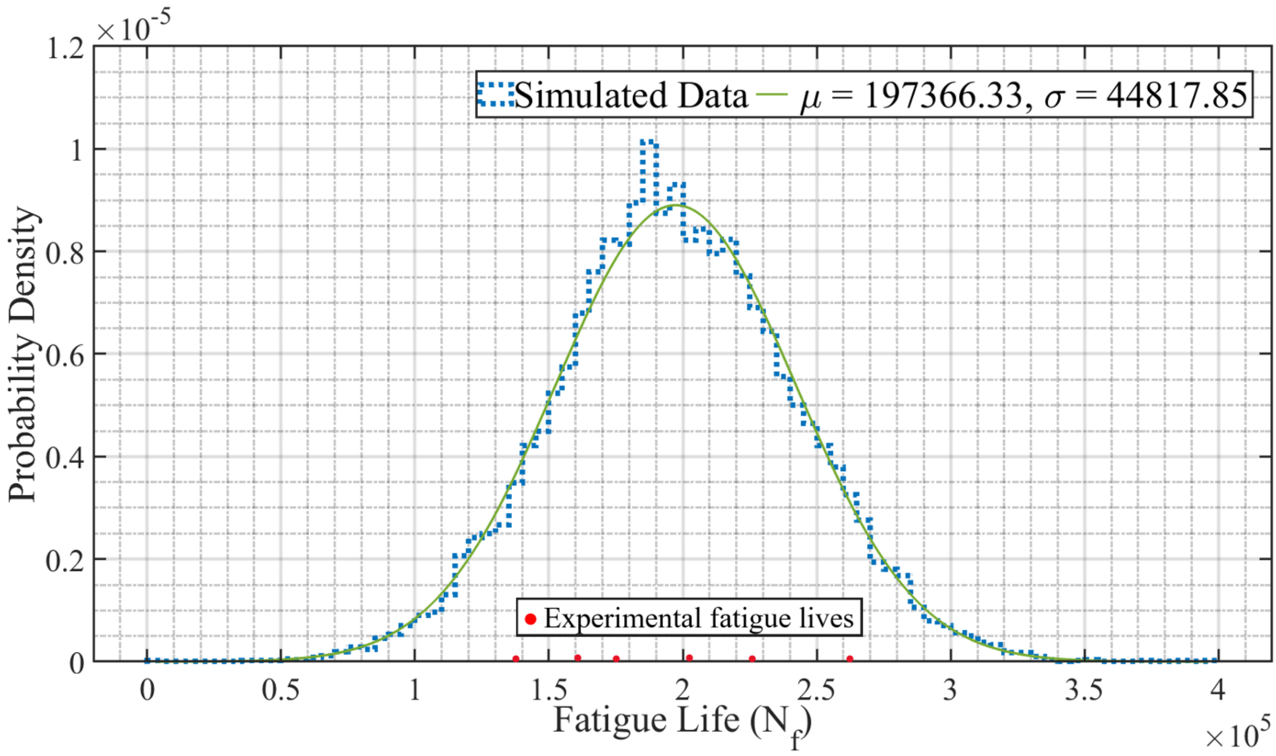

Figure 4: Simulated fatigue life.

- Generate random samples of C and m for a specified number of Monte Carlo simulations.
- For each realization, compute the fatigue life by numerically integrating the crack growth equation using the variable $\Delta K(a)$.
- Analyze the distribution of predicted fatigue life values to obtain mean, standard deviation, and confidence intervals.

This program is executed using MATLAB 2024b version. The simulated fatigue life for 10000 simulations is plotted in Figure 4. The plot shows a probability density function (PDF) for the simulated fatigue life (N_f) of a material, represented by the blue histogram overlaid with a fitted distribution curve. The fatigue life data follows a bell-shaped distribution, likely a normal (Gaussian) distribution, as indicated by the smooth green curve. The mean and standard deviation are $\mu = 197366.33$ and $\sigma = 44817.85$, respectively of the simulated fatigue lives of the GPC beams. Red markers along the x-axis indicate experimentally observed fatigue life values. These experimental points generally lie close to the peak of the simulated distribution, suggesting that the simulation captures the central behavior of the material's fatigue life well. The histogram indicates some spread in the simulated data, which aligns with the variability in fatigue life due to material and experimental uncertainties.

5 CONCLUSIONS

This study investigated the fatigue performance of Geopolymer Concrete (GPC) through a combination of experimental and statistical analyses. The size adjusted Paris' law parameters (C and m) derived from the fatigue crack growth rate curve of $\log(da/dN)$ versus $\log(\Delta K/K_c)$ demonstrated that GPC exhibits consistent crack growth behavior across different beam specimens. The experimental results highlighted that the average values of $C = 1.60 \times 10^{-7}$ and $m = 1.26$ are representative of the fatigue characteristics of GPC [10], providing a reliable baseline for modeling crack propagation. The simulated fatigue life distribution further validates the experimental findings, with the majority of experimentally observed fatigue lives aligning well with the simulated data. The probability density function (PDF) of the fatigue life, characterized by a mean of $\mu = 197366.33$ cycles and a standard deviation of $\sigma = 44817.85$, indicates that GPC demonstrates predictable fatigue performance under cyclic loading conditions. The agreement between experimental and simulated results validates the statistical model employed in this study. These experimental findings underscore the potential of GPC as a durable material for structural applications subjected to cyclic loading. The proposed fatigue crack growth equation offers a mathematical framework for predicting crack propagation trends, facilitating informed decision-making for the design and maintenance of GPC-based infrastructure. Overall, this study contributes to the growing body of knowledge on the long-term performance of GPC, supporting its adoption as a sustainable alternative to conventional concrete in construction applications.

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