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MODEL UNCERTAINTIES OF CONCRETE CRACKING RESISTANCE MODELS BASED ON PROBABILISTIC SIMULATIONS

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Abstract: In the paper, the probabilistic assessment of cracking resistance of concrete flexural members is presented. The aim of the performed analysis was to verify an alternative formula for cracking resistance calculation and to compare the proposed method with two standard methods. The experimental investigation performed at Lublin University of technology was used to verify the design models. The accuracy and reliability of the calculation methods was assessed by analyzing the model uncertainty θ. When model uncertainty $\theta > 1.0$ the prediction model yields a lower value of cracking resistance and is thus conservative, while a value of $\theta < 1.0$ implies that the prediction model yields higher cracking resistance than is actually available in the structure and is thus un-conservative. A high model uncertainty $\theta = 1.53$ was found when the cracking resistance was calculated by standard method assuming a linear distribution of normal stresses over the cross section and taking the maximum tensile stress as the concrete axial tensile strength. When applying the flexural tensile concrete strength defined in Eurocode 2 instead of axial tensile strength in a cracking resistance formula, the model uncertainty decreased to $\theta = 1.15$ but the model was still conservative. The best prediction of cracking resistance was obtained for the proposed method in which the influence of a size effect and fracture properties of concrete on cracking moment were included. In this case the model uncertainty was close to 1.0 ($\theta = 0.94$) with a relatively small scatter.

1 INTRODUCTION

When designing civil engineering structures, the trend is observed to fulfill higher economical requirements [1]. However, the safety of a structure must be on the first place. This is of paramount importance especially in case of unreinforced concrete structures, where the nature of the failure is brittle. The example of structural members which can be designed as unreinforced concrete members are foundations. As a result of extreme weather events caused by climate change, such as floods and landslides, the ground conditions may change, which in turn may affect the performance of foundations and increase the probability of the failure.

In order to connect structural safety with economical requirements, it has become more common in the last years to use a reliability analysis of civil engineering structures and structural members. In the context of applying more rational methods in the assessment of civil engineering structures, the current trend is to verify and calibrate design procedures. The consequence is the refinement of calculations in terms of more accurate reliability estimates, which necessitates consideration of uncertainties and a probabilistic computation. The application of a complex simulation software can help to analyze the correctness and reliability level of existing design code formulas vs. newly proposed methods [2, 3].

In the paper, the probabilistic assessment of cracking resistance of concrete flexural members is presented. The aim of the analysis is to verify an alternative formula for cracking resistance calculation that reflects the size effect and fracture properties, and to compare the proposed method with standard methods of a cracking moment estimation.

2 CRACKING RESISTANCE

The calculation of cracking moment is of paramount importance as it decides of a cracking resistance and a load carrying capacity of unreinforced concrete members. When the cracking moment is calculated assuming a linear distribution of normal stresses over the cross section, where the maximum tensile stress is taken as the concrete tensile strength f_{ctm} (Equation 1) too low values are obtained comparing to test results. Experimental findings, achieved for instance by Słowik [4], Dąbrowski [5] and Knauff [6], have shown that the experimental cracking resistance is $1.5 \div 2.0$ times higher than the cracking moment calculated based on the Equation 1:

$$M_{cr,T} = W_c \, f_{ctm} \tag{1}$$

where: f_{ctm} – mean axial tensile strength of concrete, $W_c = bh^2/6$ – section modulus, b – width of the cross section, h – height of the cross section.

Researchers have pointed that the cracking resistance of concrete structural members is influenced by the member's size [7-9]. In the European standard Eurocode 2 [10] this finding was included in the Equation 2 for calculating a flexural tensile concrete strength. As the consequence, the modified formula for the cracking resistance of flexural concrete member can be written as the Equation 3 (where h is the total height of cross section in meters):

$$f_{ct,fl} = \max\{f_{ctm}(1.6 - h); f_{ctm}\} \tag{2}$$

$$M_{cr,EC2} = W_c \; f_{ct,fl} \tag{3}$$

It has been well recognized that concrete exhibits size effect. Nonlinear fracture mechanics can capture deterministic energetic size effect. However, uncertainties related to the strength of the material indicate statistical size effect and the application of statistics and probability is required when taking into consideration this source of size effect [11, 12]. The application of nonlinear fracture mechanics for analyzing cracking forces in concrete members has shown that the increase of cracking forces takes place when the member's dimensions decreases. The size effect was extensively analyzed for example by Bažant and Pfeiffer [13], Hillerborg, et al. [14], Carpinteri [15].

Furthermore, experimental investigations and numerical simulations have shown that a more brittle character of failure can appear in members made of a high strength concrete [16]. Therefore, the size effect as well as fracture properties of concrete should be taken into account in order to correctly calculate cracking resistance of concrete members. The comprehensive approach to the issue was proposed in the research [14] in which several numerical simulations were presented on the development of failure crack in flexural concrete members. The numerical analyses were performed using nonlinear fracture mechanics approach and applying the nonlinear characteristic of concrete in tension in the post-critical phase (softening of concrete in tension). The coefficient κ has been derived by performing a regression analysis of calculations results which were read from nomograms presented in [14]. The coefficient characterizes the influence of concrete parameters and the member's size on cracking resistance of concrete. Coefficient κ has been described by two equations: 4a (when h/l_{ch} < 0.5) and 4b (when $h/l_{ch} \geq 0.5$):

$$\kappa = -3.68\cdot(h/l_{ch})^3 + 7.35\cdot(h/l_{ch})^2 - 4.81\cdot(h/l_{ch}) + 2.48 \quad (4a)$$

$$\kappa = -0.002\cdot(h/l_{ch})^3 + 0.04\cdot\cdot(h/l_{ch})^2 - 0.24\cdot(h/l_{ch}) + 1.58 \quad (4b)$$

Graphical representation of the relation between the coefficient κ and the ratio of cross section height to characteristic concrete length l_{ch} is presented in Fig. 1.

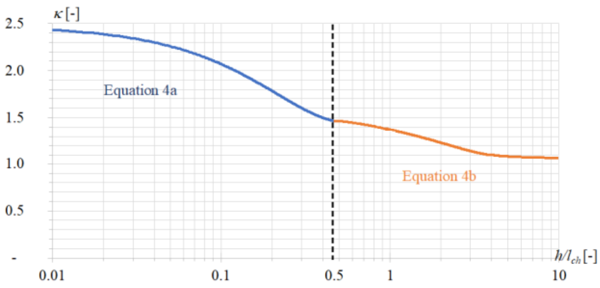

Figure 1: Coefficient κ with respect to h/l_{ch} ratio.

Coefficient κ takes into account the size effect described by a total height of cross section and concrete fracture parameters like G_F – fracture energy, E_{cm} – Young's modulus and f_{ctm} – tensile strength of concrete which are summarized in one parameter – the characteristic concrete length l_{ch}. The characteristic concrete length is the basic parameter defined based on nonlinear fracture mechanics (Eq. 5) and it describes concrete fracture properties in a complex way.

$$l_{ch} = \frac{G_F E_{cm}}{f_{ctm}^2} \quad (5)$$

Based on coefficient κ, which takes into account the size effect and fracture properties via the characteristic concrete length, a new method (Equation 6) is proposed:

$$M_{cr}^{\kappa} = W_c\, \kappa\, f_{ctm} \quad (6)$$

The reliability analysis of design models described by the Equations 1, 3 and 6 has been performed in order to verify the correctness of calculating cracking resistance of flexural concrete member. The experimental investigation performed at Faculty of Civil Engineering and Architecture, Lublin University of Technology by Marta Słowik was used for the comparative analysis.

3 EXPERIMENTAL INVESTIGATION

The experimental research presented hereafter was carried out to determine a cracking moment of flexural concrete members. Six concrete beams were tested, with the rectangular cross-section and the following dimensions: width – $b = 0.15$ m; height – $h = 0.30$ m; total length – $L = 3.00$ m; span – $l = 2.70$.

The experiments were performed using the four point bent specimens and special designed test equipment. The stand was constructed in such a way as to enable the observation of the beams' work in post-critical range. Beams were loaded by two concentrated forces, which were applied from bottom to top by hydraulic jacks. During the test, the displacement-controlled experimental procedure was used in order to slow down the cracking process and to observe crack formation more precisely. The static scheme of the test specimen is shown in Fig. 2. During the subsequent load stages of the test, beam's deflections, applied external forces and concrete strains were measured.

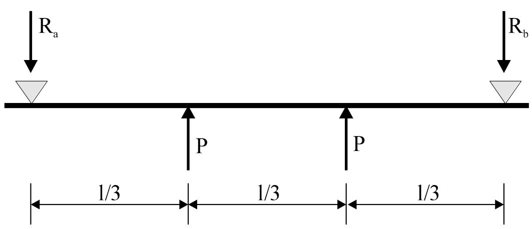

Figure 2: Static scheme of tested beams.

The beams and additional specimens for testing concrete parameters were made with the same concrete mixture. Concrete parameters: concrete compressive strength f_{cm}, concrete tensile strength f_{ctm} and Young's modulus E_{cm} were tested by standard methods. Concrete compressive and Young's modulus were tested on cylindrical specimens. Concrete tensile strength was tested on cubic specimens by splitting test f_{ctsp} and then the axial tensile strength was recalculated as $f_{ctm}= 0.9\, f_{ctsp}$.

The fracture energy G_F was estimated from the equation proposed in the CEB-FIP Model Code [17]:

$$G_F = \alpha_F\, f_{cm}^{0.7} = 82.95 \text{ N/m} \tag{7}$$

where: α_F - coefficient, which depends on the maximum aggregate size d_{max} ($\alpha_F = 4$ when $d_{max} = 8$ mm; $\alpha_F = 6$ when $d_{max} = 16$ mm; $\alpha_F = 10$ when $d_{max} = 32$ mm); f_{cm} – mean compressive strength of concrete.

Because of the reversed load scheme and the influence of the weight of the beam, the highest values of the bending moment were obtained in the sections of the applied forces. The destructive crack in the tested beams formed near the cross-section where one of the forces was applied. The crack always propagated perpendicular to the beam axis. A picture of the failure crack for an example beam is shown in Figure 3.

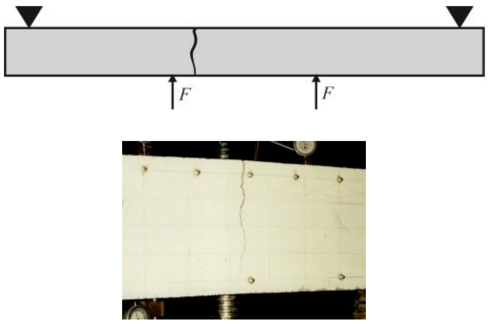

Figure 3: Picture of the failure crack - an example.

The following cracking moments were obtained in tested beams: 4.447 kNm, 4.549 kNm, 5.019 kNm, 5.116 kNm, 5.379 kNm, 5.473 kNm. The mean value of experimental cracking moment was $M_{cr,E} = 5.00$ kNm. Such results were a stimulation to verify it in a probabilistic way including models according to Equations 1, 3 and 6.

4 STOCHASTIC MODEL

The multipurpose probabilistic software FReET has been used for statistical, sensitivity and reliability analysis of the design problem connecting with the cracking resistance of concrete flexural members [18,19]. When performing the reliability analysis, the parameters included in the design formulas of cracking resistance of concrete members were treated as random variables:

1. Modulus of elasticity E_{cm} ;
2. Concrete compressive strength f_c;
3. Concrete axial tensile strength f_{ctax};
4. Total height of the member cross section h;
5. Width of the member cross section b;
6. Cracking moment obtained from experiments $M_{cr,exp}$.

The experimental database used in determining the model uncertainty was compiled from the experimental investigation. The type of the distribution and its statistical moments (descriptive statistics) of individual variable were adopted on the basis of experimental results. They are summarized in Table 1.

Table 1: Statistical charcterestics for considered variables

Variable	Distri bution	Mean value	Standard deviation	Coefficient of variation
f_c [MPa]	LN	27.67	2.769	0.100
$f_{ct,ax}$ [MPa]	LN	1.472	0.1585	0.108
E_{cm} [MPa]	LN	22118	1927.1	0.087
h [m]	N	0.30	0.009	0.030
b [m]	N	0.15	0.0045	0.030
$M_{cr,exp}$ [kNm]	LN	4.997	0.4220	0.084

As it is pointed in [20] statistical correlations among basic random variables should be taken into account. Hovewer, in most cases, there is a lack of information on statistical correlation [21]. In the performed analysis, the statistical correlation among random variables was included only according to concrete parameters with respect to previously performed tests reported in the literature [1, 22-24]. The correlation matrix is defined in Table 2.

Table 2: Correlation matrix

	E_{cm}	f_c	f_{ctax}
E_{cm}	1	0.4	0
f_c	0.4	1	0.4
f_{ctax}	0	0.4	1

Model predictions were provided by the use of Latin Hypercube Sampling (LHS). The probabilistic mean sampling scheme was employed to performe 30 random simulations. The statistical parameters were evaluatedand and histograms were built for all the dependent variables, i.e., fracture energy G_F, ratio h/l_{ch}, experimental cracking moment R($M_{cr\ exp}$), and cracking resistance calculated based on

Eq. 1 – R($M_{cr\ T}$)=(bh^2/6) f_{ctax},

Eq. 3 – R($M_{cr\ EC2}$)=(bh^2/6) f_{ctfl} and

Eq. 6 – R($M_{cr\ kappa}$)=(bh^2/6) κf_{ctax}.

Model analysis was performed using prepared samples of random variables presented in Tab. 1 and the sumarized statistical results are presented in Table 3. The obtained probability density distributions for experimental cracking moment R($M_{cr\ exp}$) and cracking moments R($M_{cr\ T}$), R($M_{cr\ EC2}$) and R($M_{cr\ kappa}$) are compared in Fig. 4.

Table 3: Statistical results

Variable	Mean value	Standard deviation	Coefficient of variation
G_F [N/m]	102.08	7.226	0.071
h/l_{ch} [-]	0.29	0.067	0.229
R($M_{cr\ exp}$) [kNm]	5.00	0.427	0.085
R($M_{cr\ T}$) [kNm]	3.31	0.414	0.125
R($M_{cr\ EC2}$) [kNm]	4.31	0.525	0.122
R($M_{cr\ kappa}$) [kNm]	5.36	0.491	0.091

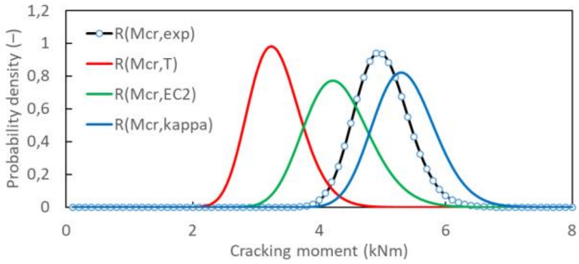

Figure 4: The comparison of probability density distribution for predicted cracking moments.

5 MODEL UNCERTAINTIES ASESSMENT

Three types of uncertainty are defined in Probabilistic Model Code [25]:

- intrinsic physical or mechanical uncertainty,
- statistical uncertainty, when the design decisions are based on a small sample of observations or when there are other similar conditions,
- model uncertainties.

Model uncertainties are not only inherent in the design basic variables, but also arise due to some uncertainties associated with the prediction model itself. Therefore, the model is also characterized by its parameters which should be considered as random variables. The model uncertainty reflecting the difference between experiment and simulation may be described by the ratio θ_i:

$$\theta_{\mathrm{i}} = R_{exp,i} / R_{cal,i} \tag{8}$$

where: $R_{exp,i}$ is the structural resistance obtained from experiment i; $R_{cal,i}$ is the structural resistance obtained from design model of experiment i.

The determination of model uncertainty is typically conducted through a statistical approach, whereby the responses derived from a set of experimental measurements are compared with the predictions of the computational model. The obtained results are then statistically calculated and the statistical parameters of the model uncertainty, i.e. the mean value μ_θ and coefficient of variation V_θ, are obtained. In line with code recommendations, the probability distribution is considered to be a two-parameter lognormal distribution.

The analysis has been determined to calculate the statistics of the model uncertainty described by the ratio θ and its variability, associated with the prediction models for cracking resistance for concrete members. Characterization of the model uncertainty was achieved by comparing ultimate cracking resistance obtained from experimental tests on representative members to the predictions of each of the design models. In effect, the statistics of three separate cases of the model uncertainty were characterized:

$\theta_1(R) = R(M_{cr\ exp})/R(M_{cr\ T})$ based on Eq. 1,

$\theta_2(R) = R(M_{cr\ exp})/R(M_{cr\ EC2})$ based on Eq. 3.

$\theta_3(R) = R(M_{cr\ exp})/R(M_{cr\ kappa})$ based on Eq. 6.

The statistical parameters for model uncertainties θi are presented in Tab. 4 (mean value μ_θ, standard deviation s_θ, coefficient of variation V_θ) and two-parameter lognormal probability density distributions for model uncertainties are sumarized in Fig. 5.

Table 4: Statistical parameters for model uncertainity

Result name	μ_θ	s_θ	V_θ
$R(M_{cr\ exp})/R(M_{cr\ T})$	1.532	0.244	0.199
$R(M_{cr\ exp})/R(M_{cr\ EC2})$	1.178	0.185	0.157
$R(M_{cr\ exp})/R(M_{cr\ kappa})$	0.940	0.120	0.128

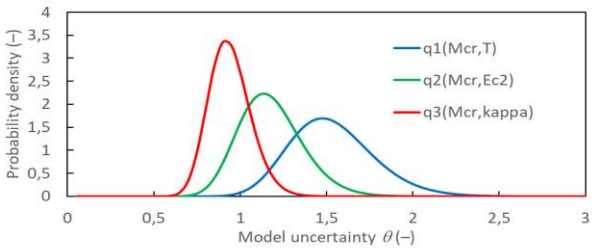

Figure 5: Probability density distributions for model uncertinties.

The study here implies to use following random variable as model uncertainty related to prediction formula:

1. $M_{cr,T} = W_c\, f_{ctax}$: $\mu_{\theta1} = 1.532$, $V_{\theta1} = 0.199$,
2. $M_{cr,EC2} = W_c\, f_{ct,fl}$: $\mu_{\theta2} = 1.178$, $V_{\theta2} = 0.157$,
3. $M_{cr,kappa} = W_c\, \kappa f_{ctax}$: $\mu_{\theta3} = 0.940$, $V_{\theta3} = 0.128$.

A value of $\theta > 1.0$ points that the prediction model yields a lower value of cracking resistance and is thus conservative, while a value of $\theta < 1.0$ implies that the prediction model yields higher cracking resistance than is actually available in the structure and is thus un-conservative.

6 CONCLUSIONS

The accuracy and reliability of the calculation method can be assessed by analyzing the model uncertainty.

The model uncertainty $\theta_1(R) = R(M_{cr\ exp})/R(M_{cr\ T})$, determined for cracking resistance based on the formula in which the axial tensile strength was applied (Eq.1), was the highest and a mean value reached $\mu_{\theta1} = 1.53$ with a relatively high coefficient of variation $V_{\theta1} = 0.20$. The model uncertainty $\theta_2(R) = R(M_{cr\ exp})/R(M_{cr\ EC2})$ decreased when applying flexural tensile concrete strength $f_{ct,fl}$ instead of f_{ctax} (Eq. 3) and the mean value was $\mu_{\theta2} = 1.18$. The coefficient of variation was lower too $V_{\theta2} = 0.16$. Although both formulas $R(M_{cr\ T})$ and $R(M_{cr\ EC2})$ appeared to be conservative but it has been observed that when using $f_{ct,fl}$ described by Eq. 2, the size effect can be taken into account and a better prediction of cracking resistance has been obtained.

In case of using the coefficient κ (Eq. 6), the mean value of model uncertainty $\theta_3(R) = R(M_{cr\ exp})/R(M_{cr\ kappa})$ was $\mu_{\theta3}=0.94$ and a lower coefficient of variation than in case 1 and 2 was noticed $V_{\theta3} = 0.13$. Although the mean value of model uncertainty $\theta_3(R)$ was lower than 1.0, which suggests the model $R(M_{cr\ kappa})$ to be un-conservative, but was close to 1.0 with a relatively small scatter.

It can be concluded that cracking resistance calculation for unreinforced concrete members based on the new formula with the inclusion of the coefficient κ showed a better agreement with test data comparing to standard design methods. When using the coefficient κ, both the influence of the size of the member and the impact of fracture properties of concrete on cracking resistance can be taken into account.

The performed analysis brought the promising results on a good prediction of cracking resistance of concrete members using the new model $R(M_{cr\ kappa})$. Further analyses are necessary to verify the model for members of different sizes and for concretes of different strengths.

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