Fracture Mechanics of Concrete Structures,
Proceedings FRAMCOS-2, edited by Folker H. Wittmann,
AEDIFICATIO Publishers, D-79104 Freiburg (1995)

# INFLUENCES OF NOTCH SIZE, ECCENTRICITY AND ROTATIONAL STIFFNESS ON FRACTURE PROPERTIES DETERMINED IN TENSILE TESTS

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## Abstract

This paper deals with the stability and validity of direct tensile tests in the determination of the cohesive stress-deformation relationship. The involved influencing factors, such as notch depth and size of specimen, eccentricity and rotational stiffness of the loading system, are investigated by means of the Fictitious Crack Model.

The main conclusions may be outlined as follows. The introduction of a notch produces negligible errors to the measured stress-deformation curve for concrete specimens of normal laboratory size, whereas an eccentricity may give rise to large errors. Rotational stiffness of the whole loading system should be sufficiently high in order to obtain a valid stress-deformation relationship, provided that specimens size is properly chosen. Steep jumps observed on experimental curves are closely related to the transition from combined tension and bending to pure tension in the fracture cross-section.

## 1 Introduction

For quasi-brittle materials such as concrete, the fracture behavior is characterized by a large cohesive Fracture Process Zone (FPZ) in front of the crack tip. Consequently, the conventional Linear Elastic Fracture Mechanics (LEFM) cannot be readily applied to fracture of concrete structures of normal size. Therefore various non-linear fracture mechanics models, for instance the Fictitious Crack Model (FCM) by Hillerborg et al. (1976), Crack Band Model by Bazant et al. (1983), have been developed and applied successfully to the analysis of plain or lightly reinforced concrete structures.

In those models the basic law for FPZ is the cohesive stress-deformation relationship and is best determined from direct tensile tests on notched or necked specimens. Such tests, however, are by no means easy to carry out and often hampered by stability problems.

There are two types of stability problems which are associated with the tensile stiffness and rotational stiffness of the whole testing system, respectively. The former problems can be solved by using the modern close-loop testing machine and proper deformation feedback directly measured over the fracture cross-section.

Another type is observed by Hordijk et al. (1987) and Hordijk (1991). Deformations are not evenly distributed across the fracture cross-section and steep jumps sometimes occur on the stress-deformation curves.

It is thus intended to investigate the effects of notch depth, eccentricity, size, rotational stiffness on the measured stress-deformation relationship and to clarify whether and how a stable and valid tensile test can be made. Further detail can be found in the licentiate's thesis by Zhou (1988).

## 2 Methods of Analysis

The Fictitious Crack Model is able to provide the most realistic simulation of crack formation and development of concrete, including both local crack growth and stresses and deformations, and global load-deformation response. Therefore it is suitable to use this model for the present study.

The material laws for FCM are the stress-deformation curve for FPZ and the stress-strain curve for the undamaged zone. The stress-strain is approximated to be linear and the stress-deformation curve to be bilinear (Fig. 1). Specimen size and basic material properties such as Young's modulus, tensile strength and fracture energy are shown in Fig. 1. The characteristic length is $l_{ch} = EG_F / f_t^2 = 0.15m$.

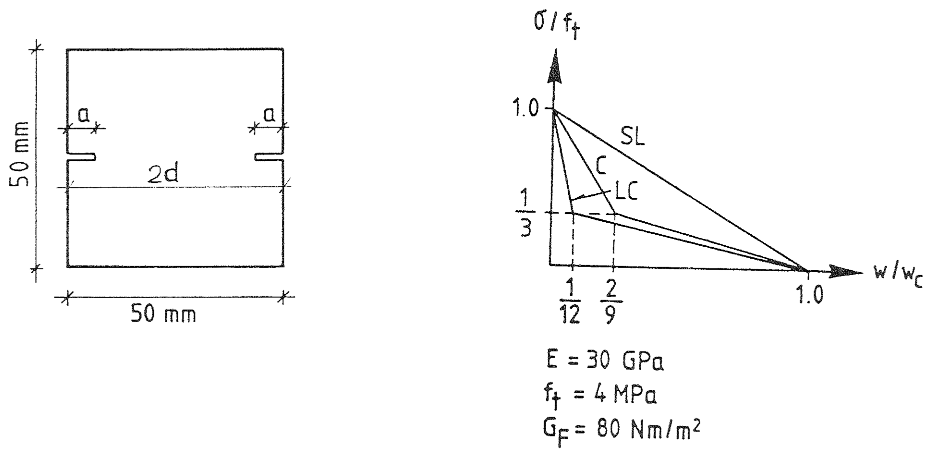


Fig. 1 Dimensions of specimen and stress-deformation curves.

## 3 Influencing Factors

In direct tensile tests for the determination of the tensile stress-deformation curve, notched or necked specimens are used to facilitate the measurement of deformation over the fracture process zone. It is thus intended to estimate possible errors caused by the introduction of a notch.

The measured stress-deformation curve is considered to be a valid material law only if the deformations are evenly distributed across the fracture cross-section in the whole test. However, a possible eccentricity, which may be caused by a deviation of loading point, non-symmetry of specimen geometry, or non-homogeneity of material properties, may give rise to uneven deformation distribution across the fracture cross-section.

Therefore, it is attempted to simulate the effects of an eccentricity on deformation distribution and the stress-deformation curve. Furthermore, it is intended to investigate how rotational stiffness of test set-ups and specimen length may help to count-act the possible rotation in the fracture cross-section.

### 3.1 Notch sensitivity

The simulated stress-deformation curve and stress distributions across the fracture cross-section are shown in Fig. 2. The geometry of specimen (notch ratio $a/d = 0.25$) and the material law C in Fig. 1 were used in the simulation. Although stress distributions are not even before the peak point, they tend to be uniform just after the point. Besides, the simulated curve deviates slightly only around the peak point from the true input curve.

Fig. 3 shows that notch sensitivity of measures strength depends on brittleness number ($d/l_{ch}$) and notch depth. If a 5% error on strength is considered acceptable, $d/l_{ch}$ should be less than about 0.3 for notch ratio

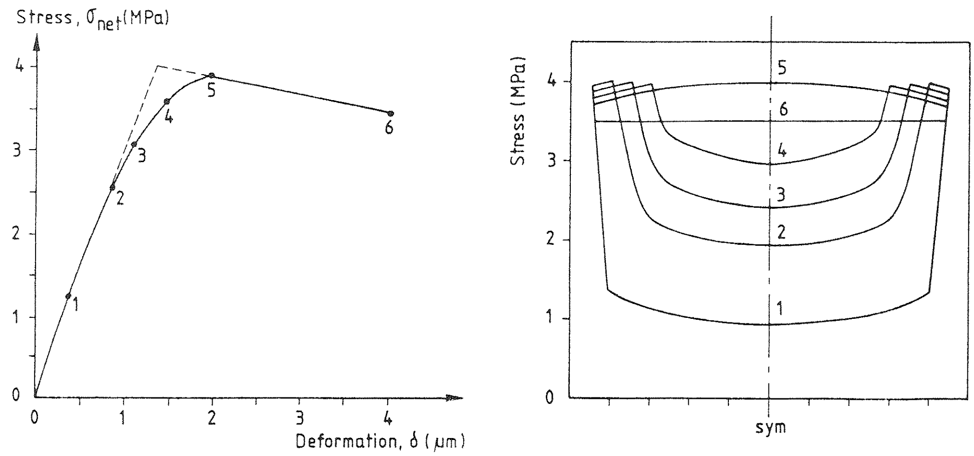


Fig. 2 Simulated (solid line) and true (dash line) stress-deformation curves and stress distributions across the fracture cross-section.

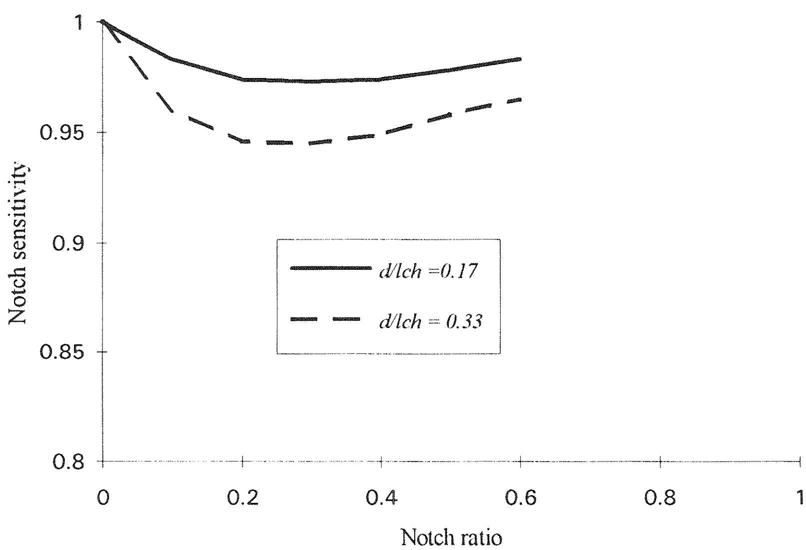


Fig. 3 Notch sensitivity of tensile strength predicted by FCM.

between 0.2 and 0.5. That is to say, specimen width ($=2d$) should be chosen to be less than about $0.6l_{ch}$. For normal concrete, $l_{ch} = 0.1 \rightarrow 0.4m$, specimen width changes from about 60 mm to 240 mm accordingly.

For large specimen size and brittle materials, LEFM is applicable to analysis of notch sensitivity. The stress intensity factor for a double notched prismatic specimen can be calculated using the equation (Bethem et al. (1973)):

$$K_I = \frac{P}{2Bd}\sqrt{\pi a}Y(\frac{a}{d}) \quad (1)$$

where $Y^P$ is a geometry function as:

$$Y^P(\frac{a}{W}) = (1-\frac{a}{W})^{\frac{1}{2}}(1.122 - 0.561\frac{a}{W} - 0.015(\frac{a}{W})^2 + 0.091(\frac{a}{W})^3). \quad (2)$$

The critical stress intensity factor can be related to fracture energy as:

$$K_{IC} = \sqrt{EG_F}\,. \quad (3)$$

The nominal strength is defined as:

$$\sigma_{net} = \frac{P_{max}}{2B(d-a)}. \quad (4)$$

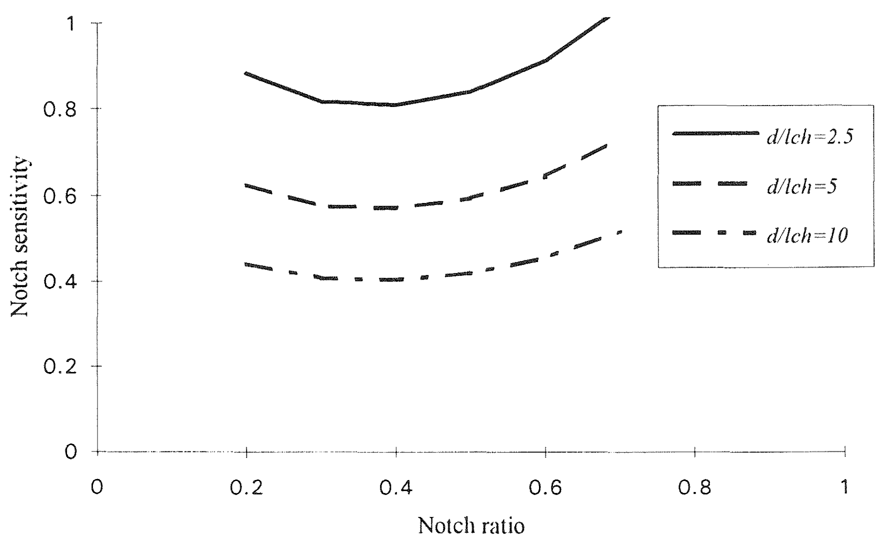


Fig. 4 Notch sensitivity of tensile strength predicted by LEFM.

The ratio between nominal strength and true tensile strength (or referred to as notch sensitivity) can be derived as:

$$\frac{\sigma_{net}}{f_t} = (\frac{d}{l_{ch}})^{1/2} / (\sqrt{\pi a / d}(1 - a/d) f(\frac{a}{d})) \quad (5)$$

The equation is depicted in Fig. 4 for $d / l_{ch}$ = *2.5, 5* and *10*. As can be observed, tensile strength tends to be more sensitive to a notch when material becomes more brittle and specimen size increases.

### 3.2 Eccentricity

The effect of an eccentricity on tensile strength and the stress-deformation curve is investigated through simulating a tensile test subject to a eccentric load by FCM. The geometry of specimen ($a/d = 0.25$) and the material law C in Fig. 1 were used in simulation. Small load eccentricities 1 mm and 2.5 mm (i.e. e/d=4% and 10%) are introduced.

The influences on stress-deformation curve and deformation distributions across the fracture cross-section are given in Fig. 5. As can be seen, the stress-deformation curve in the case of eccentric loading may differ strongly from the true curve. Deformation distribution across the fracture cross-section tends to be uneven as the average deformation increases and the fracture cross-section tends to bend instead of pure tension.

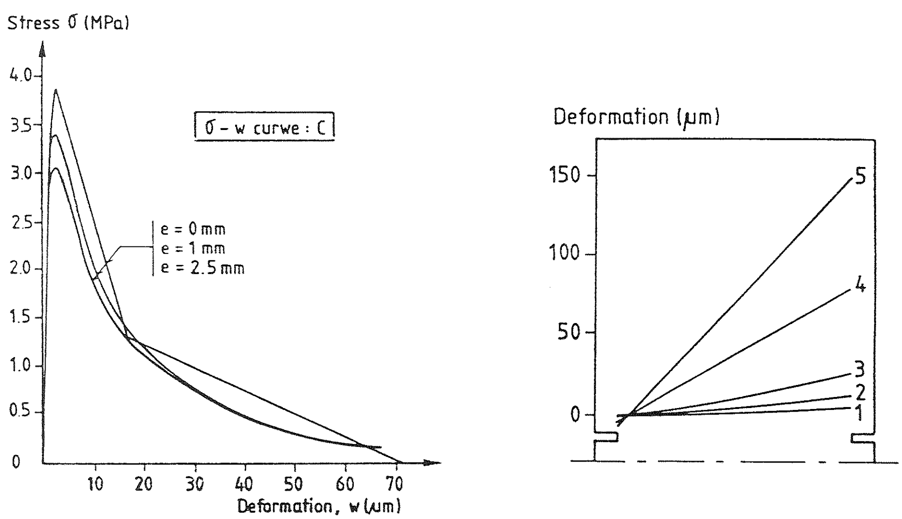


Fig. 5 Influence of eccentricity on the stress-deformation curve and deformation distribution across the fracture cross-section.

The errors on measured tensile strength caused by eccentricity are observed in Fig. 5. An eccentricity of 1 to 2.5 mm may cause 15% to 23% errors on strength. Although the increase of specimen size may reduces possible eccentricity and the consequent errors, notch sensitivity impose a limit on the increase in size.

### 3.3 Rotational stiffness

If rotational stiffness of the test set-ups is sufficiently high, the rotation in fracture cross-section may be partly or completely prevented. To illustrate such effects, different rotational stiffness values are imposed at the ends of specimen (eccentricity = 4%). The geometry of specimen and material law LC in Fig. 1 were used.

The result is shown in Fig. 6. When the rotational stiffness is high enough, the simulated curve is almost the same as the true curve except for small deviation around the peak-point.

On the other hand, when the rotational stiffness is zero and the ends of the specimen are allowed to rotate freely, the stress-deformation curve differs much from the true curve. However, the simulated curve appears to be smooth and no sudden stress jumps on the curve connected with the observed unstable effect.

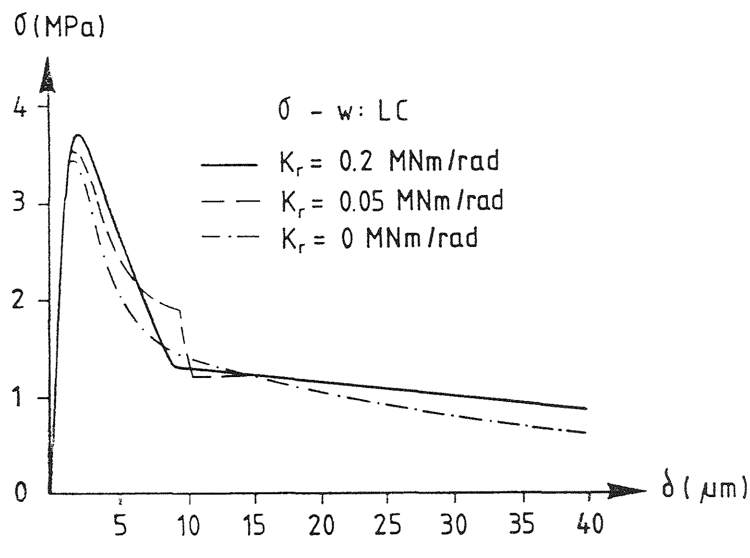


Fig. 6 Effect of rotational stiffness on tensile tests.

A stress jump is observed only in case of the intermediate stiffness. It may be inferred that this phenomenon is closely related to a transition from combined tension and bending to pure tension in the fracture cross-section.

### 3.4 Specimen length

It should be pointed out that not only the rotational stiffness but the stiffness of specimen are also very important. If a specimen is too long and thin, the possible rotation across fracture cross-section cannot be prevented by whatever stiffness of the test-ups, as shown in the work by Rots et al. (1989). This can be illustrated by a criterion for valid tensile tests proposed by Hillerborg (Hazzandeh et al (1987), Hillerborg (1989)).

The criterion is expressed as:

$$(\frac{1}{K_r}+\frac{h}{2EI_0})^{-1} \succ 2I_c(-\frac{d\sigma}{dw})_{\max} \qquad (6)$$

where $K_r$ is the rotational stiffness, $E$ is Young's modulus, $I_0$ and $I_c$ are moment of inertia for unnotched section and notched section, respectively. $(-\frac{d\sigma}{dw})_{\max}$ is the steepest slope in the stress-deformation curve. $h$ is length of specimen.

The maximum length may be found through assuming the stiffness as infinite:

$$h_{\max} \prec (I_0 / I_c)E / (-\frac{d\sigma}{dw})_{\max}. \qquad (7)$$

For the often used bilinear stress-deformation curves, $(-\frac{d\sigma}{dw})_{\max}$ is the initial slope and is equal to $f_t^2 / CG_F$ where C is a constant. Therefore, the Equation above can be changed into:

$$h_{\max} \prec (I_0 / I_c)Cl_{ch}. \qquad (8)$$

As C is about 1, the maximum length should not be greater than the characteristic length which is from 0.1 to 0.4 m for most concrete materials.

## 4 Concluding remarks

Stability and validity of direct tensile tests on notch specimens for the determination of the cohesive stress-deformation relationship have been analyzed. The effects of notch depth, eccentricity, rotational stiffness have been investigated. The following conclusions may be drawn.

The introduction of a notch imposes some errors on the measured tensile strength and the stress-deformation curve if a notched specimen is used. However, the error is acceptable for concrete specimen of normal laboratory size.

Eccentricity can give rise to large errors if the specimen is allowed to rotate freely. If the stiffness of loading system is high and specimen is not too long, this error can be reduced to an acceptable extent.

A steep jump on the stress-deformation curve is closely related to a transition from bending to pure extension.

## 5 References

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