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# A METHOD TO EVALUATE TENSION SOFTENING MODEL FOR CONCRETE

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## Abstract

A convenient method to determine the parameters of bi-linear tension softening model by using a standard fracture energy test have been studied. With introducing new empirical parameters, brittleness index (BRI) and initial fracture work ($g_\beta$), it is indicated that the first and second slope, $T_1$ and $T_2$, of the bi-liner model can be determined by $g_\beta$-$T_1$ relationship and BRI-$T_1T_2$ relationship. In addition, with the fracture energy and a tensile strength all four parameters of bi-linear model is capable of assessing. A verification using the results of three-point bending test and wedge splitting test was made.

## 1 Introduction

For evaluating the tension softening of concrete, several methods have been reported in empirical and/or theoretical approach such as Cornelissen et al. (1986), Roelfstra et al. (1986), Li et al. (1987) and Hu et al. (1989). From the numerical point of view, however, those methods does not seem to be convenient to determine the model for an numerical analysis such as finite element analysis. The object of this paper is to describe an empirical and simpler method to evaluate the tension softening of concrete. Basic concept is to determine the parameters of bi-linear type tension softening model based upon a fracture energy test result.

## 2 Outline of procedure

### 2.1 Procedure

When we adopt the bi-linear tension softening model in a numerical analysis, it is necessary to determine the four parameters of the bi-linear model, $F_t$, $s_1$, $w_1$ and $w_2$ (Fig. 1). In this case the $F_t$ in not a tensile strength but an internal local stress level where local softening takes place. To determine them, a fracture energy ($G_F$) test, a tensile strength ($f_t$) test and a static elastic moduli test are of use. The step of procedure is:

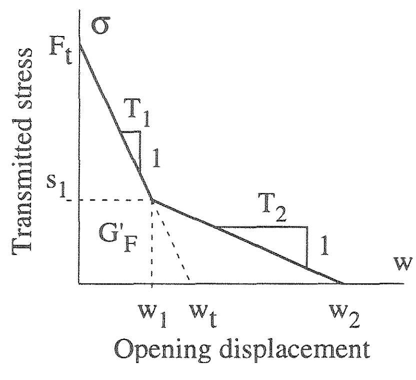


Fig. 1. Parameters notation in bi-linear tension softening model

1. $F_t$ is determined by using $F_t$-$f_t$ relation which is an empirical relationship.
2. $s_1$, $w_1$ and $w_2$ are not directly determined. They are calculated from some medium parameters, $T_1$, $T_2$ and $G'_F$ (Fig. 1). $T_1$ and $T_2$ is the first

and the second slope of the bi-linear model respectively, and $G'_F$ is fracture energy calculated from the bi-linear model.

3. $T_1$ is determined by using $g_\beta$-$T_1$ relation which is established by numerical experiments. Here, $g_\beta$ is a new empirical parameter related to the initial part of fracture work.
4. $T_2$ is determined by using BRI-$T_1T_2$ relation which is an empirical relationship. Here, BRI is a new empirical parameter related to the brittleness of material.
5. $G'_F$ value is considered to be the same to $G_F$ obtained by an actual fracture energy test.

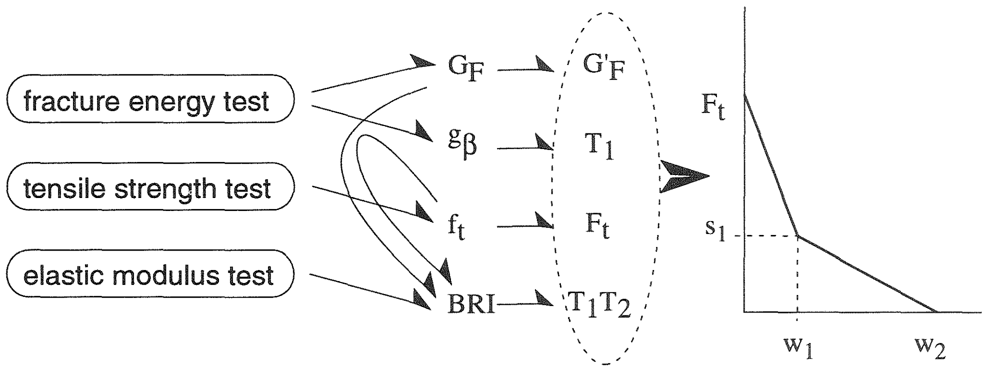


Fig. 2. Procedure to determine parameters

## 2.2 Relationship between parameters

The parameters are related to each other. $T_1$, $T_2$ and $G'_F$ are defined by the expression (1).

$$T_1 = \frac{w_1}{F_t - s_1}$$

$$T_2 = \frac{w_2 - w_1}{s_1} \qquad (1)$$

$$G'_F = \frac{1}{2}(F_t w_1 + s_1 w_2)$$

If $F_t$ is a known parameter, then:

$$s_1 = (2G'_F - w_t F_t)^{0.5} / (T_2 - T_1)^{0.5}$$

$$w_2 = w_t + (2G'_F - w_t F_t)^{0.5} (T_2 - T_1)^{0.5} \qquad (2)$$

$$w_1 = w_t - T_1 (2G'_F - w_t F_t)^{0.5} / (T_2 - T_1)^{0.5}$$

$$; w_t = T_1 F_t$$

Fig.2 illustrates the procedure of determining the softening model parameters from conventional tests and the concerned relationship.

## 3 Experimental verification

In order to verify the concept of determining the parameters in bi-linear softening model, our experimental results previously presented (Mihashi (1992), Kim (1992)) were of use. In the previous study, three different strength of concrete with varying a maximum aggregate size had been tested.

Table 1. Mix proportion and test results

| dmax | W/C | W | C | S | G | Si | s.p. | fc | ft | Es | Ed | Vp | BRI |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (mm) | | (kg) | | | | | (cc /CW) | (MPa) | | (GPa) | | (mm$^3$/g) | (m$^{-1}$) |
| 25 | 0.25 | 111 | 444 | 721 | 1006 | 111 | 35.5 | 96.4 | 6.42 | 41.5 | 54.4 | 29.9 | 7.91 |
| 25 | 0.24 | 111 | 472 | 721 | 1006 | 83 | 40.6 | 72.8 | 6.88 | 45.6 | 51.4 | 27.8 | 7.80 |
| 25 | 0.47 | 160 | 340 | 721 | 1006 | 60 | 12.1 | 60.2<br>68.8 | 4.70<br>5.18 | 34.0<br>38.0 | 43.5<br>44.7 | 60.8<br>46.6 | 4.44<br>5.78 |
| 25 | 0.60 | 188 | 313 | 721 | 1006 | 0 | 0.0 | 32.2 | 3.14 | 32.0 | 37.7 | 78.9 | 2.32 |
| 25 | 0.65 | 227 | 349 | 721 | 1006 | 0 | 0.0 | 21.4 | 2.88 | 27.8 | 34.5 | 100.6 | 1.59 |
| 5 | 0.25 | 111 | 444 | 721 | 1006 | 111 | 57.0 | 105.9 | 6.08 | 39.8 | 46.6 | 22.6 | 9.61 |
| 5 | 0.47 | 160 | 340 | 721 | 1006 | 60 | 28.0 | 58.8 | 3.82 | 35.3 | 41.8 | 49.7 | 3.74 |
| 5 | 0.65 | 227 | 349 | 721 | 1006 | 0 | 0.0 | 20.9 | 2.35 | 22.7 | 27.4 | 94.4 | 2.82 |
| - | 0.25 | 111 | 444 | 721 | 0 | 111 | 57.0 | 110.7 | 6.13 | 34.5 | - | 35.8 | 18.97 |
| - | 0.47 | 160 | 340 | 721 | 0 | 60 | 28.0 | 46.3 | 3.84 | 27.5 | - | 48.2 | 6.4 |
| - | 0.65 | 227 | 349 | 721 | 0 | 0 | 0.0 | 33.8 | 2.67 | 21.0 | - | 98.1 | 7.1 |

The mix proportion and the test results of compressive strength, splitting tensile strength, static and dynamic elastic moduli, porosity and BRI are given in Table 1. The values in the table are average ones of three specimens for each series.

Three point bending test (3PB) with 100x100x840 mm and wedge splitting test (WS) with varying the size of specimen had been carried out. In this paper, the results of 3PB and the results of WS for 200 mm(height) x 240 mm(width) x 120 mm(thickness) were used. The three point bending test was followed the RILEM recommendation (1985).

## 4 Results and discussion

### 4.1 $F_t$-$f_t$ relation

The $F_t$ of each series estimated by data fit method (Roelfstra (1986)) versus the tensile strength $f_t$ is shown in Fig. 3. The $F_t$ is proportional to $f_t$ up to about 3MPa. However, in the high stress range over 4MPa $F_t$ is 30% higher than $f_t$. It is considered that while the $F_t$ represents local tensile strength around a crack tip the $f_t$ is an averaged stress calculated using a nominal section area of fractured specimen.

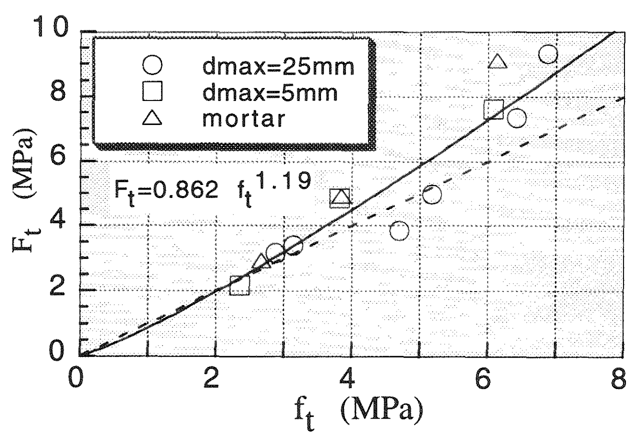


Fig. 3. $F_t$-$f_t$ relation

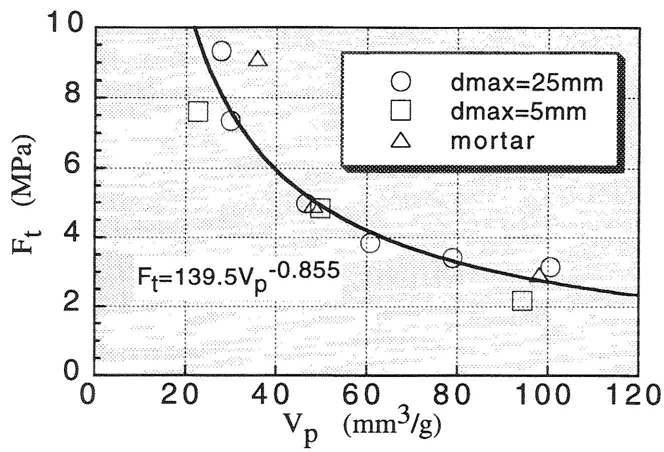


Fig. 4. $F_t$-Vp relation

The relationship between $F_t$ and the porosity Vp of the tested concrete is shown in Fig. 4. The diagram indicates that the local tensile strength $F_t$ is significantly attributed to the microstructure of concrete in terms of the porosity. From these results, if the empirical $F_t$-$f_t$ relation is provided for the test method of tensile strength, $F_t$ can be determined by the relation.

### 4.2 $g_\beta$-$T_1$ relation

A numerical calculation of the three-point bending test has been carried out with varying the $T_1$ value and with keeping $T_2$ value for several $F_t$. The $g_\beta$ is defined as the area of load-deflection curve, illustrated in Fig. 5, which is a work at the beginning of fracture. When $\beta$ takes the value 0.5 the numerical calculation provides a relationship between $g_\beta$ and $T_1$ as shown in Fig. 6. The $g_\beta$ is divided by the thickness of specimen, b. $g_\beta/b$ for each $F_t$ is clearly described by power functions. If the coefficients of the power function are represented by the function of $F_t$, $g_\beta/b$ is described by both $T_1$ and $F_t$. In the present case, $g_\beta/b$ is expressed as:

$$g_\beta/b = 0.0892F_t^{3.49}T_1^{(0.213+0.0762Ft)} \qquad (3)$$

The experimental $g_\beta/b$ measured by using load-deflection records are plotted in Fig. 5 with the value of $F_t$. The measured data is approximately agree with the numerical $g_\beta/b$-$T_1$ relationship. In an actual procedure of determining the softening parameters, while $F_t$ is already obtained by $F_t$-$f_t$ relation, measured $g_\beta/b$ provides a corresponding $T_1$.

The $g_\beta/b$-$T_1$ relation has to be prepared for the type of fracture energy test to be adopted. Fig. 7 demonstrates another $g_\beta/b$-$T_1$ relationship in the case of our wedge splitting test.

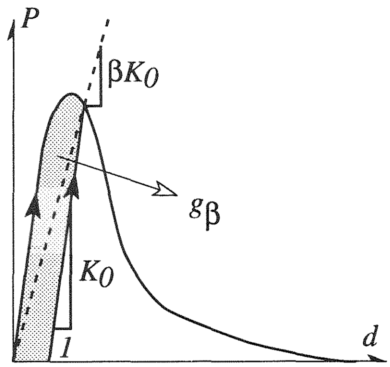


Fig. 5. Definition of $g_\beta$

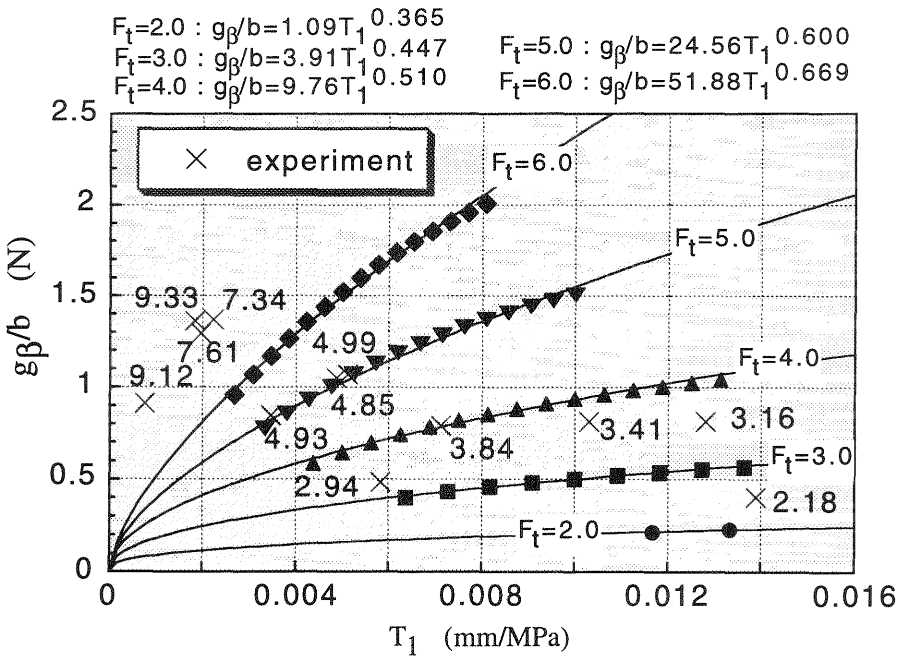


Fig. 6. $g_\beta/b$-$T_1$ relationship for three point bending test

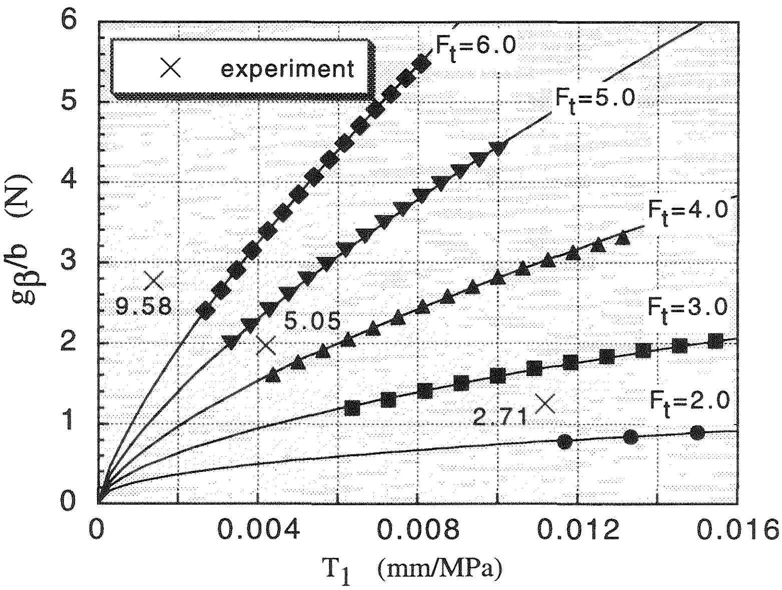


Fig. 7. $g_\beta$/b-$T_1$ relationship for wedge splitting test

### 4.3 BRI-$T_1T_2$ relation

The brittleness of material is represented both by elastic energy at a peak load and by the energy dissipation after the peak load. In the case of concrete, the ratio of the volumetric elastic energy $f_t^2/2E_s$ to the fracture energy $G_F$ is capable of introducing:

$$\gamma = f_t^2/2G_FE_S$$

If we defined a brittleness index (BRI) as 2γ, then:

$$BRI = f_t^2/G_FE_S \tag{4}$$

BRI can be reversed to get characteristic length $l_{ch}$ by Hillerborg (1983). In the case of BRI, the greater heterogeneity of material gives the higher value of BRI.

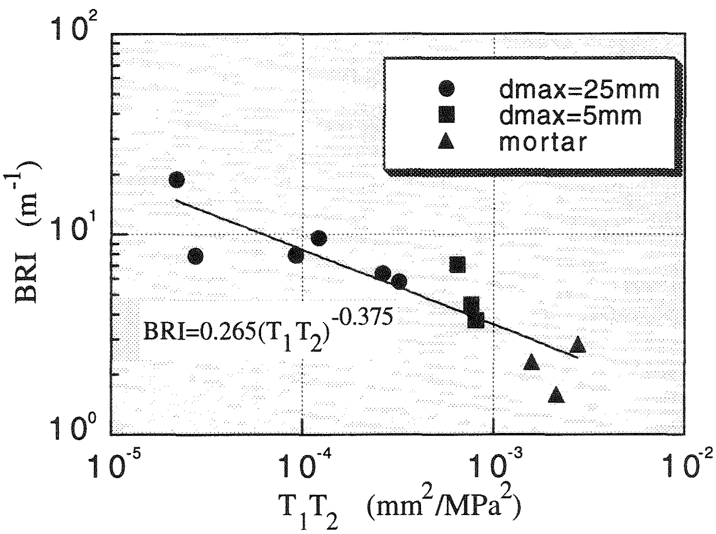


Fig. 8. Relation between brittleness parameters

BRI is expected to be a function of $T_1$ and $T_2$ because BRI indicates entire brittleness of whole material, as well, $T_1$ and $T_2$ represent the brittleness of tension softening model. BRI obtained from experimental data is shown in Fig. 8. BRI is expressed as the product of $T_1$ and $T_2$:

$$BRI = 0.265(T_1T_2)^{-0.375} \tag{5}$$

Once the relation of BRI-$T_1T_2$ is established like Eq. (5), BRI obtained from a fracture energy test provides the value of $T_1T_2$, which is one of condition to calculate the parameters expressed as Eq. (2).

## 5 Conclusion

A simple method of determining the parameters of bi-linear tension softening model of concrete is described. It is shown that new empirical parameters,brittleness index BRI and initial fracture work $g_B$ in a fracture energy test, are related to the slopes of bi-linear model. Consequently, the parameters of bi-linear tension softening model are capable of evaluating in terms of the relationship and by using several interrelation of the parameters.

## Acknowledgement

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