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A METHOD FOR DETERMINING FRACTURE PROPERTIES OF CONCRETE THROUGH A SINGLE TEST

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Abstract

The paper proposes a method for the determination of fracture properties of concrete, i.e. Young's modulus, E, tensile strength, f_t, fracture energy, G_F, through the same bending test on a notched beam.

Firstly, fracture energy is evaluated from the area under the load-deflection curve from the bending test, as suggested in the RILEM Recommendation. Secondly, the Young's modulus is calculated from the initial compliance in the same curve. Finally, the net flexural strength is evaluated from the peak load in the same curve and tensile strength can be derived from the relationship between net flexural strength/tensile strength ratio and brittleness number, or the generalized size-effect law, predicted by the Fictitious Crack Model.

The proposed method is verified using some experimental data available in the literature. The determined values of young's modulus and tensile strength are about 10% lower than the corresponding data evaluated from other separate experimental tests. From a practical point of view, the agreement may be satisfactory. The method might be a good and simple

practical testing method for the determination of fracture properties of normal concrete, though further assessment is needed.

1 Introduction

Nonlinear fracture mechanics models such as Fictitious Crack Model (FCM) proposed by Hillerborg et al. (1976), Crack Band Model by Bazant et al. (1983), have been proven to give a realistic description of fracture behavior of concrete. As a result, those models predict size effects of flexural strength and shear strength of plain and lightly reinforced concrete structures and are much better than other approaches based on strength theory or Linear Elastic Fracture Mechanics (LEFM).

The main feature of those models is the introduction of the tensile stress-COD relationship for the fracture process zone (FPZ), apart from the stress-strain law for the undamaged zone. However, it is very difficult to determine this relationship in direct tensile tests and great care should be taken to testing set-ups and specimen size in order to obtain stable and valid tests as shown in the licentiate's thesis by Zhou (1988). Fortunately, the stress-strain curve and stress-COD curve for most concrete materials of normal aggregate are quite similar and can be simplified to be linear and bilinear curves. Therefore, it is possible to determine these two relationships indirectly and only Young's modulus, E, tensile strength, f_t and fracture energy, G_F, are needed to determine. Even so, the determination of E, f_t and G_F still involves two or three types of tests.

It is thus intended to propose a more simple method for determination of fracture properties through a single bending test on notch beams as recommended for fracture energy test by RILEM. This method will be verified by comparing it with some experiments available in the literature.

2 Fracture mechanics properties

For the Fictitious Crack Model and other similar models, the constitutive laws include a stress-strain curve and a stress-COD curve (Fig. 1). For most plain concrete materials of normal aggregates, the shapes of these two curves are quite similar and are often simplified to be linear or bilinear. One of the most often used simplifications is proposed by Petersson (1981) and shown in Fig. 2. Therefore, these two curves can be obtained indirectly if Young's modulus, tensile strength and fracture energy are determined.

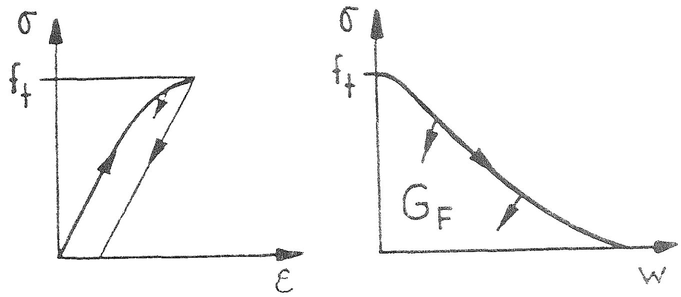

Fig. 1 General stress-strain curve and cohesive stress-COD curve

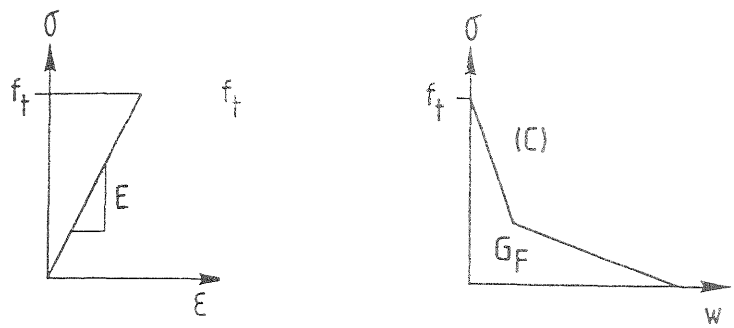

Fig. 2 Simplified linear stress-strain curve and bilinear stress-COD curve. The critical COD, w_c, is $18G_F/5f_t$. The stress and the COD at the knee point are $f_t/3$ and $2w_c/9$.

2.1 Fracture Energy

According to the RILEM Recommendation (1985), fracture energy is determined by means of three-point bending tests on notched beams (Fig. 3) and calculated by the following equation:

$$G_F = \frac{A1 + Mg\delta_0}{b(d-a)} \tag{1}$$

where A and δ_0 are shown in Fig. 3. b, d and a are thickness, height and initial notch depth, respectively. M and g are mass and gravity factor respectively.

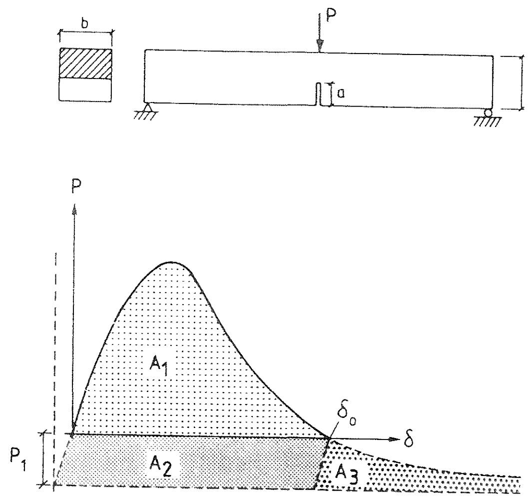

Fig. 3 RILEM Recommendation test for fracture energy (1985).

2.2 Young's modulus

The Young's modulus can be determined from compressive or tensile tests. A simple way is to use the resonance frequency method to determine dynamic modulus e.g. Petersson (1981). Besides, Young's modulus is also derived from the initial compliance in the load-CMOD curves, e.g. Shah (RILEM (1990)) or the initial compliance in the load-deflection curve, e.g. Karihaloo (1991).

2.3 Tensile strength

Tensile strength can be determined by direct tensile tests on notched or unnotched specimens. However, direct tensile tests are more difficult to perform and require sophisticated testing machines. Alternatively, tensile strength can be obtained from splitting tests. If there are no initial stresses it is believed that splitting tensile strength should be more or less higher than tensile strength.

3 Proposed method

In this method, fracture energy, Young's modulus and tensile strength are determined from the same three-point bending test on a notched beam (Fig. 4). The fracture energy is as usual evaluated from the area under the load-deflection curve. Young's modulus is evaluated from the initial compliance of the same curve whereas the tensile strength can be derived from the peak point load.

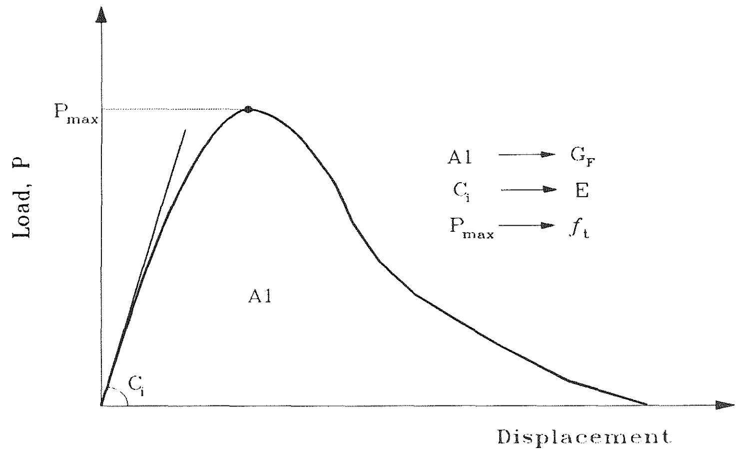

Fig. 4 Proposed method for the determination of fracture energy, Young's modulus, tensile strength from a single bending test on a notched beam.

3.1 Young's modulus

Young's modulus may be evaluated from the initial compliance in the load-deflection curve obtained from a bending test on notched beam.

Total deflection at the load application point is composed of the deflection due to the existence of a crack and the deflection due to the deformation of the beam when no crack exists:

$$\delta = \delta_{cr} + \delta_{ncr}. \tag{2}$$

The second term on the right-hand side may be approximately calculated using the following equation from the beam theory:

$$\delta_{ncr} = \frac{PS^3}{4Ebd^3} \quad . \tag{3}$$

The former can be calculated from the equation (Tada et al (1975)):

$$\delta_{cr} = \frac{3PS^2}{2Ebd^2} V(\frac{a}{d}). \tag{4}$$

Where the last term on the right-side of the equation is a geometry function. For $S/d = 4$, it is expressed as:

$$(\frac{a}{d}) = (\frac{a/d}{1-a/d})^2 (5.58 - 19.57(\frac{a}{d}) + 36.82(\frac{a}{d})^2 - 34.94(\frac{a}{d})^3 + 12.77(\frac{a}{d})^4). \tag{5}$$

For $S/d=8$, it is slightly different as:

$$(\frac{a}{d})=(\frac{a/d}{1-a/d})^2(5.755-19.63(\frac{a}{d})+36.98(\frac{a}{d})^2-35.39(\frac{a}{d})^3+12.345(\frac{a}{d})^4). \qquad (6)$$

Inserting Eqs. (3) and (4) into Eq. (2), we can obtain:

$$\delta=\frac{3PS^2}{2Ebd^2}V(\frac{a}{d})+\frac{PS^3}{4Ebd^3}=\frac{PS^2}{4Ebd^2}(6V(\frac{a}{d})+\frac{S}{d}). \qquad (7)$$

Young's modulus can thus be evaluated from the equation below:

$$E=\frac{P}{\delta}\frac{S^2}{4bd^2}(6V(\frac{a}{d})+\frac{S}{d})=\frac{1}{C_i}\frac{S^2}{4bd^2}(6V(\frac{a}{d})+\frac{S}{d}), \qquad (8)$$

where C_i is the initial compliance.

3.2 Tensile strength

Net flexural strength in bending tests can be related to specimen size, tensile strength, Young's modulus and fracture energy as shown in Fig. 5.

Net flexural strength is defined for three-point bending tests as:

$$f_{net}=\frac{3(P_{max}+(S-L/2)bd\rho g)S}{2bd^2} \qquad (9)$$

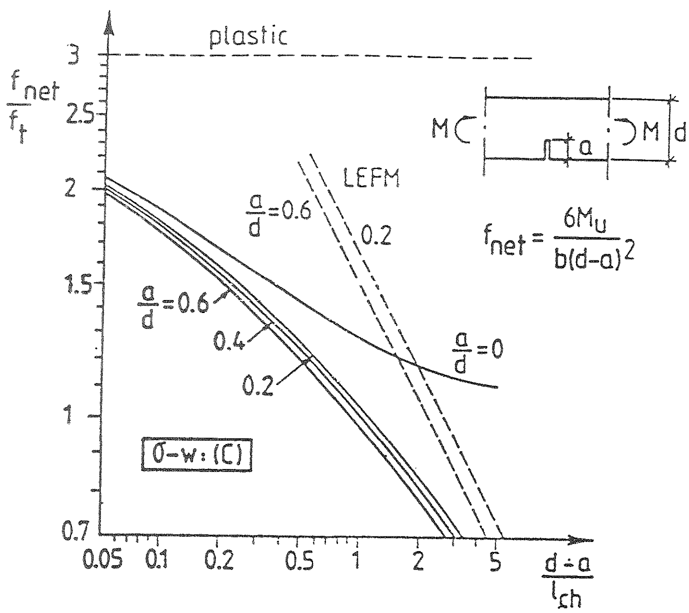

Fig. 5 Flexural/tensile strength ratio predicted by Petersson (1981).

where P_{max} is peak-point load, and S, L, b and d are span and length of the beam. ρ and g are density of the concrete and gravity factor.

Based on the data (Table 1) read from Fig. 5, a simple best-fitting equation is obtained to facilitate the calculation of tensile strength from net flexural strength.

The curves in Fig. 5 may be fitted by:

$$\frac{f_{net}}{f_t} = \alpha(1+\beta(\frac{d-a}{l_{ch}})^r)^{-1/2r}. \tag{10}$$

It can be converted into:

$$(\frac{f_{net}}{f_t})^{-2r} = \alpha^{-2r} + \alpha^{-2r}\beta(\frac{d-a}{l_{ch}})^r \equiv \alpha_1 + \beta_1(\frac{d-a}{l_{ch}})^r \tag{11}$$

which is much easy to be used to fit the curves. As suggested by Bazant (1986), $r = 0.44$ could make a very good fitting.

The bets-fitting equation is found for $a/d = 0.2$ as

$$(\frac{f_{net}}{f_t})^{-0.88} = 0.390 + 0.624(\frac{d-a}{l_{ch}})^{0.44} \tag{12}$$

and for $a/d = 0.6$ as:

$$(\frac{f_{net}}{f_t})^{-0.88} = 0.382 + 0.575(\frac{d-a}{l_{ch}})^{0.44} \tag{13}$$

Table 1 Data read from Fig. 5 used for obtaining the best-fitting equation.

$\frac{d-a}{l_{ch}}$	$\frac{f_{net}}{f_t}$			
	$a/d = 0.2$		$a/d = 0.6$	
	FCM	Best-fitting	FCM	Best-fitting
0.05	1.97	1.94	2.07	2.03
0.1	1.74	1.73	1.82	1.82
0.2	1.51	1.51	1.58	1.59
0.5	1.20	1.20	1.27	1.28
1.0	0.98	0.98	1.04	1.05
2.0	0.78	0.78	0.85	0.84

The comparisons between the best-fitting equations and the corresponding data read from the curves are given in Table 1.

For any a/d between 0.2 to 0.6, the following equation is obtained by interpolating linearly between $a/d = 0.2$ and $a/d = 0.6$.

$$(\frac{f_{net}}{f_t})^{-0.88} = (0.382 + \frac{0.390-0.382}{0.6-0.2}(0.6-\frac{a}{d})) + (0.575 + \frac{0.624-0.575}{0.6-0.2}(0.6-\frac{a}{d}))(\frac{d-a}{l_{ch}})^{0.44} \tag{14}$$

From the equation above, tensile strength can be expressed as a function of net flexural strength, Young's modulus and fracture energy as:

$$f_t = f_{net}(0.394 - 0.02\frac{a}{d})^{1.136}\left(1-(0.648-0.122\frac{a}{d})(\frac{d-a}{EG_F})^{0.44} f_{net}{}^{0.88}\right)^{-1.136} . \tag{15}$$

4 Verification of the proposal

4.1 Young's modules

The determination of Young's modulus from Eq. (8) is checked using the test data by Karihaloo et al. (1991) and the comparisons are made in Table 2. The Experimental values of Young's modulus were obtained from compressive tests. It is observed that the calculated values are around 10% lower than the experimental ones.

Table 2 Comparison of the Young's modulus values evaluated by the proposed method and the experimental results by Karihaloo et al . (1991).

Dimensions $S*b*d$ (mm)	Notch ratio a/d	Compressive strength f_c (MP_a)	Initial compliance C_i $(\times 10^{-8} m/N)$	E (GP_a)	E^{cal} (GP_a)
800*81*203	0.295	26.8	1.36	24.62	22
800*81*203	0.296	39.0	1.17	33.80	26
800*80*203	0.295	49.4	1.05	34.65	29
800*81*204	0.293	67.5	0.92	37.20	33
800*81*203	0.293	78.2	0.83	40.30	36

4.2 Tensile strength

Tensile strength values determined using Eq. (15) are compared with the experimental results as shown in Table 3.

The calculated tensile strength values are about 10% lower than the tests. In the data by Planas et al (1986) tensile strength was determined from splitting tests. As splitting tensile strength is generally higher than direct tensile strength, the calculated tensile strength values may be quite close to the true strength values.

Table 3 Comparisons of tensile strength values determined by the proposed method and by the experiments.

Data	$S*b*d$ (mm)	a/d	f_c (MP_a)	E (GP_a)	G_F (Nm/m^2)	f_{net} (MP_a)	f_t (MP_a)	f_t^{cal} (MP_a)
(1)	800*100*100	0.50	37	24.3	123	4.73	3.10	2.81
	1131*100*200	0.50	33	21.5	125	3.73	2.80	2.52
	1386*100*300	0.50	38	25.0	118	3.60	3.15	2.75
	400*100*100	0.33	38	34.1	104	4.45	3.24	2.67
	760*100*190	0.33	38	34.1	118	4.29	3.24	3.05
	1440*100*360	0.33	38	34.1	151	3.86	3.24	3.00
(2)	450*100*100	0.50		20	113	3.7	2.4	2.2

Note: data (1) and (2) are taken from Planas et al. (1986) and Komerling et al. (1983), respectively.

5 Conclusions

The paper proposes a method for the determination of all fracture properties of concrete, i.e. Young's modulus, E, tensile strength, f_t, fracture energy, G_F, through the same bending test on a notched beam.

The proposed method was verified by using some experimental data available in the literature. The determined values of Young's modulus and tensile strength are slightly lower than the experimental ones. However, from a practical view of point, the agreement may still be considered satisfactory. Therefore the method might be a good and simple practical testing method for the determination of fracture properties of normal concrete, though further assessment is needed.

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