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SIZE EFFECT IN CONCRETE AND REINFORCED CONCRETE STRUCTURES

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Abstract
There is much experimental evidence on the existence of the size effect in concrete and reinforced concrete (RC) structures. The problem has two aspects – statistical and deterministic. Although the statistical aspects are not negligible, the size effect on the nominal strength is controlled by the structural energy release due to concrete cracking. If a stable crack growth before reaching peak load is possible, strong size effect may be expected. If this crack growth is not possible, structure fail at crack initiation with no size effect. In any small concrete and RC structure relatively stable crack growth is possible. As a consequence, any small structure exhibits a relatively ductile behavior and size effect which is at least in a limited size range always present. There is no general size effect law, however, it may be useful to classify structures in two classes: (1) Structures of positive geometry – size effect strong only in a limited size range and (2) Structures of negative geometry – size effect strong in a broad size range.

1 Introduction

The size effect problem has two aspects: (1) Statistical and (2) deterministic (mechanical). Although the statistical aspects (Weibull, 1939) are not negligible, it has been generally agreed that the main reason for the size effect lies in concrete cracking and related structural energy release (Bažant, 1984). From the deterministic point of view, the formulation of the size effect relationship has been in the past principally treated in two different ways: (1) Based on experimental results, for each particular problem, without any theory behind and (2) based on a theory, general size effect laws have been formulated which in a simple close form relate the nominal structural strength for any geometry type with it's size. The first approach is strictly practice oriented and it covers only a limited size range for a certain problem type. The second (theoretical) approaches rely on the assumptions which need not to be generally fulfilled.

Currently, two major completely opposite types of theoretical scaling laws for concrete structures exist. The first type is essentially derived using linear elastic fracture mechanics (LEFM), nonlinear fracture mechanics, the cohesive crack model or simple energy balance considerations between the structural energy release and the concrete energy consumption capacity. These approaches deal with a single crack and an a priori assumption on a constant or proportionally scaled initial flaw. They are important from the theoretical point of view and useful for determination of the material fracture properties. However, for concrete and reinforced concrete structures the single crack growth assumption is unrealistic i.e. in most structures more than one crack always exists.

Applying one of the above methods and assuming: (1) The crack length proportionality at peak load and (2) the size of the concrete fracture process zone (FPZ) different from zero, all these approaches essentially yield to Bažant size effect law (Bažant, 1984):

$$\sigma_N = B f_t (1+\beta)^{-1/2}; \quad \beta = d/d_0 \tag{1}$$

where σ_N= nominal strength, d= structure size, f_t = tensile strength of concrete, B and d_0 are two constants to be determined either experimentally or by a more sophisticated analysis. According to (1), the size effect is transitional between the yield limit (plasticity, no size effect) and the size effect of linear elastic fracture mechanics (maximal size effect).

The assumption of a constant and size independent concrete fracture energy is for concrete structures approximately true. However, the hypothesis on the crack length proportionality at peak load is generally not fulfilled. As a consequence, the validity of Bažant's size effect law is limited.

The second type of the scaling law, recently introduced by Carpinteri (1994), is based on the multifractal aspects of damage – multifractality of the crack surfaces. Practically, the concept relies on the homogeneity (inhomogeneity) of the material i.e. in a small concrete structure the aggregate size is large relative to the structure size and, therefore, the inhomogeneity is maximal and the size effect strong. On the contrary, in a large concrete structure the aggregate size is small relative to the structure size and the material is close to be perfectly homogeneous. As a consequence the size effect disappears. According to the fractal damage concept, the size effect law (MFSL) is of the form (Carpinteri, 1994):

$$\sigma_N = (A + \frac{C}{d})^{1/2} \qquad (2)$$

where A and C are two constants obtained by fitting of test or calculated data. As can be seen from (2), if $d \rightarrow \infty$ the nominal strength yields to a constant value different from zero (strength limit). On the contrary, when $d \rightarrow 0$, $\sigma_N \rightarrow \infty$. This means that the size effect for any concrete structure is strong only in a limited size range.

Many experimental and numerical results for concrete and RC structures can be interpreted using this non-mechanical concept (Ožbolt, 1995). However, in the concept the basic mechanical background is missing – the homogeneity (inhomogeneity) of the strain field. For small concrete structures the strain inhomogeneity normally coincides with the material inhomogeneity. In larger structures the material inhomogeneity disappears, however, the inhomogeneity of the strain field generally does not. Namely, by the nature of the problem the strain inhomogeneity may be present even when $d \rightarrow \infty$ (all proportionally notched structures). Therefore, the same as (1), Eq. (2) has a limited range of applicability.

2 Scaling laws – new crack propagation approach

The structural size effect is caused by cracking and structural energy release as a consequence of cracking. Therefore, to understand the size effect and to recognize structures which are size effect sensitive it is important to distinguish different types of crack propagation. After the crack initiates ($\sigma = f_t$), it's growth is controlled by energy balance between the structural energy release rate ($\Delta U/\Delta a$) and the concrete energy consumption capacity (G_f), with $U=$ energy accumulated in the structure and $a=$ crack length. Depending on the energy balance, two principally different types of crack propagation may be distinguished: (1) $\Delta U/\Delta a \geq G_f$ — unstable crack propagation and (2) $\Delta U/\Delta a < G_f$ — stable crack propagation.

When unstable crack growth takes place the structural energy release caused by crack growth can not be consumed by concrete (there is no energy equilibrium). Therefore, the load must decrease after the crack initiates. This means that the structure fails when in the critical cross-section the concrete tensile strength is reached. With this failure type the structure exhibits a strong sensitivity on the variation of the tensile strength and no sensitivity on the variation of the concrete fracture energy i.e. the ultimate load is a function of the tensile strength only and there is no size effect on the nominal strength (expect statistical).

At stable crack growth, the structural energy released as a consequence of cracking can be consumed by concrete (energy equilibrium is possible). Depending on the extend of cracking, the energy consumed by concrete may significantly contribute to the ultimate load. This contribution is the main reason for the existence of the size effect on the nominal strength. Therefore, with stable cracking the ultimate load is mainly controlled by the cracking process rather than by the tensile strength. Structures which exhibit a stable type of fracture are sensitive to the variation of the concrete fracture energy and insensitive to the variation of the tensile strength. Consequently, the sensitivity of any structure to the size effect can be checked by checking the sensitivity of the structural response to the variation of G_f.

2.1 Scaling laws for typical concrete and RC geometries

According to LEFM, with respect to a single crack growth, two typical structure configurations (geometries) exist: (1) Positive configurations (geometries) – after crack initiation an unstable crack growth takes place (see Fig. 1a) and (2) negative configurations (geometries) – after initiation the crack grows in a stable manner (see Fig. 1b).

The above mentioned geometries are two extreme cases. However, in concrete and RC structures one has to account for some additional aspects such as: (1) Finite size of the FPZ, (2) existence of a number of cracks before reaching failure, (3) concrete nonlinearity and related change of the failure mode when increasing the size and (4) influence of reinforcement. Consequently, with respect to crack growth, the concrete and RC structures may be classified in three typical categories: (1) Single crack growth – positive geometry, (2) single crack growth – negative geometry and (3) multiple crack growth – complex type of geometry.

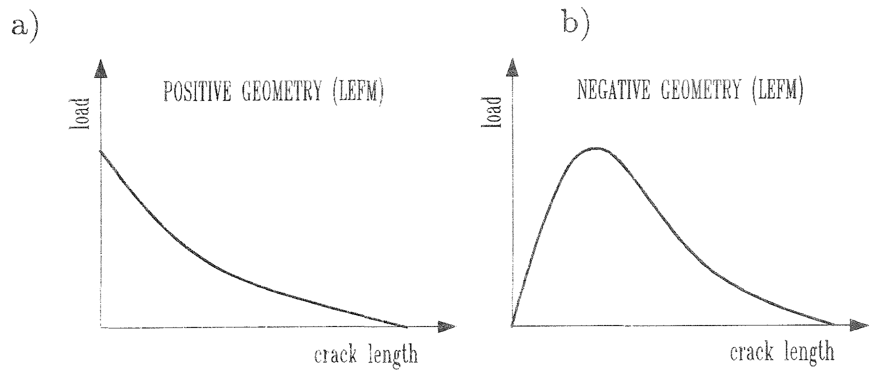

Fig. 1 Structural response for typical fracture geometries:
a) Positive (normal) geometry; b) Negative geometry.

2.1.1 Single crack propagation – positive concrete geometry

As discussed above, positive geometries exhibit unstable crack growth i.e. the load decreases immediately after crack initiation. However, FPZ in concrete has a finite size, different from zero. Therefore, for a relatively small structure the formation of a stable damage zone is possible. Generally, when $d \to 0$ it can be shown that in the critical cross-section the strain gradient $(\Delta\epsilon/\Delta x)$ approaches infinity i.e. a strong strain localization exists. For such a case $U \to 0$ and also $\Delta U/\Delta a \to 0$. Since G_f is a constant different from zero, $G_f/(\Delta U/\Delta a) \to \infty$, and theoretically $\sigma_N \to \infty$. However, practically $d = 0$ has no physical meaning and, therefore, $\sigma_{N d\to 0} = \sigma_{N plasticity}$. This nonlinear effect (relatively large concrete FPZ) disappears when the size of the FPZ becomes negligible in comparison to the structure size $(d \to \infty)$. Consequently, in the critical cross section $\Delta\epsilon/\Delta x \to 0$, $\Delta U/\Delta a \to \infty$ and $G_f/(\Delta U/\Delta a) \to 0$. This means failure at crack initiation and no size effect.

The size effect law for positive concrete geometries may be approximately described by an empirical equation of the form:

$$\sigma_N = Bf_t(1+\gamma)^{\alpha} \leq \sigma_{plasticity}, \quad \gamma = d_0/d \tag{3}$$

where B and d_0 are two constants which depend on the problem type and material fracture properties (similar as in the case of Bažant's size effect law), α is a problem dependent constant between 0 and 1.

The general shape of the curve from (3) is plotted in Fig. 2a in a double logarithmic scale and in Fig. 3 in normal scale. The shape is essentially the same as predicted by (2) although with a completely different physical background. The nonmechanical arguments, exploited in the multifractal damage theory, coincide with the mechanical arguments discussed above. However (3) is in contradiction with Bažant size effect law (1). For positive geometries the crack length proportionality at peak load does not hold. Only in special cases when the size of the initial flaw is proportional to the structure size (e.g. an artificially notched beam) Bažant's size effect law applies. However, in such a case the crack length proportionality is not a structural property but an imposed boundary condition on the crack length.

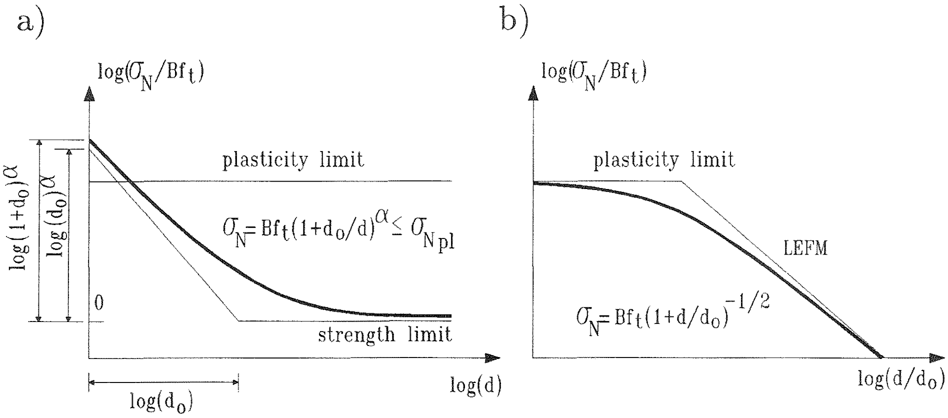

Fig. 2 Size effect laws: (a) positive geometry and (b) negative geometry.

2.1.2 Single crack propagation – negative concrete geometry

For negative geometries (e.g. in case of a headed stud embedded in a large concrete block) a stable crack growth before reaching the ultimate load is possible and G_f significantly contributes to the ultimate load. For extremely large structures ($d \rightarrow \infty$) the relative size of the FPZ yields to zero. It can be demonstrated that for such a case the crack length at peak load increases approximately proportionally with the structure size (Elices and Planas, 1992) i.e. a strong strain localization in a broad size range exists. Therefore, Bažant's size effect law approximately applies (see Fig. 2a and 3).

Strictly speaking, in the limit case ($d \rightarrow \infty$) the contribution of the tensile strength to the ultimate load is always larger than the contribution of the concrete fracture energy. Therefore, theoretically, the nominal strength yields to a constant value different from zero. However, in contrast to positive geometries, this takes place for a relatively large structure size and is not interesting from the practical point of view. Namely, the ratio between the "residual nominal strength" ($d \rightarrow \infty$) and the maximal nominal strength ($d \rightarrow 0, \sigma_{N_{plasticity}}$) is negligible small (see Fig. 3).

In the past, the upper limit on the nominal strength (mode-I single crack) was often considered as a *strength* or *plasticity* limit (Bažant, 1984). This is principally not correct since there is an essential difference between strength and plasticity limit. Namely, the plasticity limit (contribution of both, strength and cracking to the ultimate load) is the highest and the strength limit (only contribution of the material strength) the lowest limit on the nominal strength i.e. they can never coincide.

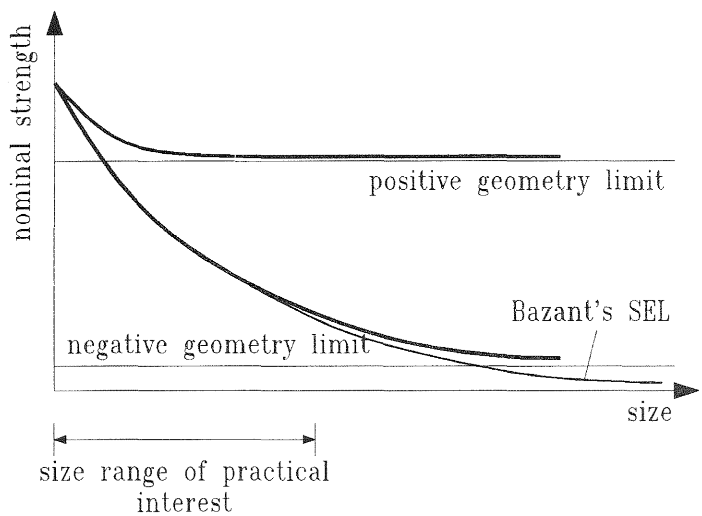

Fig. 3 Schematical plot of the nominal strength as a function of the size for positive and negative geometries.

2.1.3 Complex concrete and RC geometries

In the case of the aforementioned positive and negative geometries it has been assumed that after crack initiation only a single crack grows. Due to the reasons mentioned before, in most RC structures more than one macrocrack grows before reaching the ultimate load. When extensive cracking in the pre-peak load-displacement response takes place the concrete fracture energy significantly contributes to the ultimate load and the size effect is strong. Due to the complexity of the failure mechanism each case must be separately studied, not using a simple single crack approach, but employing a more sophisticated nonlinear numerical fracture analysis. If possible, numerical results should be checked by experiments. This is, however, not simple since systematic tests on large concrete and RC structures are usually related with extremely high costs. Principally, structures with a complex crack growth may exhibit a size effect on the nominal strength either as a structure of positive geometry, negative geometry or as a combination of both.

3 Example – Diagonal shear failure of slender RC beams without shear reinforcement

To demonstrate the size effect for a typical complex geometry, the results of a numerical study using the nonlocal microplane FE code (Ožbolt and Bažant, 1994) for slender RC beams without shear reinforcement which fail in diagonal shear are shown in Fig. 4. The calculated nominal shear strength obtained for a broad size range are plotted and compared with test results and Bažant's size effect law. The fit of the calculated data, using linear regression analysis, with the size effect law proposed for positive geometry (3) is also plotted. The fit agrees well with the calculated data and the experimental results in the whole size range. On the contrary, for larger beam depths, the calculated and test data show an obvious disagreement with Bažant's size effect law which underestimate test and calculated data.

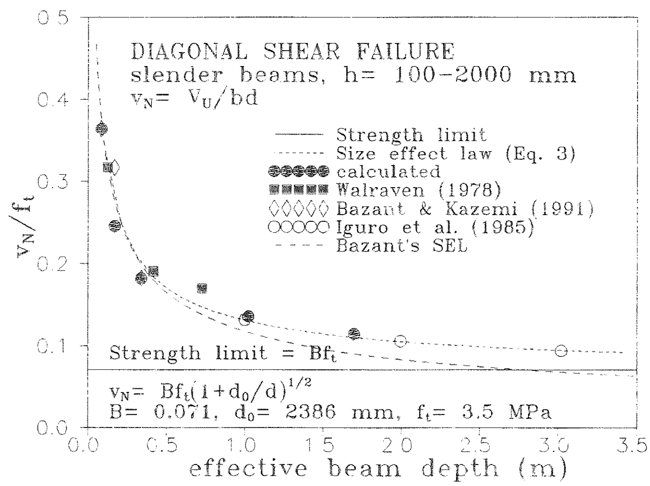

Fig. 4 Size effect on the nominal shear strength – comparison between calculated data, test data and Bažant's size effect law.

Summarizing the numerical results, it may be pointed out that the size effect on the nominal diagonal shear strength is in a size range up to approximately $h=$ 1000 mm strong and close to that obtained by LEFM. Therefore, in a limited size range the beams act as a structure of negative geometry i.e. stable crack growth is possible. For large beams the size effect disappear and the nominal shear strength yields to a constant value different from zero. In the limit case ($d \rightarrow \infty$) the structure acts as a structure of positive geometry i.e. the beam fail at initiation of the first bending crack. This means that the failure mode is changing when increasing the beam depth from diagonal shear to failure at crack initiation. The nominal diagonal shear strength must yield to a constant value different from zero, otherwise, large beams would fail in diagonal shear before any bending crack at the beam bottom initiates, what is impossible.

4 Conclusions

1. The main reason for the size effect on the nominal strength in concrete structures is due to concrete cracking and the related structural energy release. If in a broad size range a stable crack propagation is possible the size effect is strong. On the contrary, if no stable crack propagation is possible the structure fails at crack initiation without size effect.

2. The ultimate load of structures which exhibit a strong size effect is a function of both, concrete fracture energy and material strength. However, if there is no size effect, the ultimate load is a function of the material strength only. Therefore, the sensitivity of any structure on the size effect may simply be checked by investigating the sensitivity of the ultimate load on the variation of the concrete fracture energy. If it is strong, the size effect must be also strong.

3. There is no general size effect law for concrete and reinforced concrete structures. However, it may be useful to distinguished two classes of structure configurations (geometries): (1) positive (normal) – stable crack growth is possible for small structures and (2) negative – stable crack growth is possible. In the first case the the size effect is strong only in a limited size range and for $d \rightarrow \infty$ the nominal strength tends to a constant value different from zero. In the second case the size effect on the nominal strength is strong in a broad size range and for $d \rightarrow \infty$ the nominal strength yields approximately to zero.

4. Due to the complexity of concrete and RC structures, most of them exhibit a transition of the failure mechanism when increasing the size i.e. in a small size range they act as a structure of negative geometry (stable crack propagation, quasi ductile behavior). On the contrary, large structures act as a structure

of positive geometry. For any concrete and RC structure the size effect always exists at least in a limited size range.

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