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CRACK SURFACE FRICTION AND SIZE EFFECT IN MODE I PROPAGATION FOR MORTAR AND CONCRETE.

A. Turatsinze and A. Bascoul
Laboratoire Matériaux et Durabilité des Constructions
Institut National des Sciences Appliquées - Université Paul Sabatier
Toulouse, France

Abstract

Scanning electron microscopy in conjunction with the replica technique gives a new insight of the fracture process zone in mode I crack propagation for mortar and concrete. Results lead to presume that during the crack opening, the friction effect across the crack surfaces is an important source of energy dissipation which can partly explain the size dependence which affects the experimental specific fracture energy.

We propose a simple model which describes the experimental specific fracture energy as the sum of the energy necessary for the crack propagation in Griffith's sense and the one due to the friction effect which induces a restrained widening of the crack.

The validation of the model has been carried out by means of three point bending tests on notched beams. We varied the length of the specimen ligament and the roughness of the crack path which governs the friction effect. The RILEM TC-50 procedure gives experimental results which are in good agreement with the ones predicted by the proposed model.

1 Introduction

Observations of the fracture process of mortar and concrete in mode I crack opening have been carried out by the replica technique and scanning electron microscopy on notched specimens in three point bending tests. Results were presented in FRAMCOS 1 by Bascoul et al. (1992), Turatsinze (1992) and by Bascoul et al. (1994). They show that inside the material, at a given load displacement beyond the peak load, the main crack path is continuous but always followed by discontinuous microcracks. However, if one can consider these microcracks as components of the fracture process zone, it is patent that the observed microcracks are not distributed at random. They mark the future path of the continuous crack. For this reason the energy dissipated for the formation of these microcracks is a part of the overall energy necessary for the formation of the future continuous crack surfaces.

Moreover several phenomenons act in opposition to the crack widening. In this paper the term restrained widening is used to designate all the mechanisms which hinder the increase of the crack opening, essentially friction due to the sinuous crack path but also interlocking, crack deflection and aggregate bridging. They induce an energy dissipation which has to be taken into account in the evaluation of the fracture energy per unit of crack surface. This article deals with experiments on mortar and concrete notched beams in three point bending tests (see figure 1).

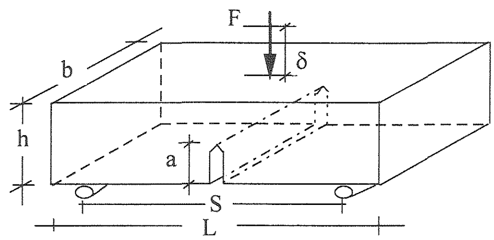


Fig. 1. Specimen geometry (h = 80, b = 50, S = 320 and L = 420 mm)

The overall fracture energy measured by means of Hillerborg's procedure (1985) includes the energy necessary for the propagation of the crack and the energy dissipated by restrained widening effect. Size dependence is generally observed on the values of the fracture energy per unit of crack surface derived from this procedure. It could be explained through the concept of restrained crack widening. The remainder of this paper deals with a model established from this concept in order to explain the variation of the fracture energy per unit of crack surface with both the length of the initial ligament and the crack opening level.

2 Restrained widening and fracture energy modeling

The model is based on the idea that the restrained widening essentially results from the crack surface roughness which may be characterized by the linear roughness R_l. Moreover we consider that the measured fracture energy E_A can be regarded as the sum of two terms E_C and E_Φ

$$E_A = E_C + E_\Phi \qquad (1)$$

where E_C is the energy necessary for the initiation and the propagation of the crack, whereas E_Φ is the energy dissipated by restrained widening during the increase of the crack opening. If the projected crack surface is S, we can deduce the fracture energy per unit of crack surface γ_A according to relation (2).

The term γ_C can be regarded as a characteristic parameter identified to the specific fracture energy in Griffith's sense to which is added the varying term γ_Φ. So, we have to show that size dependence can be justified by the term γ_Φ. For this purpose, we consider a sample whose width is equal to the unit with a periodic crack profile characterized by the step length α (see figure 2) and by the linear roughness R_l. Any 'i' step of the crack profile can be located by Y_i.

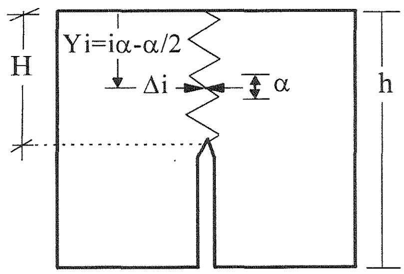


Fig. 2. Crack profile modeling

The specimen ligament length H is greater than the length of one step (relation 3) and we assume that at every step, a resultant force Φ perpendicular to the direction of the mean crack plane acts in opposition to the increase of the crack widening (relation 4).

$$\gamma_A = E_C/S + E_\Phi/S = \gamma_C + \gamma_\Phi \tag{2}$$

$$H = n.\alpha \quad \text{with } n >> 1 \tag{3}$$

$$\Phi = K.(R_l - 1) \tag{4}$$

K is a coefficient which has a force dimension. It allows us to characterize crack profiles with different shapes but with the same linear roughness. So we can ascertain that Φ is zero for a strictly plane crack ($R_l = 1$).

The concept is similar to the one of closure pressure used in fictitious crack models where more sophisticated relations are always assumed according to the crack widening. These relations rely on the softening branch which is observed in direct tension tests. Here, it must be emphasized that even in direct tension tests there is always propagation of a crack across a section. Thus softening is not representative of an intrinsic material behavior since it results from a structure mechanism. So the constant force Φ may be regarded as a rough but realistic approximation.

Considering Δ_i as the mean crack opening of the step i, the energy E_Φ dissipated for the whole crack profile is like

$$E_\Phi = K.(R_l - 1).\Sigma\Delta_i \tag{5}$$

2.1 Crack opening along the ligament

Tests have been carried out by Turatsinze (1992) to determine the distribution of the crack openings along the ligament. The measured parameters and the location of the used gages (G1 to G5) are illustrated in figure 3. The same figure gives an example of recorded curves. Beyond the peak of the load - deflection curve, it has been noticed that Δ_i is a linear function of Y_i. This is in accordance with the kinematic block theory in fracture mechanics of mortar and concrete. In the following sections we will always deal with specimens loaded up to obtain linear crack opening distributions

$$\Delta_{i(\Delta)} = C_{(\Delta)}.Y_i \quad (6)$$

where $C_{(\Delta)}$ depends on the crack opening level Δ. It does not depend on the ligament length H. Δ is measured under the notch by the LVDT whose distance from the upper fiber of the specimen is marked d (figure 3). If Δ_0 is the crack opening at the tip of the notch we can write

$$C_{(\Delta)} = \Delta/d = \Delta_0/H \quad (7)$$

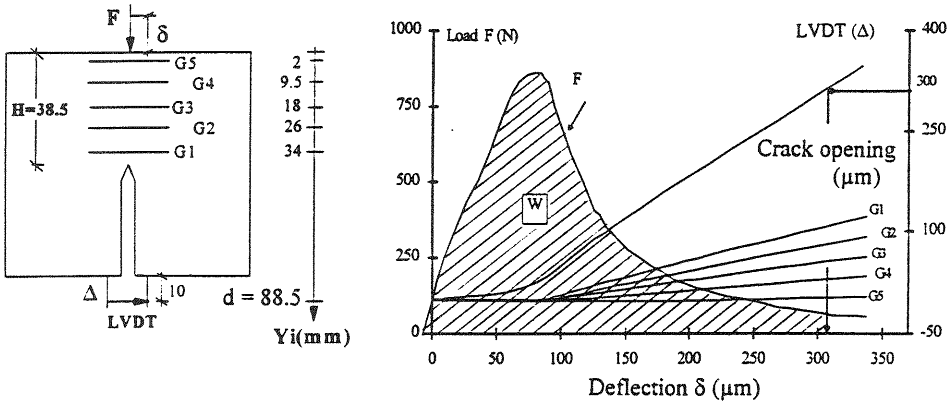


Fig. 3. a) Location of the points of measurement
b) Load and crack opening distribution according to the deflection δ

2.2 Relationship between the fracture energy and the specimen ligament length

$$\Delta_i = \Delta_0.Y_i/H \quad (8)$$

Two cases have to be considered :

- As long as there is no effective separation of the two crack surfaces equation (5) can be written like

$$E_{\Phi(H,\Delta)} = K.(R_l - 1).\Delta_0.\Sigma Y_i/H = K.(R_l - 1).(\alpha+2.\alpha+3.\alpha+...+n.\alpha - n.\alpha/2).\Delta_0/H$$

$$= K.(R_l - 1).(\Delta_0/H).H^2/2\alpha \quad (9)$$

Thus, according to relation (7)

$$E_{\Phi(H,\Delta)} = \beta.C.H^2 \qquad \text{where } \beta = K.(R_l - 1)/2\alpha \quad (10)$$

$$\gamma_{A(H,\Delta)} = \gamma_C + E_{\Phi(H,\Delta)}/H = \gamma_C + \beta.C.H \quad (11)$$

- This linear relation is no longer valid if the crack opening exceeds a critical value Δ_c characteristic of the beginning of an effective separation of the two crack surfaces. For the part of the crack whose opening reaches or exceeds the critical value Δ_C, the restrained widening effect vanishes and an additional increasing of the crack opening does not induce a dissipation of energy. In these conditions (figure 4), we must consider that restraining effects remain acting on a part of the ligament X along which relation (11) is still valid. For the other part of the ligament, whose length is (H-X), a constant value of energy ω by unit of ligament length has been dissipated because of the restrained widening occuring when the crack opening increased from zero to the critical value Δ_C. Then $E_{\Phi(H,\Delta)}$ and $\gamma_{A(H,\Delta)}$ are respectively expressed by relations (12) and (13).

$$E_{\Phi(H,\Delta)} = \beta.C.X^2 + \omega.(H - X) \qquad (12)$$

$$\gamma_{A(H,\Delta)} = \gamma_C + E_{\Phi(H,\Delta)}/H = \gamma_C + (\beta.C.X - \omega).X/H + \omega \qquad (13)$$

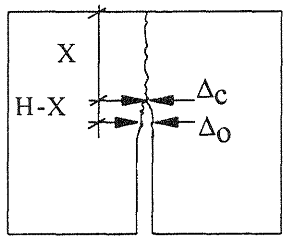


Fig. 4. Critical crack opening Δ_C and effective separation

For a given mix, the model involves the parameters ω, β, and Δ_C to be linked. When the crack opening exceeded the critical value Δ_C along all the ligament it can be easily shown that :

$$\omega = 2.\beta.\Delta_C \qquad (14)$$

2.3 Theoretical variation of the apparent specific fracture energy

According to relations (11) and (13), γ_A depends on both the initial ligament length H and the crack opening level. C characterizes the crack opening level Δ (relation 7). The parameters β and Δ_C can only be deduced from the experimental results.

Let us accept these parameters as given. According to the crack opening level Δ, the length X concerned by the restrained widening can be calculated

$$X = \Delta_C.d/\Delta \qquad (15)$$

Two cases may occur :

- $H \leq X$. The critical crack opening Δ_C is not reached anywhere along the ligament. Then, the fracture energy per unit of crack surface γ_A is a linear function of H calculated according to relation (11).

- H≥X. The crack opening reaches or exceeds the critical value Δ_c on a part of the crack path. Then γ_A is an hyperbolic function calculated according to relation (13). X decreases as Δ increases and γ_A tends to the constant value $\gamma_C + \omega$.

β can be deduced from the slope of the linear branch equal to βC. The intersection with the γ_A axis determines the term γ_C. This is illustrated in figure 5. It has to be noticed that the dependence of γ_A on Δ is to be related with the influence of the "P-δ tail cutting" on the measured fracture energy by means of the RILEM TC-50 procedure (Elices et al., 1992). Using this procedure a part of the fracture energy is neglected. However it has to be noticed that we introduce an effect of widening restraint which takes place as soon as the crack propagates and acts until the crack opening reaches the critical value Δ_c. Consequently, the influence of the restrained widening on the fracture energy cannot be limited to the energy neglected by "P-δ tail cutting".

3 Experimental results and model validation

To validate our model, it was necessary to compile test data about fracture energy determined for various ligament lengths and various roughness conditions. Here we present the results of tests performed for this purpose. Our mechanical device allowed us to vary the ligament length from 20 mm to 70 mm for beams 80 mm in depth. Concerning the roughness conditions, it is known that the path of the crack in mortar and concrete is generally located at the transition zone (Maso, 1982). So, the profile of the fracture surfaces depends on the size of the aggregate and it was assumed that both the sinuosity of the crack path and the critical crack opening Δ_c increase with the maximum aggregate size d_{max}.

Three mixes (I, II and III) have been studied. Crushed marble has been used as aggregate and the mix proportions were chosen in order to obtain the same maniability. Their main characteristics are given in table 1.

Table 1. Studied mixes

Mix	I	II	III
Water (l/m^3)	352	296	204
Cement CPA HP (kg/m^3)	664	557	385
Aggregate (kg/m^3)	1169	1455	1629
d_{max} (mm)	0.4	3.15	16
Compressive strength (MPa)	45.7	47.5	49.9
Young modulus (GPa)	24.1	27.1	36.7

For each test 3 or 4 samples were available. During the test the load F, the deflection δ and the crack opening level Δ were recorded. Some values of Δ were fixed and for each one the corresponding deflection δ was determined. A dissipated energy E_{Ameas}

(= w + mgδ/2) was determined according to these values of δ (figure 3). An apparent fracture energy per unit of crack surface γ_{Ameas} = E_{Ameas}/S was deduced. The energies dissipated by crushing γ_P and by friction γ_F at the supports were substracted from γ_{Ameas} to obtain the fracture energy per unit of crack surface $\gamma_A = \gamma_{Ameas} - \gamma_P - \gamma_F$.

The energy dissipated by crushing γ_P was evaluated according to the conclusions of Ramoda (1987). The energy γ_F dissipated by friction at the supports was determined according to Guinea et al. (1992) by considering a simplified rigid-body kinematics. For our specimens γ_A may be expressed as:

$$\gamma_A = (0.67 + 0.22.a/h)\gamma_{Ameas} \qquad (16)$$

Three crack opening levels have been considered : Δ = 100, 200 and 300 µm for mixes I and II, 200, 300 and 400 µm for mix III. For mixes I and II, samples generally broke for Δ values between 300 and 400 µm. Concerning mix III, a crack opening inferior to 200 µm did not ensure a linear distribution of the crack opening along the specimen ligament. The energy per unit of crack surface γ_A was calculated for each mix according to these crack opening levels. Results are given in table 2.

3.1 Determination of the parameters β, Δ_c and ω

For each mix the parameters have been determined from the results corresponding to the lower crack opening Δ (100 µm for mixes I and II, 200 µm for mix III). In these cases the first experimental points allowed the fitting of a linear relation and the parameter β could be deduced from the slope of the linear branch equal to β.C. Next, the least square method was applied to fit the hyperbolic branch and to determine the parameters X and ω. A gap between the linear branch and the hyperbolic curve was then possible. Some readjustments on β value and thus the slope of the linear branch were consequently necessary. Indeed, taking into account equations (14) and (15), the parameters X and ω had to satisfy the following relation $X = \omega.d/2.\beta.\Delta$.

Table 3 gives the values of γ_C, β, Δ_C and ω determined by this procedure. For mix III the corresponding curve is drawn in figure 5.

3.2 Validation

The model must be able to predict the variation of the energy per unit crack surface γ_A versus the initial crack length H for any higher Δ value. For a given crack opening level Δ, the factor C is calculated from relation (7) and the length X can be deduced from relation (15) according to the critical value Δ_c of the mix (see table 4). Then, applying relations (11) and (13) new curves of variation of γ_A versus the initial crack length H can be drawn. This is illustrared by the dotted curves in figures 6a and 6b for mix III and respectively Δ = 300 µm, Δ = 400 µm. The experimental results have been reported in these figures. They are in good agreement with the prediction of the model.

Table 2. γ_A (N/m), experimental results

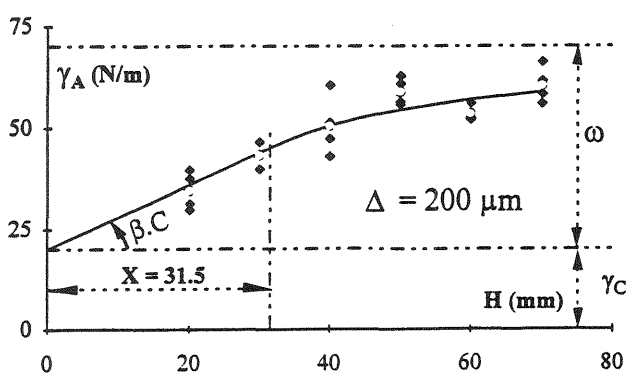
MIX	I			II			III		
Δ (μm) H (mm)	100	200	300	100	200	300	200	300	400
20	15.5	21.4		25.4	32.0	35.0	29.8	34.2	
20	17.4	22.4	24.9	24.6	29.8	30.3	37.4	42.6	50.3
20	16.1	20.4	22.7	23.0	28.9	30.2	31.2	35.9	44.0
20	11.8	16.1	18.5				39.5	46.7	56.3
30	19.9	22.1	23.4	28.5	33.9		43.3	48.6	57.9
30	18.3	21.0	22.6	25.7	29.4		39.8	44.6	53.9
30	19.8	21.9	23.9	28.8	34.0	35.6	43.4	47.4	54.9
30	18.3	20.7	22.0				46.5	51.7	60.8
40	13.9	15.7	15.8	25.1	30.2	32.5	60.5	66.5	73.3
40	14.3	16.7	17.2	25.5	30.3	31.8	47.2	51.4	57.2
40	12.0			27.2	37.2	41.2	51.1	55.7	61.2
40							42.8	46.7	52.0
50	22.8	23.7	23.7	33.2	35.4	36.0	55.6	58.9	60.7
50	22.6	24.3	24.5	30.2	32.8	33.4	62.7	67.1	69.7
50	27.0	28.2	28.4	40.2	43.1	43.8	60.8	66.1	68.9
50	22.6	24.1	24.4				56.3	60.0	62.2
60	22.8	25.7	27.0	35.1	36.1		52.2	54.7	55.7
60	24.9	27.6	28.5	29.6	32.6	33.2	56.0	57.8	58.6
60	18.1	20.2	21.5	32.0	34.1	33.6	52.0	54.5	55.9
60	22.3	25.1	26.2						
70				33.8	36.8	38.1	66.2	69.9	71.7
70				37.1	40.7	42.3	58.2	61.5	62.9
70				30.4	34.2	35.8	61.5	65.1	66.7
70				41.8	45.3	47.1	55.9	58.6	60.2

Table 3. Values of the model parameters for the three mixes

Mix	I	II	III
γ_C (Nm^{-1})	6	15	20
β (MPa)	0.41	0.38	0.36
ω (Nm^{-1})	20.8	26.4	50.1
Δ_C (μm)	28	36	70

Table 4. Length X (mm) still subjected to restrained widening

X (mm)			
Mix / Δ (μm)	I	II	III
100	25.2	32.5	
200	12.6	16.2	31.5
300	8.4	10.8	21.0
400			15.8



Mix III

γ_C (Nm^{-1})	20
β (MPa)	0.36
ω=(Nm^{-1})	50.1
Δ_C (μm)	70

Fig. 5. Determination of the model parameters (mix III)

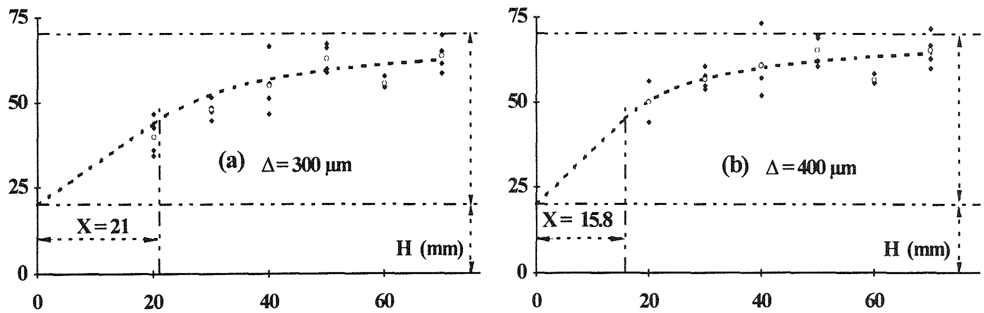


Fig. 6. (a), (b) Application of the model (mix III)

4 Conclusions

During the crack opening, restraining forces are induced and an energy is dissipated. We assumed that this energy is added to the one necessary for the crack propagation in Griffith's sense. So, we have proposed a simple model of the dissipation of the fracture energy including the part due to the restrained widening effect. Then, the fracture energy per unit of crack surface γ_A is the sum of two terms : the specific fracture energy in Griffith's sense (γ_C) and the term due to the restrained widening effect (γ_Φ).

We tried to validate this theoretical approach. For this purpose, the fracture energy was measured from three point bending tests by means of the area under the load-deflection curve. The aggregate size and the ligament length were varied.

The specific fracture energy in Griffith's sense γ_C is an average value including the energy necessary for the crack propagation in the matrix and at the interfaces. If we consider the specific fracture energy as intrinsic to the material, one can notice that this energy is not directly accessible by using the RILEM TC - 50 procedure. It is included in the fracture energy per unit of crack surface in which the part due to the restrained crack widening depends on the kinematic block separation. γ_C must depend on the proportion and the nature of the mix components. So, the observed increase of γ_C with the maximum aggregate size (see table 3) cannot be generalized.

The ideal way to point out the influence of the roughness of the crack on the energy dissipated by restrained widening would be to test specimens with the same matrix but different maximum aggregate sizes. In such conditions, it would be sensible to consider γ_C as constant and it would be possible to impute the observed variation on γ_A solely to the variation of the maximum aggregate size. Unfortunately, even if we leave out the variation of the maniability of the mix, the microstructure of the matrix is changed when the aggregate size is changed. Indeed, the water/cement ratio of the transition zone is different that the one of the matrix (Barrioulet et al., 1977). If the aggregate size is varied, the developed matrix-aggregate interface is changed as is the water/cement ratio of the matrix.

Concerning our experimental results, β values are very close each other as it can be seen in table 3. This indicates that the variation of the studied mix components slightly influences the parameters β. On the other hand, table 3 shows that the critical crack opening Δ_c logically increases with the maximum aggregate size. So, according to relation (14), restraining effect must be more significant for the mix with the greater aggregate size. It must vanish more quickly for a mix with a small aggregate size since in that case the critical crack opening Δ_c is lower. All these observations are consistent with the experimental data.

So, it can be concluded that the restrained crack widening seems to be a main source of the size dependence in mode I crack propagation for mortar and concrete. However, the proposed model has to be improved. Particularly, the restrained crack widening forces have been considered as constant during the crack opening. This is a rough approximation. We also subjectively considered the restrained widening effect and the critical crack opening as linked. We estimated that they increase with the roughness which increases with the maximum aggregate size. This argument is not fully justified. Indeed, one can presume a prominent interlocking effect with a minor roughness effect. For these reasons, it is fundamental to develop work about the measurements of the roughness and a more realistic evaluation of these restrained crack widening forces.

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