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AN R-CURVE APPROACH FOR PULL-OUT OF FIBERS FROM A MATRIX

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Abstract

A fracture mechanics model was developed to predict the pull-out behavior of fibers. Stable debonding prior to the peak pull-out load was described by a fracture resistance curve (*R*-curve). For a given interfacial condition, the pull-out of fibers with the same diameter but different embedded lengths can be predicted using the same *R*-curve. The *R*-curve for the pull-out of aligned fibers was extended to inclined fibers which also undergo bending during the pull-out. The fiber bending energy was incorporated into the energy balance for the stable interfacial crack propagation.

1 Introduction

Response of fiber-matrix interface is usually characterized based on pullout of fibers from a matrix as shown in Fig. 1. Typical pullout force and slip curves for steel fiber-cementitious matrix interface are shown in Fig. 2. The pullout load initially almost linearly increases with the slip. Nonlinearity in the pullout load and slip curves, which is often regarded

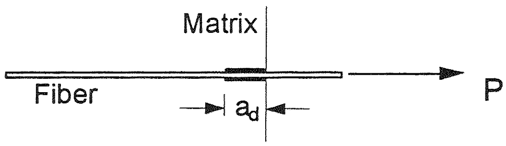

Fig. 1 Pullout of a fiber from a matrix

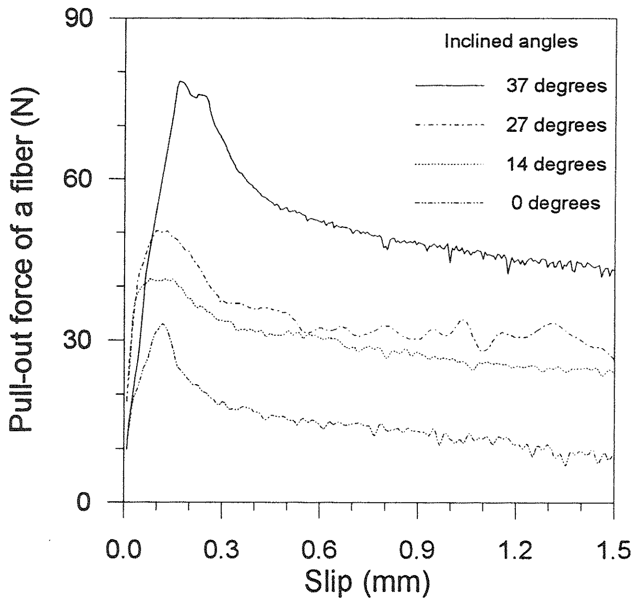

Fig. 2 Pullout force vs. slip curves for fibers with different inclinations

as an indication of propagation of an interfacial crack, is usually observed before the peak load. After the pullout load reaches the maximum value it decreases slowly with increasing slip. This result may indicate that when fibers are pulled-out from a cementitious matrix, an interfacial crack initiates at some point before the peak, and the pullout-slip response is governed by propagation of this interfacial crack. A fracture mechanics R-curve approach is proposed to describe the pullout-slip response.

2 R-curve approach for pullout of an aligned fiber

Application of load to a structure with an initial crack of size, a_0, produces strain energy, U. The rate of strain energy release with respect to crack length, a, is denoted by G, which is a crack driving force. On the other hand, the crack propagation at the crack tip needs to consume some energy, W. The rate of change of W with respect to crack length, a, is denoted by R, and is termed the fracture resistance. Based on fracture mechanics, the failure (the peak load) of a quasi-brittle material is governed by the following two conditions:

$$G = R \quad and \quad \frac{\partial G}{\partial a} = \frac{\partial R}{\partial a} \tag{1}$$

If the pullout of a fiber from a cementitious matrix is considered as a mode II fracture problem, Eq. (1) can be used to describe this problem. However, both G and R should be known before Eq. (1) can be used. Stang et al. (1990) have proposed a G-curve for the fiber-matrix interface debonding based on a shear-lag analysis. Since only pull-out behavior up to peak load is concerned here, the effect of frictional stress is ignored. As a result, this G-curve is expressed as:

$$G = \frac{P^2 \coth^2[\omega(L-a)]}{4E_f \pi^2 r^3} \tag{2}$$

where P is the pull-out load of the fiber, a is the debonded length at fiber-matrix interface, L is the fiber embedded length, r is the radius of the fiber, E_f is the modulus of elasticity of the fiber, and ω is a parameter to quantify stiffness of weak interface layer between the fiber and the matrix and is defined as:

$$\omega = \sqrt{\frac{k}{E_f \pi r^2}} \tag{3}$$

where k is the shear stiffness of the matrix which is modeled as a shear-lag. Li et al.(1991) experimentally confirmed that the ω value is basically constant regardless of fiber embedment length.

Ouyang and Shah (1991) have proposed a general R-curve by solving a differential equation derived from Eq. (1). For the fiber pullout problem, this R-curve is,

$$\begin{aligned} R &= \beta(a-a_0)^d \\ d &= \frac{1}{2}+\frac{\alpha-1}{\alpha}-\left[\frac{1}{4}+\frac{\alpha-1}{\alpha}-\left(\frac{\alpha-1}{\alpha}\right)^2\right]^{1/2} \end{aligned} \tag{4}$$

where α and β are constants to be determined. Substituting Eqs. (3) and (4) into Eq. (1) results in

$$\frac{a_c^*-a_0}{d} - \frac{1}{2\omega}\coth[\omega(L^*-a_c^*)]\ \sinh^2[\omega(L^*-a_c^*)] = 0 \tag{5}$$

where L^* and a_c^* are the fiber embedded length and the corresponding critical debonded length at the maximum load for a reference specimen. To determine the parameters α and β, one needs to test a reference specimen with the embedded fiber length of L^* to obtain its maximum pull-out load, P_c^*. The value of a_c^* can be obtained by substituting the value of L^* into Eq. (5) and solving this nonlinear equation. The value of α $(= a_c^*/a_0)$ can thus be calculated. The value of β can then be determined by substituting Eqs. (2) and (4) into Eq. (1) based on the measured value of P_c^*:

$$\beta\left(a_c^*-a_0\right)^d = \frac{(P_c^*)^2\coth^2[\omega(L^*-a_c^*)]}{4E_f\pi^2 r^3} \tag{6}$$

After knowing both G- and R-curves, the behavior of pull-out of aligned fibers from cement-based matrices can be predicted. From the condition of $R = G$, the relationship between the pull-out load and the debonded length is obtained:

$$P = \left\{ \frac{4E_f \pi^2 r^3 \beta (a-a_0)^d}{\coth^2[\omega(L-a)]} \right\}^{1/2} \tag{7}$$

Eq. (7) predicts that the pull-out load increases with increasing debonded length up to the peak load for an aligned fiber. The peak pull-out load is automatically obtained from maximizing the P values based on Eq. (7). For a given matrix, the peak pull-out loads for a series of fibers, which have the same diameters but different embedded lengths can be predicted by substituting different L values into Eq. (7).

Somayaji and Shah (1981) have reported a series of results on pull-out of aligned steel fibers from mortars. The diameter of the fiber used was 1.59 mm, and the modulus of elasticity for the fiber was 210 GPa. The embedded lengths of the fibers varied from 38 mm to 83 mm. Based on Li et al. (1991), the value of ω for pull-out of the steel from the mortar at the age of 28 days was assumed as 0.075 1/mm. To predict peak pull-out loads for the fibers with different embedded lengths, the R-curve should first be determined based on a reference pull-out specimen. The specimen with 83 mm fiber embedded length was chosen as the reference sample, and the corresponding peak pull-out load was equal to 729 N (L^* = 83 mm and P_c^* = 729 N). By substituting these values of L^* and P_c^* into Eq. (5), the critical debonded length for the reference specimen, a_c^*, was solved. As a result, the value of $\alpha = a_c^*/a_0$ was obtained. It is noted that the initial debonded length, a_0, depends on microstructure of fiber-matrix interface. A small value of $a_0 = 0.02$ mm was used in this study for all predictions. The obtained value of α was used in Eq. (4) to calculate the value of d. These quantities of L^*, a_0 , a_c^* , d as well as P_c^* were substituted into Eq. (6) to obtain the value of β. The R-curve for this series of specimens with different embedded lengths was then determined from Eq. (4), which is shown in Fig. 3. The peak pull-out loads for three different fiber embedded lengths are obtained (points A, B, and C correspond to L = 38, 64 and 83 mm, respectively). The G-curves corresponding to those peak loads, calculated from Eq. (2), are also shown in Fig. 3. The G-curves for L = 38 mm, 63 mm and 83 mm become tangent to the R-curve at points A, B, and C, respectively, which correspond to the peak loads for different specimens and were determined

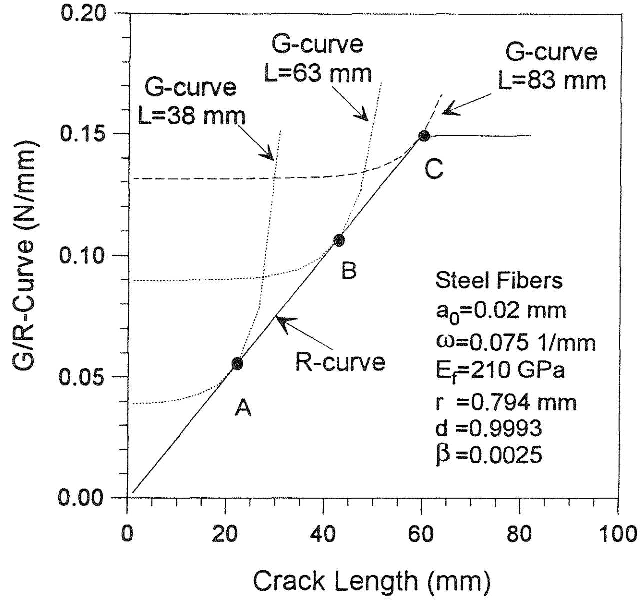

Fig. 3 Relationship between G-curves and the proposed R-curve

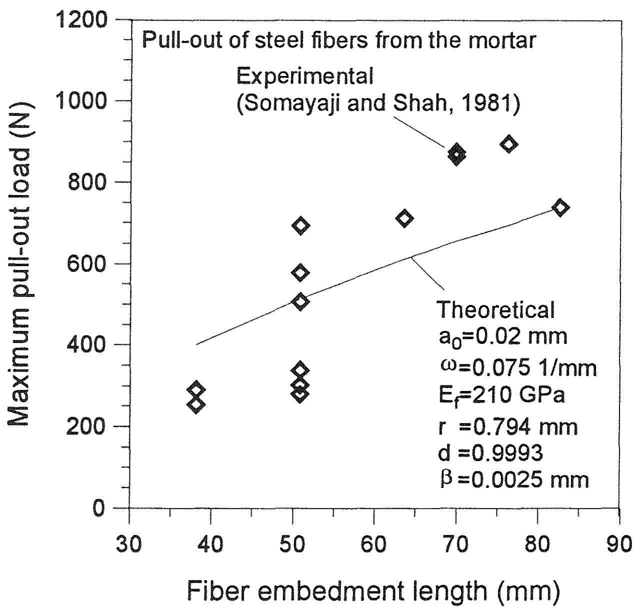

Fig. 4 Predicted maximum pull-out loads of aligned steel fibers and experimental data reported by Somayaji and Shah (1981)

from Eq. (1). These critical values of G (at points A, B and C) vary with the fiber embedded lengths. The predicted peak pullout loads are compared with those experimentally obtained in Fig. 4. The proposed R-curve can reasonably predict the effect of the fiber embedded length on the peak pullout load.

3 Pullout of an inclined fiber

When a fiber is pulled out from a matrix at an inclined angle, the pull-out load, P_θ, can be divided into two components, P_x and P_y , as shown in Fig. 5. The component, P_x , results in pull-out of the fiber from the matrix, whereas the component, P_y , causes bending of fiber at its exit point to the matrix.

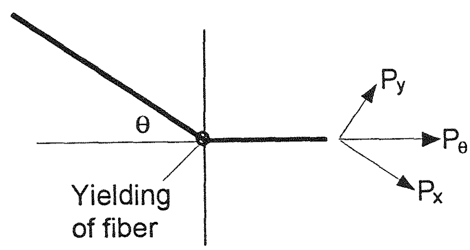

Fig. 5 Pullout of an inclined fiber from a matrix

In order to predict change of load capacity due to inclination, additional energy due to fiber bending should be taken into account. This additional strain energy release rate due to bending of the fiber is (Ouyang et al., 1994)

$$G_b = \frac{mP_x}{16\pi r}\left(\frac{\sigma_{fu}}{E_f}\right)^2\left\{1+\frac{1}{\sinh^2[\omega(L-a)]}\right\} \tag{8}$$

where σ_{fu} is ultimate stress of the fiber, and m is a constant accounting for the yielding length of the fiber. Since the pullout load, P, should be replaced by the load component, $P_x = P_\theta cos\theta$, in Eq. (2) for the inclined fiber, the strain energy release rate for the inclined fiber, G_θ, is

$$G_\theta = \frac{P_x^2 \coth^2[\omega(L-a)]}{4E_f\pi^2 r^3} = \frac{(P_\theta \cos\theta)^2 \coth^2[\omega(L-a)]}{4E_f\pi^2 r^3} \tag{9}$$

The energy balance, G_θ - G_b = R, for the pull-out of the inclined fiber then results in

$$\frac{P_\theta^2 \cos^2\theta \coth^2[\omega(L-a)]}{4\pi^2 r^3 E_f} - \frac{mP_\theta \cos\theta}{16\pi r}\left(\frac{\sigma_{fu}}{E_f}\right)^2 \left\{1 + \frac{1}{\sinh^2[\omega(L-a)]}\right\} - \beta(a-a_0)^d = 0 \tag{10}$$

It is assumed that the R-curve does not change with fiber inclination in Eq. (10). For each value of debonded length, a, the corresponding value of P_θ can then be solved from Eq. (10). Since Eq. (10) includes the bending mechanism, it is valid only when the fiber is yielded. It can be expected that the minimum inclination angle for fiber yielding should be very small for metals based on small deformation theory in mechanics of materials. The above analysis may not apply to the case that the failure of matrix wedge also occurs when the fiber is pulled-out. This usually occurs when $\theta > 45°$.

The theoretical predictions are compared to the experimental results reported by Ouyang et al. (1994) in Fig. 6. The value of $m = 20$ was used. The values of $a_0 = 0.02$ mm and $\omega = 0.048$ 1/mm were used. The value of $P_c = 33.5$ N for the aligned fiber was used as the reference value for the theoretical prediction. Both the values of $\sigma_{fu} = 1700$ MPa and $\sigma_{fu} = 0$ were used for theoretical prediction. The value of $\sigma_{fu} = 0$ corresponds to the case where the bending effect on fiber pullout load is negligible. As a result, the bending effect can be separated as shown in Fig. 6. This bending effect depends on the diameter and the yield strength of the fibers used.

4 Summary

A fracture mechanics model has been developed to predict the pull-out behavior of fibers. Stable debonding prior to the peak pull-out load was described by a fracture resistance curve (R-curve). The effect of fiber embedded length on the pullout response can be described by the proposed R-curve. For a given interfacial condition, the pull-out of fibers with the same diameter but different embedded lengths can be predicted

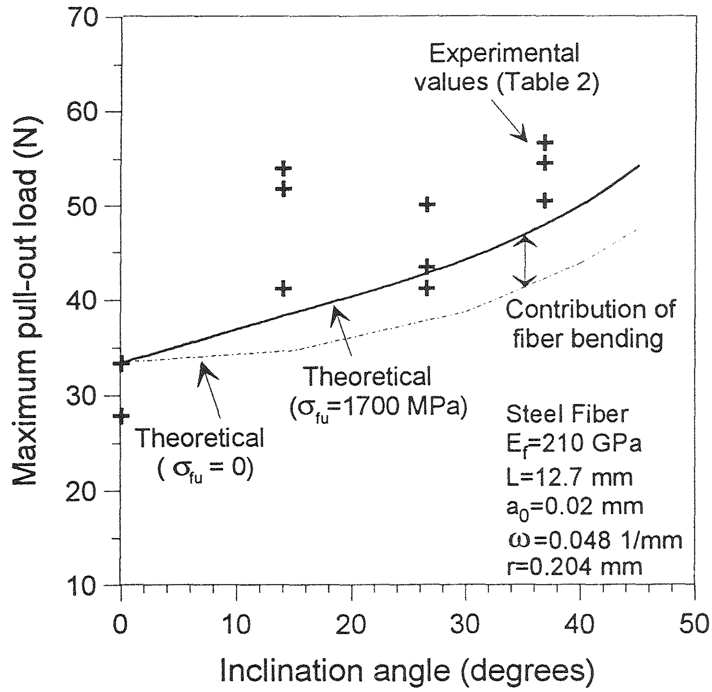

Fig. 6 Experimental and theoretical comparison of maximum pull-out load for steel fibers

using the same *R*-curve. The *R*-curve for the pull-out of aligned fibers has been extended to inclined fibers which also undergo bending during the pull-out. The fiber bending energy was incorporated into the energy balance for the stable interfacial crack propagation. The proposed *R*-curve approach can reasonably predict the pullout response of both aligned and inclined fibers.

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