FRACTURE OF FIBRE-REINFORCED CONCRETE BEAMS WITH LOW FIBRE CONTENT

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Abstract

A number of beams made of steel fibre-reinforced concrete was subjected to four point bending test in order to investigate the ultimate load as well as the post peak behaviour. A numerical model has been developed, which takes into account the action of concrete matrix and steel fibres separately. The model is based on the discrete crack. The behaviour of concrete matrix is approximated by means of linear fracture mechanics. Steel fibres are activated in the crack and the carried force depends on the pull-out of individual fibres. A small theoretical study shows the dependence of the ultimate load on the size of the beam. The effect of fibres is also size dependent.

1 Introduction

Concretes reinforced with different types of fibres are investigated at many research institutions. The aim of such research is often to get a new material with better mechanical properties. The idea followed here should improve the currently used concretes by adding a relatively small amount of fibres (~50 kg/m^3). Therefore a concrete of usual composition with rather big aggregate size is used (32 mm). Steel fibres which are rather cheap increase the quality of concrete subjected to tensile stresses without substantial increase of costs. The low amount of fibres makes it possible

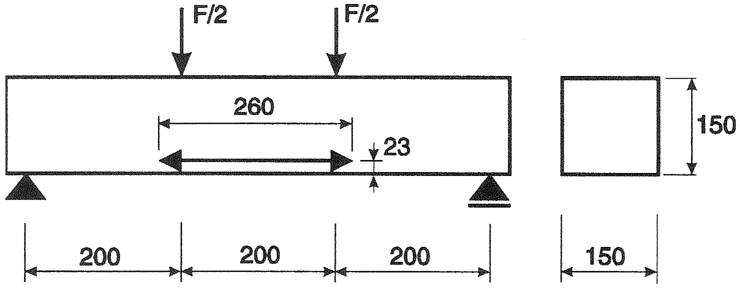


Fig. 1. Layout of the experiment

to avoid the problems with mixing of concrete. Such a material has not of course any extremely good properties, however, the advantage of its wide use without substantial difficulties makes it interesting for an efficient use in building industry. Such fibre-reinforced concrete is mainly used for industrial floors, tunnel linings, pile heads or foundations. In particular in the case of industrial floors the distance of expansion joints may be almost doubled and also the damage due to concentrated loading is significantly lower than in the case of a floor made of classical reinforced concrete.

A series of experiments of beams reinforced by different types of steel fibres has been carried out at the Czech Technical University in Prague. The individual fibres and differences in the results has been reported elsewhere e.g. Goldau et al. (1994). This paper deals with a theoretical model based on a discrete crack and the action of fibres in the crack. A small study showing a size effect serves as a basis for future experimental work.

2 Experimental program

The beams 150/150/700 mm have been subjected to four point bending test. The scheme of the test is illustrated in Fig. 1. The deflection was increased with a constant speed 0.6 mm/minute. The most important measured values included the deflections at the midspan and under the two forces and the longitudinal displacement crossing the crack in the tensile zone at the middle of the beams. The measuring length of the gauge was 260 mm. The longitudinal displacement is considered as the most suitable basis for comparison of measured and calculated results.

The fibres cut from steel sheets and profiled, smooth steel fibres and fibres with hooks at the ends (Dramix) were used in individual beams. The composition of concrete was in all cases exactly the same (with exception of fibres). The advantage of profiled fibres is mainly in better workability of concrete. Very good results were achieved in the tests with smooth steel fibres of the length 45 mm and diameter 0.45 mm. These fibres showed a stable and reliable behaviour in the post peak zone.

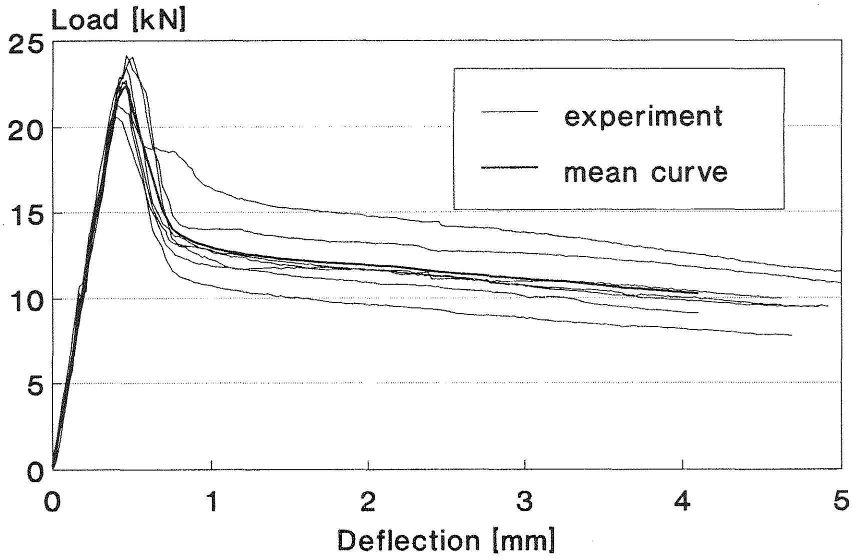


Fig. 2. Load-deflection diagram

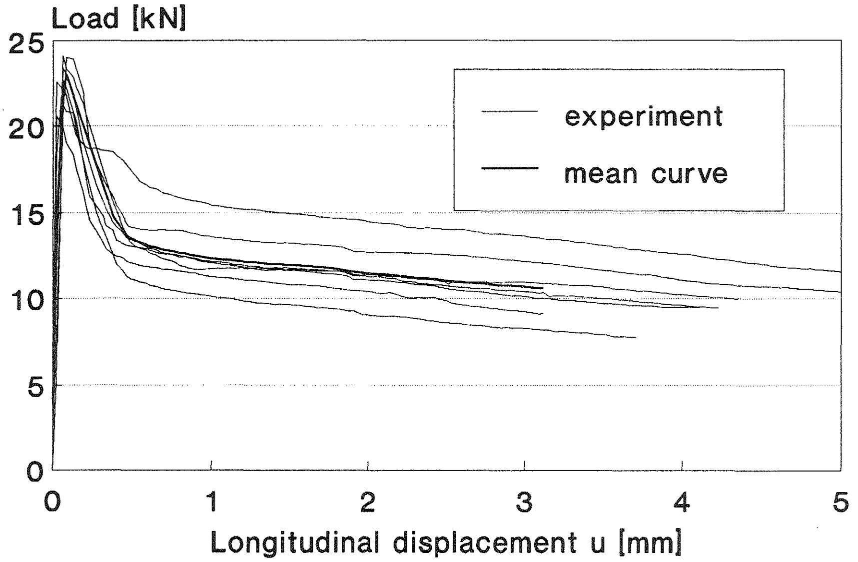


Fig. 3. Displacement in tensile zone

Almost 90 specimens were tested in series including from 3 to 9 beams. Additional tests were carried out on cubes and prisms in order to obtain the compression strength and splitting tensile strength. A small series of three beams was made of plain concrete of the same composition and was subjected to the equal loading procedure. A typical load deflection diagram for the beams reinforced by smooth steel fibres 0.45/45 mm is plotted in Fig. 2. Fig. 3 shows the dependence of load on the longitudinal displacement in the tensile zone. Although the diagrams look very similar there is a difference in the initial elastic portion. The deflections which show lower initial stiffness represent a displacement between the beam and stiff external structure connected with supports. This arrangement could influence the measured displacements, therefore the values from the Fig. 3 were taken as a basis for the development of a numerical model.

3 Numerical model

An analysis is based on the discrete crack approach. The crack is assumed to originate in the central area of the beam between the concentrated loads. The fibre-reinforced concrete is considered as a two phase material. The first phase is represented by a concrete matrix and the second one by fibres. The fibres are not active earlier than the crack opens. The behaviour of concrete matrix is assumed on the basis of linear fracture mechanics. The forces in fibres develop in dependence on the crack opening.

3.1 Concrete matrix

A model describing the deformation of concrete matrix contains three components - elastic strain, effect of cracking and plastic strain. A similar model has been used also by Bažant et al. (1994). The individual components of displacement are illustrated in Fig. 4. Elastic strain is described by the Young modulus which may be measured directly on specimens. Effect of cracking was analysed using a finite element analysis on the principles of linear fracture mechanics (LEFM) (Bažant et al. (1991)). The compliance function *C(a)* describing the dependence of flexibility of the beam on the crack length was approximated by the expression

$$C(a) = \frac{c_f}{(h-a)^m} - \frac{c_f}{h^m} \tag{1}$$

where a is a crack length, h is a depth of the beam and c_f and m are constants, which may be determined from the data obtained from a finite element model. The ultimate load F carried by the beam calculated according to the LEFM if the crack grows, is given as follows

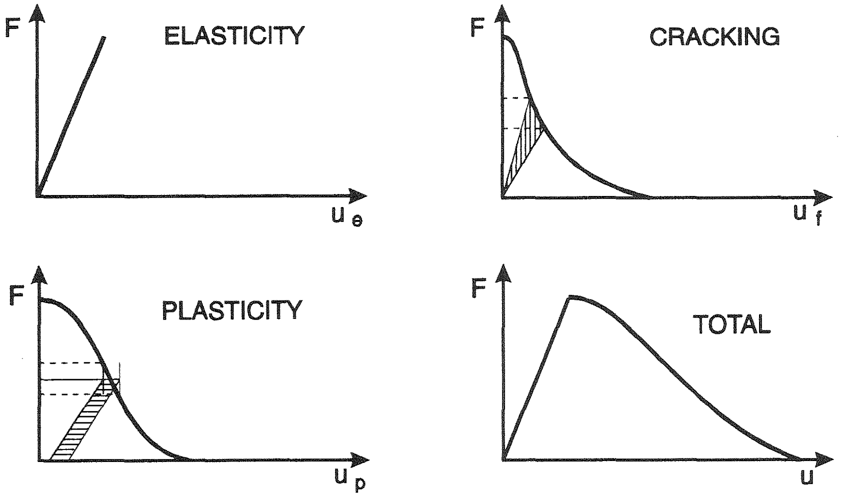


Fig. 4. Displacement components in the model

$$F = \sqrt{\frac{2G_f b}{\frac{\partial C}{\partial a}}} \tag{2}$$

where G_f is a fracture energy, b is a width of the cross-section and a is a crack length. The last component of the displacement is a plastic portion u_p, which is dependent on the loading force. In the presented analysis a parabolic - linear diagram has been used with the parameters determined from the test results. The total displacement is a sum of the three components $u = u_e + u_f + u_p$.

The ultimate load carrying capacity of the beam (the peak load) is approximated under the following assumtions.

- The peak load corresponds to the state of the crack origin, i.e. if the tensile stress is equal to the tensile strength over the critical length l_{cr} (Fig. 5), the linear elastic-plastic diagram is assumed. (In this study l_{cr}= 56 mm.)

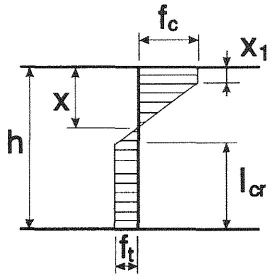
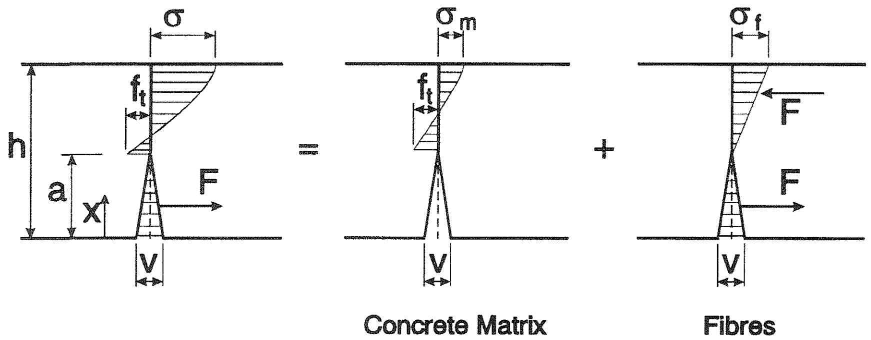


Fig. 5. Stress distribution in the cross-section at the start of cracking



a) b) c)

Fig. 8. Stress distribution in the cracked section

- The peak load is calculated according to the LEFM, if an infinitely short crack is assumed.

The lower of the two values is taken as an ultimate load of the beam, i.e. the load when the crack starts to progress.

The model briefly shown was used for the modelling of plain concrete beams. The comparison of experimental values and analytical values is presented in the diagram in Fig. 6. Only the last portion of the diagram is shown, since the values are rather small in comparison with the ultimate load.

3.2 Effect of fibres

The fibres are assumed to become active if the crack starts to originate. Due to the plasticity even a small crack opening activates the fibres, which increases the ultimate load of beams. The analysis is based on the pull-out force of one fibre. Experimental results of Naaman (1991), who tested similar fibres, were used as a basis for the force displacement diagram of one fibre. The diagram showing the force in the fibre in dependence on the slip in a pull-out test is plotted in Fig. 7. The friction of the fibre preserves the force almost constant after a certain slip.

In the crack the fibers are not all perpendicular to the crack plane. The fibres parallel to the crack plane have no effect on the tensile force carried by fibres in the crack. The fibres in some inclined position (related to the crack plane) may have higher strength than those in the perpendicular direction. A correcting factor was taken into account. Also the fibres are cut by a crack in a general position not exactly in the middle of the fibre. The anchoring length of fibers is then in average lower than a half of the fiber length, which leads to the decreasing of the force carried by fibres crossing the crack.

A very simple model of fibre effect is plotted in Fig. 8. The total action (Fig. 8a) may be assumed as a sum of the concrete matrix action (Fig. 8b)

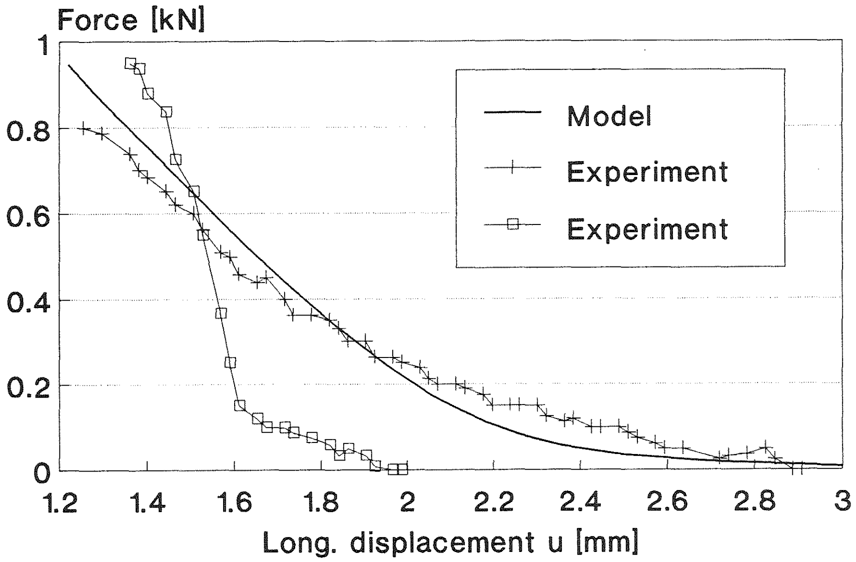


Fig. 6. Concrete matrix - diagram

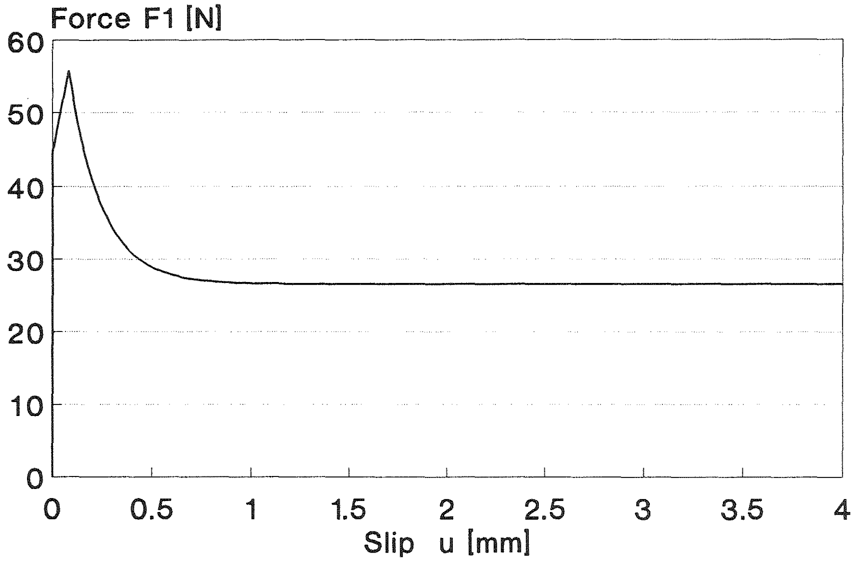


Fig. 7. Pull out of 1 fibre

and the fibre action (Fig. 8c). The tensile force carried by fibres is calculated in dependence on the crack opening *v*, crack length *a* and the number of fibres in the cross section. The bending moment carried by fibres may be calculated by integrating of forces in individual fibres crossing the crack over the crack area.

4 Experimental calibration

The set of experimental curves showing the displacement in the tensile zone was compared to the curve obtained by the model. It has been shown that all the components of displacement are necessary for the description of the experiment. The LEFM model alone is not able to fit the experimental curve, the plasticity is necessary to be included in particular in the case if the crack is long. In Fig. 9, there are plotted the highest and lowest experimental values, the average of the experimental curves and the curve obtained form the described model. The model gives a little lower values than the average of experiments, however, the shape of the curve is sufficiently accurate. The peak load is given as a sum of the load carried by a concrete matrix (19.7 kN) and the contribution of fibres in the crack (3.3 kN), which is just starting to open, what well correspond to the average peak experimental value (23.1 kN). The long post peak curve is a result of slow pull out of fibres from the crack surface. The number of fibres crossing the crack in the whole cross-section varied between 325 and 450. The fibres were counted at all the specimens and not only the number of fibres, but also their distribution in the cross-section influences the load-displacement diagram.

5 Size effect

The load carrying capacity of the beams depends on the action of concrete matrix as well as on the fibre action. The model presented here has been used to show the influence of the size of a specimen on the load carrying capacity as well as on the development of fibre action. The contribution of concrete matrix to the ultimate load is essential at all sizes. The effect of fibres is decreasing with increasing size. The opening of the crack at large specimens is small and contribution of fibres at this early stage negligible. The size effect is illustrated in Fig. 10. The horizontal lines correspond to plastic limit and elastic limit of plain concrete, which are independent on the size of a specimen. The inclined line shows the decreasing load carrying capacity calculated according to the LEFM. The load carrying capacity calculated according to the assumption of a critical length shows a size effect varying between plastic and elastic limits (triangles). Therefore the both criteria (acc. to Fig. 5 and acc. to LEFM) are taken into acount in the analysis. The last curve (squares) shows the size effect of fibre-reinforced concrete. The plastic limit is a little higher,

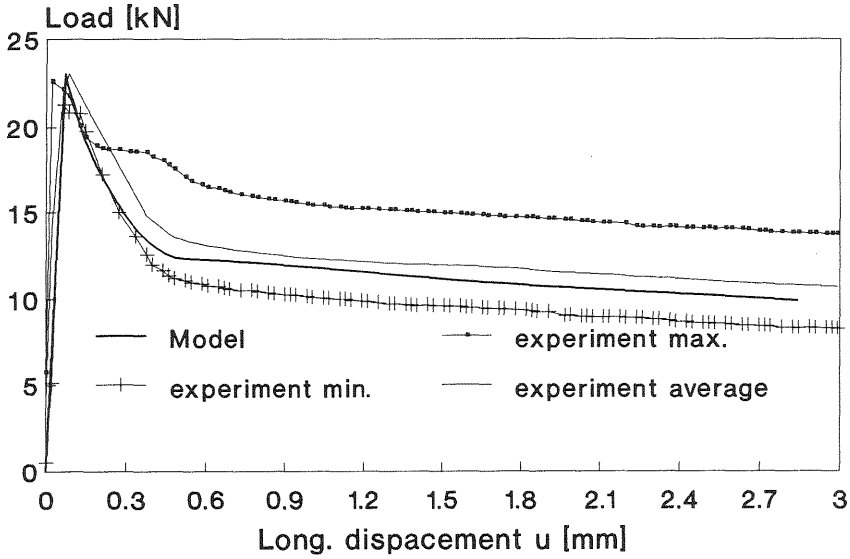


Fig. 9. Comparison Model-Experiment

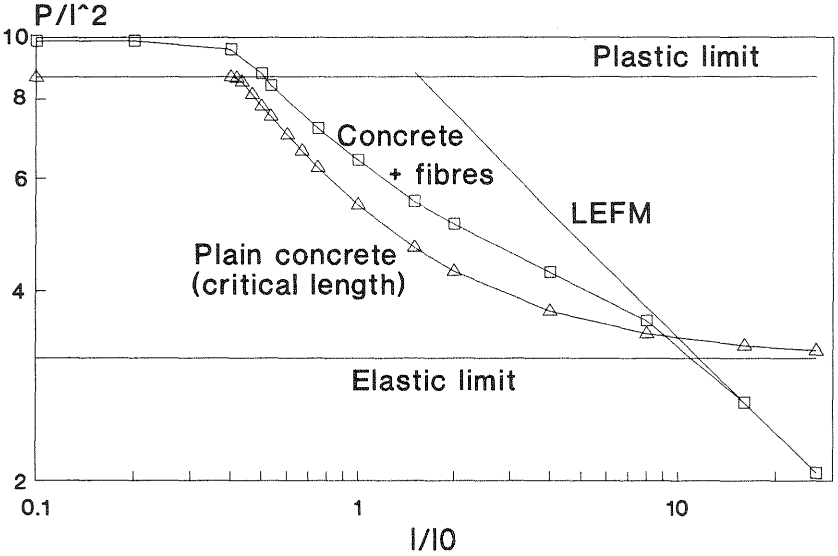


Fig. 10. Size effect

since the compression strength of fibre-reinforced concrete is higher than that of plain concrete.

6 Conclusions

A two phase numerical model of steel fibre-reinforced concrete has been developed. The displacement of the concrete matrix is considered as a sum of elastic deformation, plastic deformation and cracking. The effect of fibres depends on the number of fibres crossing the crack, on their pull-out resistance, on the crack length and crack opening. The model has been compared with experimental results and a sufficient accuracy has been achieved. The ultimate load carrying capacity shows the size effect. The effect of fibres on the ultimate load is also dependent on the size of a specimen. At small members the effect of fibres increases the peak load and at large elements this effect becomes negligible.

7 Acknowledgment

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