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A THERMO-MECHANICAL DAMAGE MODEL FOR CONCRETE AT ELEVATED TEMPERATURES

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Abstract
This paper gives an outline of a thermo-mechanical damage model for the behaviour of concrete under biaxial stress and elevated temperatures. The model is formed within the consistent framework of thermo-dynamics for non-isothermal conditions, and is coupled to the heat conduction equations for a general analysis of deformational response.

1 Introduction

The behaviour of concrete at elevated temperatures is important for an assessment of integrity (strength and durability) of structures exposed to a high temperature environment, in applications such as fire exposure, smelting plants, nuclear installations. This paper presents an overview of a thermo-mechanical damage model developed for concrete at elevated temperatures. The model has been derived within the consistent framework of thermodynamics, drawing on the iso-thermal damage models of Ortiz (1985) and Yazdani & Schreyer (1988) and the thermo-mechanical coupling aspects of Simo & Miehe (1992). In addition, account has been taken of the known stress-temperature dependence of concrete through the descriptions of thermal and thermo-mechanical damage, and thermal softening. Mechanical damage is related directly to compliance, with additional flexibility due to thermal damage. Explicit expressions have been derived for the free energy including elastic energy, damage due to microcrack formation, thermal-mechanical coupling and thermal energy.

The damage function is shown to be flexible in being able to capture the temperature dependent shape and size of failure surfaces: the model generally incorporates features of anisotropic damage, dilatation and inelastic strain responses. In a wider context, the damage model presented forms part of a study aimed at the development of a completely generalized analysis of concrete at transient elevated temperatures, including the coupling of damage, hygral diffusion and heat conduction through the material.

2 Dissipation and coupled thermo-mechanics

2.1 General framework of thermo-dynamics

Complete details of a general framework of thermodynamics can be found from recent developments in damage mechanics (Krajcinovic (1989)), plasticity (Maugin (1992)), coupled plasticity and damage (Hansen and Schreyer (1994)), and thermoplasticity (Armero & Simo (1992), Simo & Miehe (1992)). Consider a continuum defined on a reference configuration Ω in a time interval [0, t], deformation gradient, ε, stress tensor σ, associated temperature field $\Theta(., t)$, entropy η, density ρ_o, with body forces, **B**. The internal dissipation is given by:

$$\chi_{inl} = \sigma : \dot{\varepsilon} + \Theta \dot{\eta} - \dot{E} \tag{1}$$

where the three terms would be interpreted respectively as stress power, thermal power and change in internal energy. We can write the internal energy in the form, $E=\hat{E}(\varepsilon, \mathbf{D}, \eta)$, where $\mathbf{D} = \mathbf{D}(\sigma, \Theta)$ is a set of micro-structural variables, such as plastic strains, hardening parameters, or damage variables, which define the mechanical state of the material. These are typically described in constitutive form as evolution laws:

$$\dot{D} = \hat{D}(\sigma, D, \Theta) \tag{2}$$

Helmholtz free energy, ψ, is found from the internal energy through the Legendre transformation from which we obtain the conjugate relations:

$$\Theta = \frac{\partial E}{\partial \eta}; \qquad \eta = -\frac{\partial \psi}{\partial \Theta} \tag{3}$$

For the complementary problem, the internal energy and Gibb's free energy are defined as:

$$W = \hat{W}(\sigma, D, \eta); \qquad G = \hat{G}(\sigma, D, \Theta) \tag{4}$$

whence we find conjugate relations analogous to (3):

$$\Theta = -\frac{\partial W}{\partial \eta}; \qquad \eta = \frac{\partial G}{\partial \Theta} \tag{5}$$

Given a description based on Helmholtz energy and stiffness, which decreases with temperature in concrete, then the negative sign in (3) ensures an increase in entropy with temperature. For a definition based on Gibb's free energy and compliance, which correspondingly increases with temperature, one again finds an increase in entropy from (5). Finally, we write the nominal stress, absolute temperature and thermodynamic 'forces' (Maugin(1992)):

$$\epsilon = \partial_\sigma \hat{G}; \qquad \eta = \partial_\Theta \hat{G}; \qquad Y = \partial_D \hat{G} \qquad \chi_{inl} = Y\dot{D} \tag{6}$$

2.2 Entropy, dissipation and the Second Law

The second law of thermodynamics requires that the rate of change of entropy should not be less than the production of entropy from heat (Maugin (1992)), which gives rise to the dissipation inequality:

$$\chi = \chi_{inl} + \chi_{con} \geq 0; \qquad x_{con} = -\frac{1}{\Theta} Q{:}\nabla\Theta \tag{7}$$

where the second term is the dissipation due to conduction. We split the internal dissipation into parts associated with the thermal production of entropy, and the contribution to dissipation of the purely mechanical theory: $\chi_{inl} = \chi_{mech} + \chi_{ther}$. Since the second law must be satisfied under non-conductive and isothermal processes, one requires separately that $\chi_{inl} \geq 0$ and $\chi_{mech} \geq 0$, where the mechanical dissipation under isothermal conditions follows from Claussius-Duhem form: $\chi_{mech} = \sigma : \dot{\epsilon} - \psi$.

The general problem of maximum dissipation leads to evolution equations for an associative process in plasticity or damage (Simo &

Miehe (1992), Hansen & Schreyer (1994)). Given a smooth damage function, Φ, extension of the isothermal principle of maximum dissipation to a thermo-mechanical one gives the evolution of entropy:

$$\dot{\eta} = \dot{\mu}\, \partial_{\Theta} \Phi \qquad (8)$$

where the consistency parameter, μ, satisfies the Kuhn-Tucker conditions: $\mu \geq 0$; $\Phi(\sigma,\mu,\Theta) \leq 0$; $\mu\Phi(\sigma,\mu,\Theta)=0$. The flow rule (8) thus controls the evolution of thermal softening. The consequence is that the thermal dissipation can be written, $\chi_{ther} = \Theta\eta$, and we note from (8) that $\chi_{ther} = 0$ for an isothermal process.

3 Isothermal damage models

3.1 Two-phase model of Ortiz

In a seminal work, Ortiz(1985) developed a damage model for the inelastic repsonse of concrete where damage was deemed to accrue through microcrack formation in the mortar. Damage was applied directly to the compliance of the mortar, with the flow of damage related to response functions for tensile and compressive modes of fracture (i.e. cleavage and splitting modes) which are dictated by the orientation of microcracks. The aggregate phase is treated with a relatively simple plasticity model, with the composite response being determined through a theory of mixtures. Beginning with the Gibb's free energy for isothermal processes Ortiz(1985) writes:

$$G = \tfrac{1}{2}\,\sigma{:}C{:}\sigma - A^{c} \qquad (9)$$

where $\mathbf{C}$ is the fourth order compliance tensor, and A^c is the free energy to form microcracks. Flexibility is then written in additive form, $\mathbf{C} = \mathbf{C}^0 + \mathbf{C}^c(\mu)$. The first term represents the elasticity tensor in the absence of microcracks, the second the flexibility due to the active microcracks, and is regarded as the damage tensor, with μ being the damage variable. Thus, the flow rule for the inelastic strain, $\dot{\varepsilon}^i = \dot{\mathbf{C}}{:}\sigma$, requires the identification of an evolution law for the tensor damage in the form:

$$\dot{C} = \dot{\mu}\, R(\sigma) \quad \Rightarrow \quad \frac{\partial C}{\partial \mu} = R(\sigma) \qquad (10)$$

where $\mathbf{R}$ is a fourth order tensor governing the direction of damage.

To evaluate $\mathbf{C}^c$, Ortiz (1985) finds the flexibility assuming *all* microcracks are open, $\bar{C}^c$, and projects this onto the positive cone of the stress tensor. The cross-effects on microcracks under compressive splitting stresses are added to the cracked flexibility, with the compressive part being projected onto the negative cone, where the spectral decomposition of the stress tensor into its positive and negative parts is: $\sigma = \sigma^+ + \sigma^-$. Using the projection operators, the free energy can be rewritten in terms of the two modes of micocracking. The rate independent damage rules are written for the two modes of cracking:

$$\bar{C}^c{}_I = \dot{\mu}\,\frac{\sigma^+\otimes\sigma^+}{\sigma^+:\sigma^+}; \qquad \bar{C}^c{}_{II} = \dot{\mu}\,\omega\,\frac{\sigma^-\otimes\sigma^-}{\sigma^-:\sigma^-} \tag{11}$$

Finally, after applying the principle of maximum dissipation, one finds from (9-11) a damage function (Ortiz (1985)):

$$\Phi(\sigma,\mu) = \frac{\partial G}{\partial \mu} = \tfrac{1}{2}\,\sigma^+:\sigma^+ + \tfrac{1}{2}\,\omega\,\sigma^-:\sigma^- - \tfrac{1}{2}t^2(\mu) \tag{12}$$

where $t(\mu)(=2dA^c/d\mu)$ is denoted the critical stress for the onset of damage. The surface $\Phi(\mu)=0$ describes a circle in both biaxial tension and compression, and an ellipse in tension-compression. Ortiz (1985) considers flexibility along the axis as $(1/E_0 + \mu)$. Then by equating the general stress-strain relation to a specific form of tension softening, one finds the critical stress, such as:

$$t(\mu) = f_t e\,\frac{\ln(1+E_0\mu)}{(1+E_0\mu)} \tag{13}$$

3.2 Composite model of Yazdani & Schreyer

Yazdani & Schreyer (1988) developed an anisotropic damage model for concrete at room temperatures, based on Ortiz (1985), but applied to the composite concrete. Their damage surface is a modification of (12) to account for the observed macroscopic behaviour of concrete.

The response function for microcracking under compressive load is comprised of two parts. The first, which corresponds to the cross-effect, uses the shifted negative cone given by: $\bar{\sigma} = \sigma^- - \lambda_{max}\mathbf{i}$; λ_{max} is the maximum eigenvalue of $\bar{\sigma}$ introduced to preclude damage in hydrostatic compression, and $\mathbf{i}$ is the second order identity tensor. The second part is a semi-empirical term included to account for the effect of lateral pressure. Those authors postulate that imperfect fracture only occurs with the compressive mode of cracking, and thus relate the inelastic strain rate to the positive and negative cones of the deviatoric stress tensor. They write evolution of permanent strains:

$$\dot{\epsilon}^P = \dot{\mu}\,\omega\,(s^- + \beta s^+) \tag{14}$$

where $\beta > 1$ for permanent deformation is a material parameter, and the tensors, $\mathbf{s}^+$ and $\mathbf{s}^-$ are the aforementioned positive and negative cones. The use of the shifted cone is certainly effective, but creates a sharp point in hydrostatic compression which is not always representative. One can vary the parameters ω,β, $\lambda_{max} \leq \lambda \leq 0$ to exploit the surface shape.

4 A Thermo-mechanical damage model for concrete

We outline here a composite model for non-isothermal conditions based on the ideas of Ortiz (1985) and Yazdani & Schreyer (1988), despite Carol & Willam's (1994) criticizm of the stress-induced anisotropy in these models. We retain the spectral decomposition concepts, but with rotating principal axes. Microcrack closure thus occurs in local axes, and though the damage tensor may rotate, the irreversibility condition $\mu \geq 0$ should ensure that no stiffness recovery occurs (locally) even if principal axes cycle back to an earlier position.

4.1 Gibb's free energy

To formulate the constitutive laws, we begin with the relationship between compliance and stress tensors, and the Gibb's free energy:

$$\frac{\partial^2 G(\sigma,\mu)}{\partial\sigma\partial\sigma} = C(\mu) \tag{15}$$

Integrating twice with respect to σ yields the free energy:

$$G(\sigma, C^c, \Theta) = \tfrac{1}{2}\,\sigma{:}C{:}\sigma + \epsilon^P{:}\sigma + \epsilon^{\Theta}{:}\sigma + T(\Theta) - A^c \tag{16}$$

which is shown as dependent on the stress, damage tensor and temperature, and where ε^{Θ} is the sum of the free thermal strain and creep strains, both isothermal and transient. The only bar to an explicit evaluation of entropy from (16) is the expansion of the final term in A^c. It is more realistic to integrate the evolution law for entropy incrementally, since that flow rule will be well defined.

4.2 Evolution of thermal damage

We propose a temperature dependence of the compliance which is incorporated within the response functions above. We will evaluate the critical stress as Ortiz (1985), but assume all increase in flexibility, thermal or mechanical, to be attributed to the damage variable μ. Thus:

$$C = C^0 + C^c(\mu(\sigma,\Theta)) \tag{17}$$

The damage compliance is increased by a temperature dependent term applied to an evolution law like those in (11). Given the inhibiting effect of compression (Khennane & Baker (1992)), and the desired anistropy under mixed states of stress, we suggest a term like (11a) scaled by a temperature dependence $(1+f(\Theta))$, where the function $f(\Theta)$ should capture experimental results. Note that even when $\sigma^+ = 0$, damage still occurs mechanically under large compression and thermal softening so that the observed beneficial effect of *reasonable* initial stress is captured.

To define $f(\Theta)$, we approximate $1/E \approx 1/E_0(1+\Theta)^2$, so that the initial flexibility can be separated out, leaving: $f(\Theta) = (2\Theta - \Theta^2)$. In order to preserve the irreversibility asssumption, damage only accrues once the maximum temperature at a point has been exceeded. Hence we substitute the maximum temperature, Θ_{max}, for Θ in the above so that no stiffening occurs on local cooling. Finally, since the term like $O(\Theta^2)$ may be excessive for a cummulative damage model, we modify maximum temperature by a factor, $0 < \gamma < 1$, where γ should be determined from numerical experiment.

For the evolution of damage, we sum the *thermo-mechanical* response function to the cross effects given by the shfited negative cone, and a form of Yazdani & Schreyer's (1988) lateral pressure term modified to reflect the biaxial nature of this work. A beneficial temperature dependence is applied to this third function:

$$\bar{C}_{III}{}^{c} = \dot{\mu} g(\Theta)\,[\,\omega\,\alpha\, H(-\bar{\lambda})]\, I^4 \tag{18}$$

where α is a factor which accounts for the lateral pressure, $\bar{\lambda}$ is the smallest eigenvalue of $\bar{\sigma}$, and $\mathbf{I}^4$ is the fourth order identity tensor. The function $g(\Theta)$ only exists under simultaneous heating and compressive loading, and in such cases, $g(\Theta) = h^2(\Theta)$. This negates the softening applied to the critical stress, except that the second response function in biaxial compression has no temperature dependence.

4.3 Damage function

The permanent strains should be included in the set of micro-structural variables which govern the mechanical state of the material, in addition to the damage tensor itself; for the flow of permanent strains, we use (14). Thus we have: $\mathbf{D}=(\mathbf{C}^c, \varepsilon^p)$, and it can be shown that the formal statement of internal dissipation leads to the form used by Ortiz (1985),

$$\chi_{inl} = \frac{\partial G}{\partial C^c}:\dot{C}^c + \frac{\partial G}{\partial \epsilon^p}:\dot{\epsilon}^p = \dot{\mu}\,\frac{\partial G}{\partial \mu} \tag{19}$$

from which we find the explicit form:

$$\Phi(\sigma,\mu,\Theta) = \tfrac{1}{2}(1+\gamma\,\Theta_{max})^2(\sigma^+:\sigma^+) + \tfrac{1}{2}\,\omega\,\sigma^-:\frac{\bar{\sigma}\otimes\bar{\sigma}}{\bar{\sigma}:\bar{\sigma}}:\sigma^- + \omega\,(s^- + \beta\,s^+):\sigma$$
$$+ \tfrac{1}{2}\,\omega\,\alpha\,g(\Theta)\,H(-\bar{\lambda})\,\sigma:\sigma - \tfrac{1}{2}t^2(\mu,\Theta)$$

4.5 Thermal softening

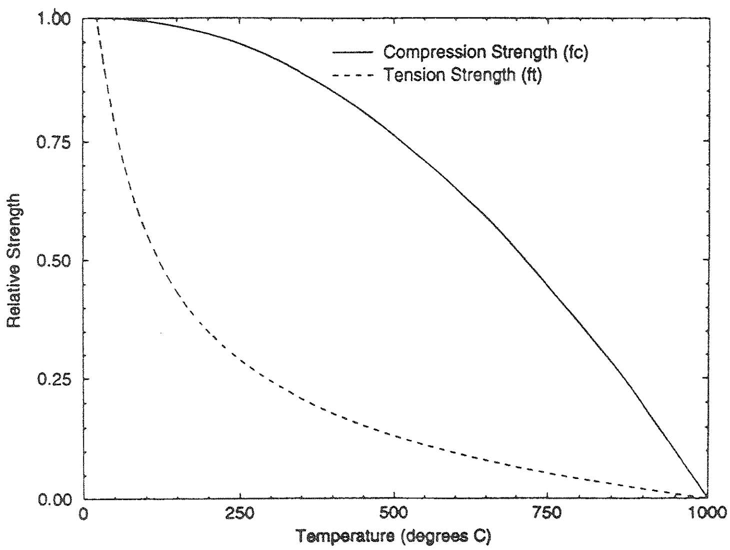

Figure 1: Thermal softening of strength

Thermal softening is here restricted to softening of the critical stress with temperature. The approach is to form a peak tensile strength that *mirrors* activation energy concepts, and a compressive strength that follows empirical observations. We define thermal softening on stress in multiplicative form: $f_t(\Theta) = f_t\, h(\Theta)$ and $t(\mu, \Theta) = t(\mu)\, h(\Theta)$, where $h(\Theta)$ is the softening function. The reduction in f_c' is essentially parabolic, with little loss up to 200°C (Schneider (1988)). The form of the damage function under uniaxial compression suggests the following:

$$h(\Theta) = [1 - (\Theta_{max}/1000)^2] \tag{21}$$

given that at 1000°C all strength has been lost. However, because of the temperature dependence of the first term in (20), the limiting damage in tension will not have the same form. For uniaxial tension, the damage function reveals a stress variation like (22) divided by $(1 + \gamma\Theta_{max})$. Figure 1 shows the curve for $\gamma = 0.01$, which clearly resembles the Arrhenius form of activation energy.

It can be shown that thermal dissipation is guaranteed non-negative, since explicit differentiation of (20) yields $\partial\Phi/\partial\Theta \geq 0$, given that we ensure that critical stress decreases monotonically with temperature.

5 Results

Despite the complexity of the damage function, it transpires that we can derive a consistent tangent operator for the mechanical phase exactly as given in Ortiz (1985), without involving the complexities of the damage function through intermediate stages. However, we defer computational details to another article. Here we restrict ourselves to results for limiting damage in cases of isothermal loading at elevated temperatures.

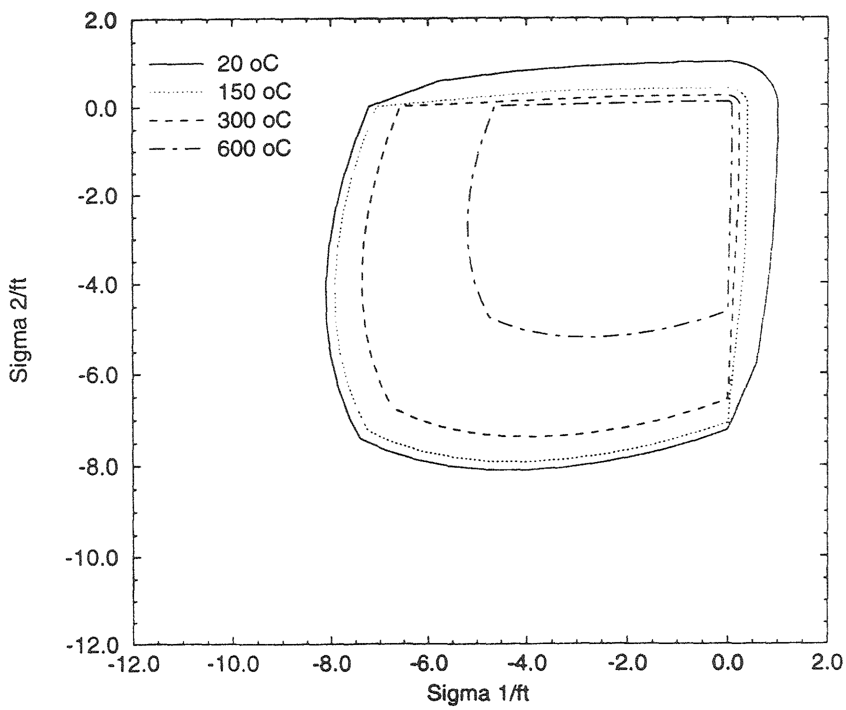

Figure 2: Thermo-mechanical damage surfaces

Figure 2 shows the limiting damage surfaces obtained with no stress during heating. The effect of the parabolic softening is to crowd the envelopes together at moderate temperatures, which is generally the case for normal strength concrete; for lightweight concrete a linear softening seems more appropriate. One can vary the shape of the envelopes as pointed out earlier by varying the factors: ω, β, λ. Similar envelopes including the inhibiting effect of initial stress during heating show a greater effect at high temperatures as expected, but overall the envelopes appear to capture the right features.

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