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# DAMAGE EVOLUTION LAWS FOR CONCRETE - A COMPARATIVE STUDY

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## Abstract
The main issue is focused on the response of different damage laws based on one parameter $k$ which is found from fracture dissipation requirements. In addition local, non-local and mesh-adapted formulations are also included. The results are more sensitive to the selected law than to the formulation adopted, however, the non-local approach is the only one which describes the failure mechanism accurately.

## 1 Introduction

Experimental observations, (Ditommaso 1984, van Mier 1984), show that the non-linear behaviour of concrete is due, basically, to the presence, growth, and coalescence of microcracks and/or microvoids depending on the loading path imposed. The development and propagation of macrocracks, leading eventually to the rupture, are the final consequence of the microcrack mechanism.

Continuum Damage Mechanics, (Kachanov 1986), has been applied successfully by several authors, (Mazars et al. 1989, Oliver et al. 1990), to simulate some of the micro-mechanisms mentioned above. This is done by introducing an internal variable $d$ which monitors the degradation of the elastic properties of the material. This paper presents a scalar damage model based on the work of Mazars (1984). The model is limited to radial or quasi-radial loading. On the other hand, the conclusions of the discrete and smeared crack models, (Petersson 1981, Rots 1988), in relation to the importance of the shape of the softening curve, inspired us to analyse the response of different tensile damage evolution laws. These laws are characterised by one parameter $k$ which is found from fracture dissipation requirements. In addition to the local formulation, an adapted mesh-material law and non-local versions are also presented.

## 2 Scalar damage model

Simplifying the material behaviour to be elastic-damaging the constitutive equation is assumed to be:

$$\sigma = (1-d)\,C : \epsilon \tag{1}$$

Thus, given a relationship $d = g(\epsilon)$ , the model is fully determined and the only requirement to assure positive dissipation is that $g(\epsilon)$ must be an increasing function. Mazars uses the constitutive Eqn. 1 and assumes that the accumulated tensile principal strains $\langle\epsilon_i\rangle_+$ are the main responsibles for crack propagation. Hence, the damage growth conditions are determined by the loading function $f$ :

$$f = \epsilon_{eq} - \kappa\,(d) \leq 0 \tag{2}$$

$\kappa\,(d)$ is the hardening-softening parameter and the equivalent strain $\epsilon_{eq}$ is defined as :

$$\epsilon_{eq} = \theta\,(\langle\sigma_i\rangle_-)\,\sqrt{\sum\langle\epsilon_i\rangle_+^2} \tag{3}$$

$\theta(\langle\sigma_i\rangle_-)$ is a weight factor introduced to improve the response of the model in multiaxial compression. Under a general state of stresses the total damage is assumed to be a linear combination of tensile damage $d^+$and compressive damage $d^-$,and in this simple way the dissymmetry of concrete is taken into account. Thus, the total damage can be expressed by :

$$d = \alpha^\beta d^+ + (1-\alpha)^\beta\, d^- \tag{4}$$

$\alpha$ is a parameter which separates, in $\langle\epsilon_i\rangle_+$, the influence of the tensile and compressive stresses. $\beta$ is a parameter introduced to describe shear dominated problems.

# 3 Damage evolution laws

## 3.1 Compressive damage law ($d^-$)

The original law, proposed in Mazars (1984), is adopted here :

$$d^- = 1 - \frac{(1-A^-)\cdot\epsilon_0}{\epsilon_{eq}} - A^- \cdot \exp\left(-B^-\left(\epsilon_{eq}-\epsilon_0\right)\right) \tag{5}$$

$\epsilon_0$ is the initial tensile damage threshold (assumed as $f_t'/E$). The parameters $A^-, B^-$ are identified by fitting to data from the uniaxial compression test on cylinders. However, from derivability conditions, at damage initiation and at the maximum compressive stress, these parameters can be related to the maximum compression stress $f_c'$ and its corresponding strain $\epsilon_{\max}$ .

## 3.2 Tensile damage laws ($d^+$)

As mentioned before, this work is focused on different kinetics of tensile damage and in particular the following laws:

- **The "Linear" Law**

$$\begin{gathered} d^+ = k\left[1 - \left(\frac{\epsilon_0}{\epsilon_{eq}}\right)\right] \Longleftrightarrow \text{if } \epsilon_0 \leq \epsilon_{eq} \leq \epsilon_u \\ d^+ = 1 \Longleftrightarrow \text{if } \epsilon_{eq} \leq \epsilon_u \end{gathered} \tag{6}$$

- **The Power Law**

$$d^+ = \left[1 - \left(\frac{\epsilon_0}{\epsilon_{eq}}\right)^k\right] \tag{7}$$

- **The Modified Mazars' Law**

$$d^+ = 1 - \exp\left[k\left(1 - \frac{\epsilon_{eq}}{\epsilon_0}\right)\right] \tag{8}$$

- **The Oliver's Law**

$$d^+ = 1 - \frac{\epsilon_0}{\epsilon_{eq}} \cdot \exp\left[k\left(1 - \frac{\epsilon_{eq}}{\epsilon_0}\right)\right] \tag{9}$$

The parameter $k$ is found from a uniaxial tensile test, using Eqn. 1 and writing the total specific dissipated energy $g_f$ (the area under the $\sigma - \epsilon$ curve) as:

$$g_f = R + E \int_{\epsilon_0}^{\infty} (1 - d^+) \cdot \epsilon \cdot d\epsilon \tag{10}$$

$R(\frac{1}{2}E\epsilon_0^2)$ is the resilience modulus (the absorbed energy in the elastic range) and $E$ the Young modulus. Through the classical definition of the crack band width $h$ and the fracture energy concept $G_f$ , (Bazant et al. 1982), it is possible to relate, (Bazant et al. 1989):

$$g_f = \frac{G_f}{h} \tag{11}$$

Now, introducing the characteristic length $\lambda$, (Ottosen 1990), as:

$$\lambda = \frac{G_f}{R} \tag{12}$$

and combining Eqn. 11 with Eqn. 12 we get:

$$g_f = \frac{\lambda}{h} \cdot R \tag{13}$$

Thus, in order to avoid snap-back behaviour the relation $\frac{h}{\lambda} \leq 1$ must be satisfied. Finally the material parameter $k$ is easily determined by substitution of Eqns. 11 and 12 in Eqn. 10 . The $k$ value for each damage tensile law becomes:

$$k_{linear} = \frac{1}{\left[1 - \frac{h}{\lambda}\right]} = \frac{\epsilon_u}{\epsilon_u - \epsilon_0} \tag{14}$$

$$k_{power} = \frac{2}{\left[1 - \frac{h}{\lambda}\right]} \tag{15}$$

$$\left[\frac{h}{\lambda} - 1\right] \cdot k_{maz}^2 + 2 \cdot \frac{h}{\lambda} \cdot k_{maz} + 2 \cdot \frac{h}{\lambda} = 0 \tag{16}$$

$$k_{oliver} = \frac{2 \cdot \frac{h}{\lambda}}{\left[1 - \frac{h}{\lambda}\right]} \tag{17}$$

The uniaxial tensile response of the analysed laws, for given $G_f$ and $h$ values, is shown in Fig. 1.

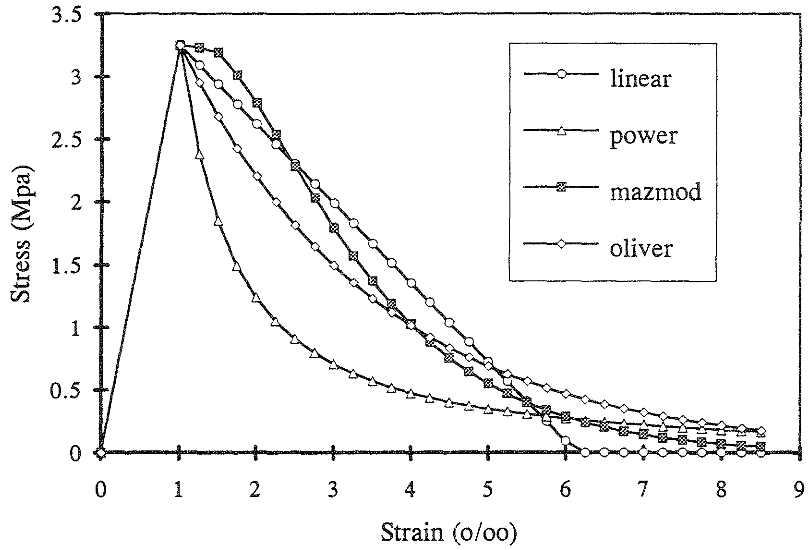


Fig. 1: Uniaxial tensile response

### 3.3 Local, non-local and mesh-adapted formulations

The local version of the previous model is completed when, in addition to the elastic properties, the tensile ($\epsilon_0$, $G_f$ , $h$ and the shape of evolution) and compressive ($f_c'$ and $\epsilon_{\max}$) damage parameters are given. However, it is well known that this approach lacks mesh objectivity. Therefore, to reduce mesh-dependence a non-local version, (Bazant et al. 1989), is presented. Strain localization is regularized by defining the equivalent strain $\epsilon_{eq}$ (Eqn.3) in a non-local (averaged) manner as:

$$\epsilon_{eqnl}\left(\overrightarrow{x}\right) = \frac{\int_{\left|\overrightarrow{r}\right|<\frac{h}{2}} \exp - \left(\frac{2\left|\overrightarrow{r}\right|}{\frac{h}{2}}\right)^2 \epsilon_{eq}\left(\overrightarrow{x}+\overrightarrow{r}\right) dV}{\int_{\left|\overrightarrow{r}\right|<\frac{h}{2}} \exp - \left(\frac{2\left|\overrightarrow{r}\right|}{\frac{h}{2}}\right)^2 dV} \tag{18}$$

Another possibility consists of averaging the scalar damage parameter $d$ in the same way as Eqn. 18 does using $\epsilon_{eq}$. The non-local formulation gives converging results to the right energy dissipation with mesh refinements (global behaviour), furthermore the strain field in the localization zone resembles closely experimental observations (local behaviour). The adapted mesh-material law is an improvement of the local version and it is done by adjusting the tensile damage parameter $k$ to the element size, keeping the

Table 1: Material parameters

| | | |
|---|---|---|
| $E$ | 37700 | Mpa |
| $\upsilon$ | 0.2(0.15) | |
| $\epsilon_{\max}$ | 1.86 | 0/00 |
| $f'_c$ | 39.45(30.0) | Mpa |
| $f'_t$ | 3.77(3.0) | Mpa |
| $G_f$ | 0.078(0.1) | N/mm |
| $h$ | 24 | mm |

compressive parameters constants. With this choice reduction of mesh-dependence is achieved, however, the size of the damage zone becomes mesh dependent. The influence of the integration scheme on the dissipated energy has not been analysed. The $k$ parameter is found in this approach using Eqns. 14 to 17 and introducing:

$$h = \sqrt{A_e} \tag{19}$$

with $A_e$ being the area of the element.

# 4 Numerical results

## 4.1 Damage laws response (splitting test)

The original analysis of this test is found in Saouridis (1988), where, after comparing local and non-local formulations, it was concluded that the maximum load is underestimated and damage was unable to propagate into the tensile region when the local model was used. The test is reanalysed here using the tensile damage laws proposed previously. The whole cylinder ($160 \times 320mm$) was modelled with 1278 linear triangular plane strain elements with one integration point. Three nodes (top and bottom) were used to impose the displacements. The material parameters assumed, as close as possible as in the original work, are given in Table 1. The global behaviour (load-displacement curve) of the analysed laws is shown in Fig. 2. The response of the different formulations is depicted in Fig. 3. The conclusion from these figures is straightforward : the results are more sensitive to the damage law used than to the formulation adopted. In this particular example the maximum experimental load reported lies between 0.47 to 0.66 kN/mm which corresponds quite well with the results

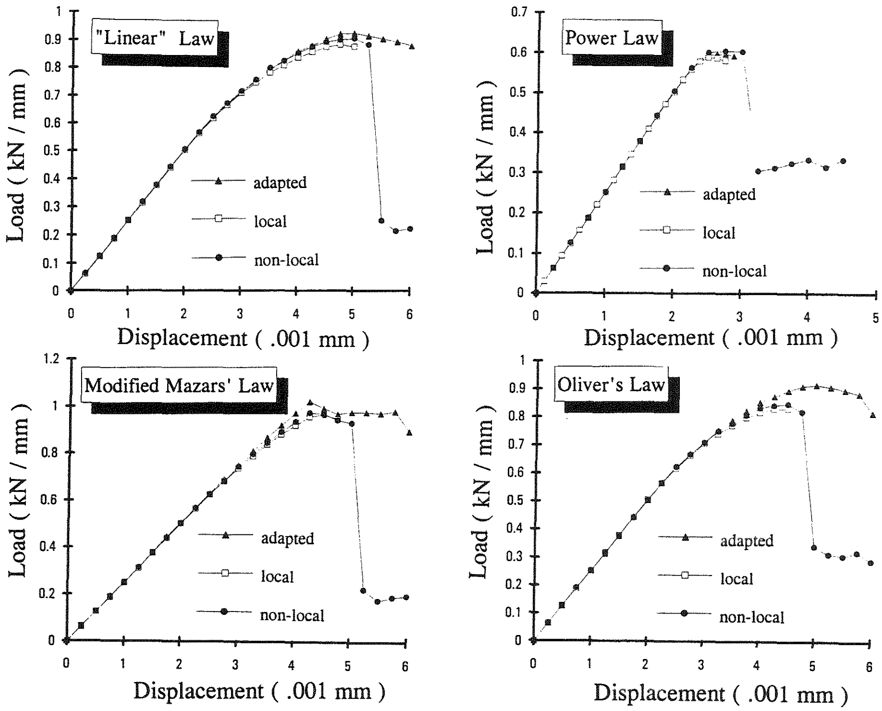


Fig. 2: Damage laws (global behaviour)

given by the power law, for all the three formulations. The other laws overestimate the maximum load. However, the correct failure mechanism is only given by the non-local formulation (a snap-back behaviour followed by yielding).

### 4.2 Mesh dependence (splitting test)

The cylinder analysed now ($80 \times 160mm$) has been modelled, (Feenstra 1993), using a composite (two yielding functions) plasticity based model. Just for comparative purposes we changed some of the material parameters shown in Table 1 (these in parentheses) and we adopted the Oliver's law to fit the input data reported by Feenstra. Next, we investigated the mesh sensitivity of the three formulations. As in the preceding analysis the same kind of elements (here plane stress), integration scheme and loading condition, were used. Three meshes were considered: coarse (322 el.), middle (1278 el.) and fine (5118 el.). The global response and the damage distribution (middle mesh) of each formulation is presented in Fig. 4. The experimental maximum load lies between 50 to 70 kN which agrees well with our numerical results (as it does too Feenstra's results). The three versions give approximately the same maximum load. However, as should be expected, the local

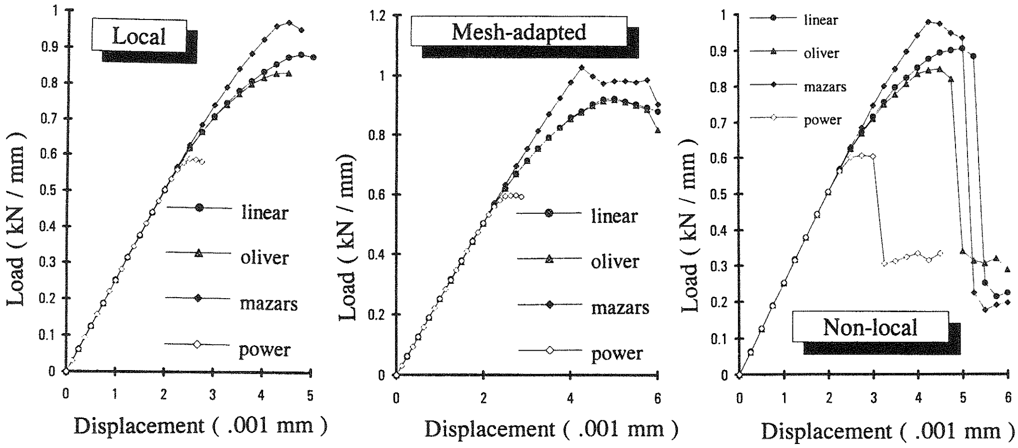


Fig. 3: Local, mesh-adapted and non-local response

approach presents a high tendency to snap-back with mesh refinements whereas the mesh-adapted and the non-local results are quite stable.

## 5 Conclusions

Under the continuum damage mechanics theory the scalar model of Mazars was presented. The numerical results of this simple model (one loading function) are in good agreement with the experimental observations. This conclusion comes from modelling the splitting test (failure in the tension-compression regime). Special attention was given to analysing the response of different damage evolution laws and the mesh dependence of the local, non-local and mesh-adapted formulations. The results were more sensitive to the damage law used than to the formulation adopted. However, the correct failure mechanism was only simulated by the non-local approach. Convergence of results was observed with the non-local and mesh-adapted formulations.

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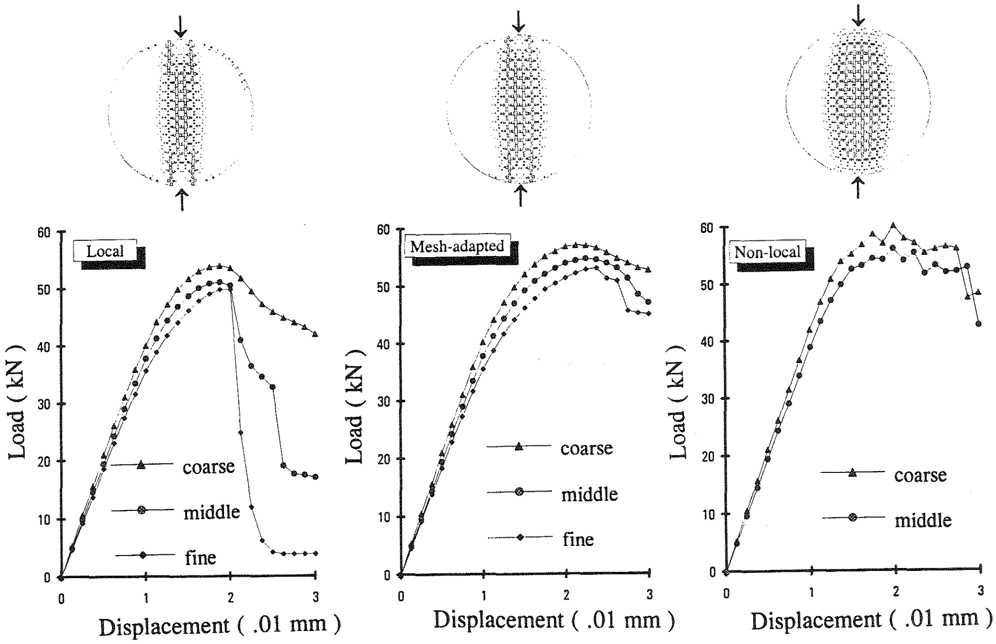


Fig. 4: Mesh sensitivity response

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