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A DAMAGE MECHANICS MODEL FOR CONCRETE SUBJECTED TO COMPRESSION

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Abstract
In this work, the crack propagation in concrete under uniaxial compression is described using a mathematical model based on damage mechanics. The pre-existing crack surfaces and fracture surface energies of concrete are calculated by means of the proposed model. The model is applied to the available test results. It is seen that there is good agreement between calculated values and experimental data.

1 Introduction

The mechanical behaviour of damaged materials due to existence, growth and nucleation of microdefects such as microcracks and microvoids have been the subject of many investigations. The presence of microcracks and the development of microcracking are the main cause of the short-term nonlinear behaviour of concrete. Initiation of bond cracks and joining of mortar cracks to form continuous crack path prior to ultimate strength has been widely investigated (Bazant and Mazars 1989, Ju et al. 1990).

Recently, in order to obtain mathematical modeling of damage in concrete, much research has been accomplished. However, adequate attention has not been given to determine how damage parameters will be obtained in concrete. Furthermore, quantitative measurements are needed for the mathematical modeling of concrete and a better understanding of fracture mechanism in the material. The main objective of this work is to obtain some fracture parameters of a quasi-brittle material such as concrete using a mathematical model based on damage mechanics.

2 The energy balance approach

Consider a solid body subjected to certain external loads as shown in Fig. 1. The cracks in the body grow and propagate under the loads. In most general case, the thermodynamic equilibrium equation of the body can be written as

$$\frac{dU}{dt} = \frac{dW}{dt} + \frac{dT}{dt} + \frac{dD}{dt} \tag{1}$$

where t is the time, U is the work done by the external load, W is the reversible (elastic) component of the stored energy, T is the kinetic energy

Fig.1. A solid body subjected to external loads

and D is the sum of all the irreversible energies such as fracture surface free energy or fracture energy, plastic work and viscous dissipation (Erdoğan 1968). Since the total dissipative energy is created near a crack (Knott 1973), the time differential of the total irreversible energies dD/dt can be written as

$$\frac{dD}{dt} = \frac{dW}{dS}\frac{dS}{dt} = \frac{dD}{dA}\frac{dA}{dt} = \gamma\frac{dA}{dt} \tag{2}$$

where $S(t) = S_o + A(t)$ in which, S_o is the total surface area of the body excluding crack surface and $A(t)$ is the crack surface at a certain time, $dD/dA = \gamma$ is the amount of energy required to create a unit area of fracture surface (fracture surface energy or strain energy release rate), and dA/dt is the rate of fracture surface energy (Erdoğan 1968).

If the cracks do not propagate when the external loads are kept constant, the system is quasistatic or quasistable and hence $dT/dt=0$. In this case, Eq.1 may be written as follows

$$\frac{dU}{dA}\frac{dA}{dt} = \frac{dW}{dA}\frac{dA}{dt} + \frac{dD}{dA}\frac{dA}{dt} \tag{3}$$

Then, the following equation can be expressed

$$\frac{d(U-W)}{dA} = \gamma \tag{4}$$

and from the integration of Eq.4

$$U-W = \gamma A_o\left(\frac{A}{A_o}-1\right) \tag{5}$$

can be derived. In this equation, A_o is the area of crack surface in the solid when $U=W=0$. The close neighbourhood of the cracks can be accepted as a stress-free region (Knott 1973). In Fig.2, v_0 is the volume which is able to cary load and V is the total volume of the solid body, $V-v_o$ and $V-v$ are the stress-free volumes before and after loading respectively.

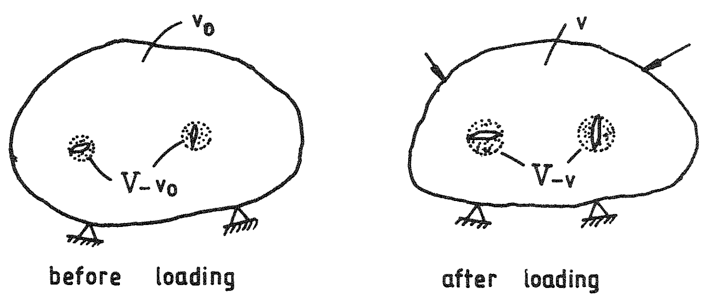

Fig.2. A solid body with flaws before and after loading

Let α_o and α the average of the surface area and h_o and h the corresponding stress-free volume of n cracks of the body before and after the loading respectively, and k be any constant. Then, the following equations can be expressed:

$$n\alpha_o = A_o,\quad V - v_o = nh_o,\quad \alpha_o^{3/2} = kh_o,\quad n\alpha = A,\quad V - v = nh,$$
$$\alpha^{3/2} = kh,\quad n\alpha_o^{3/2} = knh_o = k(V - v_o),\quad n\alpha^{3/2} = knh = k(V - v) \tag{6}$$

From this set of equation we have the following expression

$$\frac{A}{A_o} = \left(\frac{V - v}{V - v_o}\right)^{2/3} \tag{7}$$

3 Damage mechanics approach

By definition damage corresponds to irreversible degredation of the cohesion of the material under internal and/or external straining. This may lead to failure of a representative volume element. In damage mechanics, the strength of a loaded structure is determined by the deterioration of the material caused by loading. This deterioration or damage may be described in terms of continuous defect field.

Let us consider the stress-strain curve in concrete under uniaxial compressive loading. If the total volume of the concrete specimen is V, the work done by external forces can be written as

$$U = \left[\int_o^{\epsilon} \sigma(\epsilon) d\epsilon\right] V \tag{8}$$

where σ and ϵ are uniaxial stress and strain respectively. The reversible (elastic) work is approximately

$$W = \frac{1}{2}\frac{\sigma^2}{E} V \tag{9}$$

where E is the modulus of elasticity. From Eqs.5, 8 and 9 we have

$$\int_o^{\epsilon} \sigma(\epsilon) d\epsilon - \frac{1}{2}\frac{\sigma^2}{E} = \gamma \frac{A_o}{V}\left(\frac{A}{A_o} - 1\right) \tag{10}$$

If we take differential of both sides in Eq.10, then we have

$$\sigma - \frac{\sigma\sigma'}{E} = \beta \frac{d}{d\epsilon}(\frac{a}{a_o} - 1) \tag{11}$$

where $a = A/V$, $a_o = A_o/V$, $\beta = a_o \gamma$.

Under uniaxial state of stress, the damage can be characterized by a scalar parameter, D (Kachanov 1986), which denotes the concentration of microdefects existing in a representative volume element of the material.

If $D=0$, the material has no deterioration which is the reference state. If $D=1$, this indicates the failure of a representative volume element in the material.

Thus, from Fig.2, D can be written as

$$D = (v_o - v)/v_o \tag{12}$$

which is equal to zero when $v = v_o$ and equal to 1 when $v=0$. Hence, the continuous damage function ψ defined as

$$\psi = 1 - D = \frac{v}{v_o} = \begin{cases} 1 \ , & \epsilon = 0 \\ \frac{v}{v_o} \ , & 0 < \epsilon < \epsilon_m \\ 0 \ , & \epsilon = \epsilon_m \end{cases} \tag{13}$$

where ϵ_m is the maximum strain, ψ is a decreasing function, as a result $\psi' < 0$. From Eq.7, a/a_o can be obtained as

$$\frac{a}{a_o} = \left[\frac{\frac{V}{v_o} - \frac{v}{v_o}}{\frac{V}{v_o} - 1}\right]^{2/3} = \left[\frac{q - \frac{v}{v_o}}{q-1}\right]^{2/3} = \left(\frac{q-\psi}{q-1}\right)^{2/3} \tag{14}$$

where $q = V/v_o > 1$ which is a constant. In this case the following equation can be expressed

$$\frac{d}{d\epsilon}(\frac{a}{a_o} - 1) = -\frac{2}{3} \frac{1}{(q-1)^{2/3}(q-\psi)^{1/3}} \frac{d\psi}{d\epsilon} \tag{15}$$

Substitution of Eq.15 into Eq.11, the kinetic equation of damage can be obtained as

$$\psi' = \frac{d\psi}{d\epsilon} = -K(q-\psi)^{1/3}(\sigma - \frac{\sigma\sigma'}{E}) \tag{16}$$

where $K=(3/2)(q-1)^{2/3}/\beta$. From Eq.16, the following equation may be given

$$Y = (\frac{\psi'}{\sigma - \frac{\sigma\sigma'}{E}})^3 = -K^3(q-\psi) \tag{17}$$

In $\{Y=(\psi'/(\sigma - \sigma\sigma'/E))^3, \psi\}$ coordinate axes, Eq.17 represents a straight line which has a slope of K^3 and intercepts ψ axes at q. If we can obtain this line from experimental results, we can then find the constant K and q, and from which the surface area of the pre-existing cracks, fracture surface energy γ and other material parameters of the concrete.

4 Determination of parameters

The experimental stress-normalized strain curve and variation of Poisson ratio with normalized strain are taken from the previous studies (Oktar 1977 and Taşdemir 1982), and given in Figs.3 and 4, respectively.

The continuous damage function ψ given in the Eq.17 can not be evaluated experimentally as it is expressed in Eq.13, but its change is in the same direction with the variation of Poisson's ratio, which can be evaluated experimentally. However, the right hand side of Eq.13 can be taken as the ratio of $(v_m - v)/(v_m - v_o)$. Thus, we obtain

$$\frac{v_m - v}{v_m - v_o} = \frac{\mathrm{v}}{\mathrm{v}_o} = \psi \tag{18}$$

where v_m and v_o are Poisson's ratios at D=1 and D=0, respectively. In this case the relation between ψ and ϵ/ϵ_m can be easily obtained from the experimental results given in Fig.4.

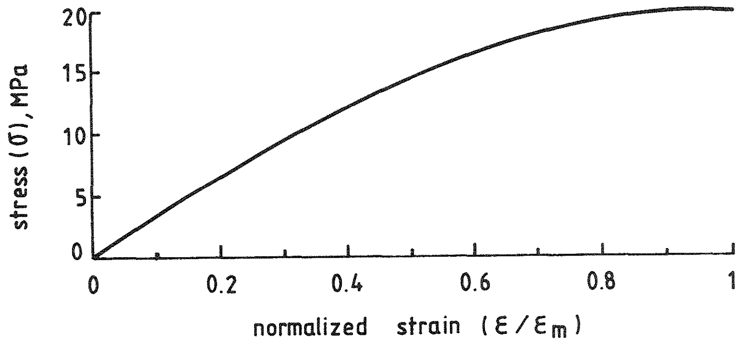

Fig.3. Stress versus normalized strain curve

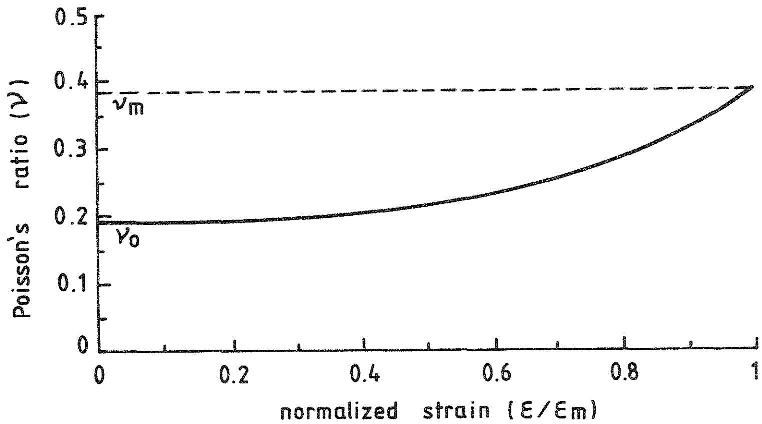

Fig.4. Poisson's ratio versus normalized strain curve

Experimental data given in Figs.3 and 5 are also used in Fig.6 for both left and right sides of Eq.17. As seen in Fig.6, there is very good linear relation between Y and ψ, and the correlation coefficient is 0.99.

The slope of the straight line (K^3) given in Fig.6 is 1445.2×10^3. At the ψ intercept, q is 1.036. Using these values in the equation $K=3/2(q-1)^{2/3}/\beta$, β can be calculated as 1.47 x 10^{-3}. Considering the equation of $q=V/v_o$, v_o can be found as $5.1\times10^6 mm^3$. Thus, the volume of crack neighbourhood in the unit volume $H_o=(V-v_o)/V$ will be 0.036 mm^3/mm^3. Let n be the number of cracks in the unit volume of concrete, then the average value of the close neighbourhood of the cracks can be

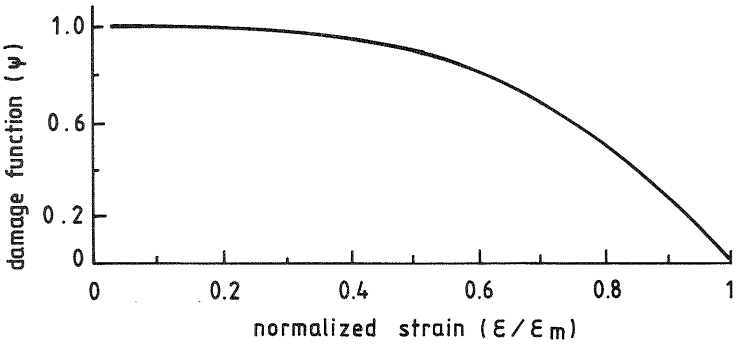

Fig.5. The variation of damage function with respect to $\varepsilon/\varepsilon_m$

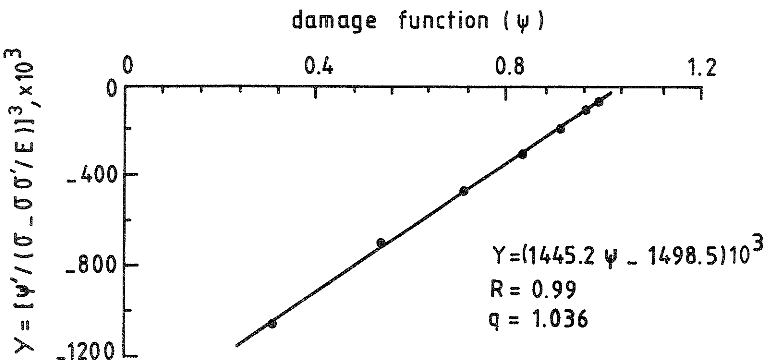

Fig.6. Y versus damage function relationship evaluated from the test results given by Taşdemir (1982)

expressed as $nh_o = H_o = 0.036\ mm^3/mm^3$. For a certain crack, if we assume the average form of the cracks as a penny shaped and consequently their neighbourhoods as a sphere, then we obtain $h_o = (4/3)\pi r^3$, where r is the radius of the sphere. Using Eq.6, $\alpha_o/2$ becomes πr^2, as a result $\alpha_o/2 = (h_o/0.75)^{2/3}$. On the other hand, it is possible to write the following $(H_o/0.75)^{2/3} = n^{2/3}(h_o/0.75)^{2/3} = n^{2/3}.h_o/2$. The total crack area in the unit volume is α_o, then $a_o = n\alpha_o$. Hence, we obtain $a_o = n\alpha_o = 2.n^{1/3}.(H_o/0.75)^{2/3}$ Using this equation, for $n=1$, $a_o = 0.0264\ mm^2/mm^3$ can be obtained. Since β is described as $\beta = a_o\gamma$, using corresponding values given above we then find $\gamma = 55.7 N/m$. It is also possible to find the other values of a_o and γ as given in Table 1. For lightweight concrete, according to the experimental data reported by Taşdemir 1982, as shown in Table 1, K^3 and q take the values of 3585×10^3 and 1.057, respectively. As calculated above, the values obtained are shown in the same table. The evaluated values using the experimental data obtained by Oktar (1977) are also given in Table 1. It is seen that the fracture surface energies decrease significantly for low number of initial crack surfaces, however, the decrease is slight for high numbers.

Table 1 Fracture surface energies calculated in this work

Source of data	K^3 $\times10^3$	q	for n=1		for n=2	
			a_o $\frac{mm^2}{mm^3}$	γ $\frac{N}{m}$	a_o $\frac{mm^2}{mm^3}$	γ $\frac{N}{m}$
Normal concrete, Taşdemir (1982)	1445.2	1.036	0.0264	55.7	0.0333	44.0
Lightweight concrete, Taşdemir (1982)	3584.8	1.057	0.0347	41.8	0.0437	33.2
Normal concrete, Oktar (1977)	2187.9	1.049	0.0316	49.3	0.0400	38.7
	1849.0	1.046	0.0300	51.5	0.0375	41.1

The results obtained in this work were compared with the experimental data available in concrete literature. As seen in Table 1, the fracture surface energies calculated in this work can be comparable with those of others (Swamy 1983).

5. Conclusion

In this paper, a mathematical model based on damage mechanics is presented and a combined theoretical-experimental approach is developed to determine some parameters of concrete. Uniaxial compression tests are used to asses the fracture surface energies of both normal and lightweight concretes. It can be concluded that obtaining Poisson's ratio as a damage parameter, comparable fracture surface energies with available experimental ones in the literature were found.

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