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# FAILURE ANALYSIS OF PRE-DAMAGED CONCRETE STRUCTURAL COMPONENTS

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## Abstract
Instead of using of fracture mechanics, which is very often employed in the analysis of cracked structural components, we present here another technique based on the equivalence between a crack, its process zone and a distribution of damage. This equivalent damage zone is an approximation of the distribution of damage around the existing crack, and it is based on the analysis of localisation with a non local damage model. The implementation of this distribution in a finite element code provides the description of the initial field of damage corresponding to the crack. After the validation of this method on a wedge splitting test, numerical calculations are presented which show a good prediction of the maximum residual carrying capacity of three point bending pre-damaged beams, made of fibre reinforced concrete.

## 1 Introduction

The evaluation of the residual carrying capacity of structural components is a problem of growing importance for civil engineering structures, which are, in particular, subjected to different external attacks leading to damage and cracks. We assume here that the cartography of these existing cracks in the structure is the input data and concentrate on the numerical evaluation of the carrying capacity within the finite element method.

Crack propagation in fracture mechanics (see e.g. Elices and Planas 1989) requires the implementation of special finite elements with adequate shape functions that represent the singular stress field at the crack tip (Ingraffea 1990, Reich et al. 1994). This is needed in order to predict correctly crack bifurcation and propagation conditions. Constitutive modelling, in the continuum sense, is a second possibility. The behaviour of the material is described with a constitutive relation aimed at representing the influence of micro cracking on the stress-strain response of the material. The goal of these continuum models of fracture is to combine the inception of cracking and the crack propagation into a single model. For example, smeared cracks models (Rots, 1988), microplane models (Bazant and Ozbolt, 1990) or continuous damage models (Mazars and Pijaudier-Cabot, 1989) can be used. Usually, these models exhibit strain softening and it is necessary to enrich the continuum description with an internal length in order to obtain physically meaningful results, and in particular fracture with dissipation of energy. Compared to the fracture mechanics approach, continuum modelling allows the description and the propagation of the crack without any modifications of the finite element mesh, and without any additional criterion such as crack orientation conditions. It is done, a priori, with a greater complexity because of the distribution of damage equivalent to an existing crack which is difficult to determine. An analytical model is developed for this purpose hereafter.

## 2 Damage model

In this analysis we use the isotropic scalar non local damage model (see Saouridis and Mazars, 1992). The constitutive relations are:

$$\sigma_{ij} = (1 - D)C_{ijkl}\varepsilon_{kl} \tag{1}$$

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the components of the stress and strain tensors respectively, $C_{ijkl}$ are the initial stiffness moduli, and $D$ is the damage coefficient. The material is initially isotropic, with its Young's modulus (E) and Poisson's ratio ($\nu$). Damage is a function of the non local effective strain defined as:

$$\tilde{\varepsilon} = \sqrt{\sum_{i=1}^{3}\left(\langle \varepsilon_i \rangle_+\right)^2} \tag{2}$$

and

$$\bar{\varepsilon}(x)=\frac{1}{V_r(x)}\int_V \psi(x-s)\tilde{\varepsilon}(s)ds \text{ with } V_r(x)=\int_V \psi(x-s)ds \qquad (3)$$

where $\langle . \rangle_+$ is the Macauley bracket and $\varepsilon_i$ are the principal strains. V is the volume of the structure, $V_r(x)$ is the representative volume at point x, and $\psi(x-s)$ is the weight function:

$$\psi(x-s)=\exp\left(\frac{\|x-s\|^2}{2l_c^2}\right) \qquad (4)$$

$l_c$ is the internal length of the non local continuum. The evolution of damage is specified according to following conditions:

$$\begin{aligned}&F(\bar{\varepsilon})=\bar{\varepsilon}-\kappa\\&\text{and}\\&\text{if } F(\bar{\varepsilon})=0 \text{ and } \dot{F}(\bar{\varepsilon})=0 \text{ then } D=f(\bar{\varepsilon})\\&\text{if } F(\bar{\varepsilon})<0 \text{ or if } F(\bar{\varepsilon})=0 \text{ and } \dot{F}(\bar{\varepsilon})<0 \text{ then } \dot{D}=0\end{aligned} \qquad (5)$$

The damage variable D results from a combination of two types of damage: $D_t$ for tension and $D_c$ for compression:

$$D=\alpha_t D_t+\alpha_c D_c \qquad (6)$$

with:

$$D_i=1-\frac{\kappa_0(1-A_i)}{\bar{\varepsilon}}-\frac{A_i}{\exp(B_i(\bar{\varepsilon}-\kappa_0))}\ , \ i=c,t \qquad (7)$$

where the constants $A_c$, $B_c$, $A_t$, $B_t$ are material parameters and

$$\alpha_t=\sum_{i=1}^{3}\left(\frac{\varepsilon_{ti}\langle\varepsilon_i\rangle_+}{\tilde{\varepsilon}^2}\right)^{\beta},\quad \alpha_c=\sum_{i=1}^{3}\left(\frac{\varepsilon_{ci}\langle\varepsilon_i\rangle_+}{\tilde{\varepsilon}^2}\right)^{\beta} \qquad (8)$$

$$\langle\varepsilon_i\rangle_+=\varepsilon_{ti}+\varepsilon_{ci} \qquad (9)$$

$\varepsilon_{ti}$ is the positive strain due to positive stresses and $\varepsilon_{ci}$ is the positive strain due to negative stresses (Poisson's effect). The purpose of exponent β is to reduce the effect of damage on the response of the material under shear compared to tension.

## 3 Distribution of damage around an existing crack

The existing crack, created in the structure by unknown loads, is to be replaced by an equivalent damage zone (Fig. 1).

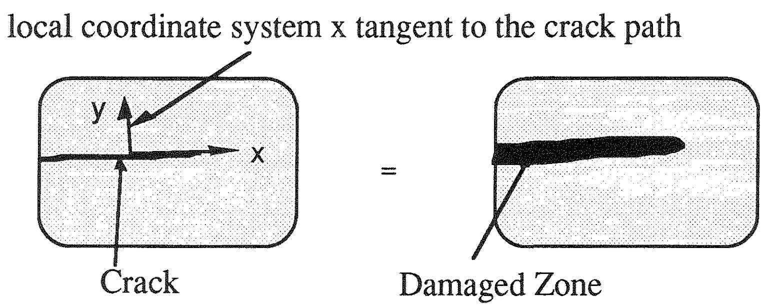


Figure 1: Distribution of damage equivalent to a crack.

It is difficult to derive an exact solution to this problem, because of the unknown loading history: the actual state of stress is not necessarily the one that has caused damage in the body. The problem is restricted here to the distribution of damage corresponding to a crack, assumed to have been created under mode I conditions. We use an approximation which is entirely analytical, and based on a bifurcation analysis (Pijaudier-Cabot and Benallal, 1993). The rate constitutive relations are:

$$\dot{\sigma}(x) = (1-D_0)C:\dot{\varepsilon} - \frac{C:\varepsilon_0}{v_r(x)}\frac{\partial f}{\partial \bar{\varepsilon}}\int_v \psi(s-x)\dot{\bar{\varepsilon}}(s)ds \qquad (10)$$

where $(\varepsilon_0, D_0)$ is the initial, homogeneous, state of deformation and damage about which the rate constitutive relations are expressed. After substitution of these relations into the equations of equilibrium, the analysis of localisation and bifurcation in a non local continuum has shown that the solution of this problem is harmonic, and its expression in the local coordinate system of the crack is:

$$\dot{u}(y) = A\exp(-i\omega\, y) \qquad (11)$$

where ω is the angular frequency of the solution. This expression does not depend on the local coordinate x, which means that the damage zone is assumed to be infinite and that the solution is purely one dimensional.

This angular frequency is given by the equilibrium condition which involves the Fourier transform of the weight function $\overline{\psi}(\omega)$:

$$\frac{(1-D_0)}{\varepsilon_{0yy}\dfrac{\partial f}{\partial \overline{\varepsilon}_0}} = \overline{\psi}(\omega, l_c) \qquad (12)$$

The angular frequency is also a function of the initial state of strain and damage. The principle of this approximation is to let the initial strain tend to infinity and to retain the angular frequency which corresponds to this case only. If the angular frequency tends to infinity, a threshold is set corresponding to the definition of a maximum strain at which failure is considered to occur. According to this assumption the maximum angular frequency corresponding to the minimum possible value of the width of the damage zone along the axis y of the local coordinate system is selected. The distribution of damage is obtained by substituting the harmonic solution in the equations of evolution (5-9), integrating the rate of damage, and rescaling the integral so that damage is equal to one along the crack path. We obtain in the coordinate system of the crack (Fig. 1):

$$D(y) = \frac{\int_{-\infty}^{+\infty} \psi(y-s)\eta(s)ds}{\int_{-\infty}^{+\infty} \psi(s)\eta(s)ds} \qquad (13)$$

$$\text{with} \begin{cases} \eta(y) = \cos(\omega_{\max} y) \text{ if } y \in \left[\dfrac{-\pi}{2\omega_{\max}}, \dfrac{\pi}{2\omega_{\max}}\right] \ \textit{and}\ D(y) \geq 0 \\ \eta(y) = 0 \text{ elswhere} \end{cases}$$

At the crack tip, the distribution is circular and variable y is replaced by the radius r defining the distance from the considered point to the tip of the crack. The accuracy of this approximation has been checked in the past by computing the fracture energy resulting from this distribution of damage and comparing to the fracture energy derived from size effect tests (Mazars et al. 1994).

## 4 Implementation and validation of the method

From a computational point of view, an existing crack in the structure is modelled as a set of broken lines. Once this set has been given, a preprocessor computes directly the initial distribution of damage at each gauss point using equation (13) along with the new threshold of damage. According to this method, damage is not equal to one at a gauss point, except if this point is located exactly on the crack line. Therefore, an element crossed by the crack has still a residual strength which seems to be non realistic. In order to circumvent this problem, we constrain damage to be equal to one in all the elements crossed by the crack line. This constraint disregard locally the analytical distribution of damage, and allows more accurate predictions.

### 4.1 Wedge splitting test

This first test is purely numerical. The structure is a plane stress model of a wedge splitting specimen made of plain concrete, where the crack mouth opening displacement is controlled (Fig. 2). The material parameters and the wave length of the damage zone are:

$$\kappa_0 = 1\ 10^{-4},\ E = 32000\ \text{MPa},\ \nu = 0.2,\ A_t = 1.,\ B_t = 1000,\ A_c = 1.4,$$

$$B_c = 1500,\ \beta = 1.,\ l_c\ =\ 1\text{cm},\ \frac{2\pi}{\omega_{max}} = 3.8 l_c$$

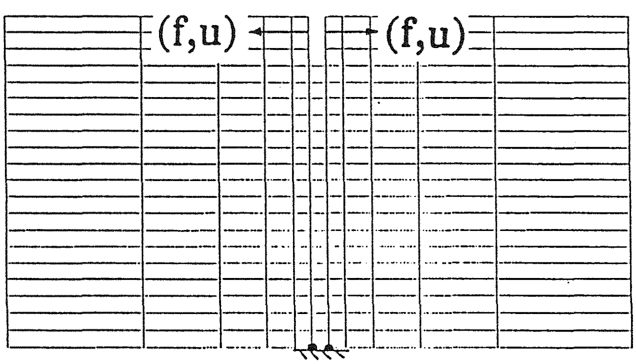


Figure 2: Wedge splitting test : mesh and boundary conditions.

We compute first the load-notch opening displacement curve starting from an initial state of zero damage. Crack propagation follows a straight line. In the course of calculation, several equivalent crack lengths corresponding to different openings of the notch are selected. In order to back calculate the crack length, the material is assumed to be completely cracked when D>0.999.

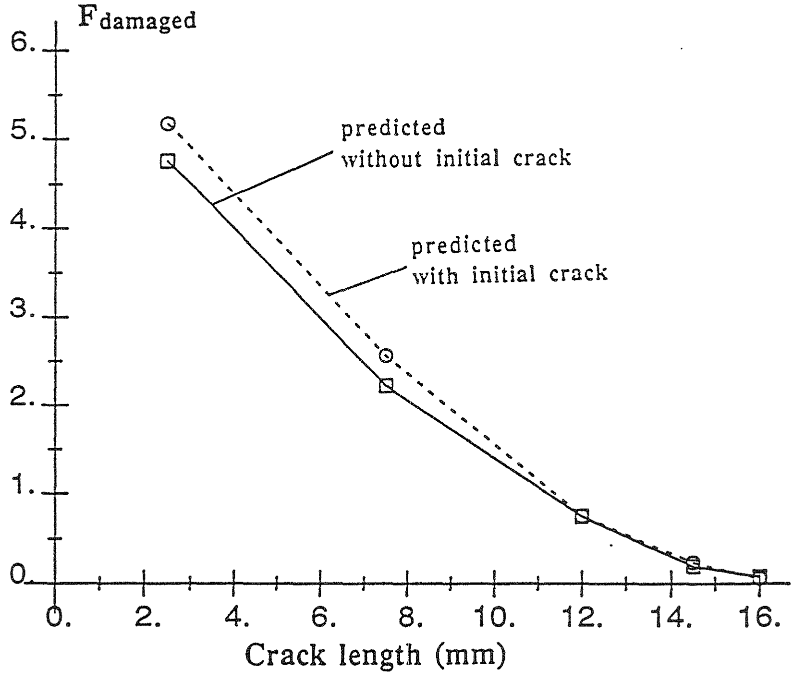


Figure 3: Wedge splitting test : residual load vs. crack length.

Once this first calculation has been performed and for each selected crack length, we use the distribution of damage according to our method, and start a new computation up to failure. Figure 3 shows the plot of the residual strength versus the crack length. The agreement between the computation starting from the uncracked plate and all the computations which started from a cracked plate with different crack lengths is quite good for this example which is purely numerical.

## 4.2 Three point bending of a fibre reinforced concrete (FRC) beam

The greater difficulty of this problem is to find a good equivalence between the existing crack and the damage zone: when damage is locally equal to one at a material point of the specimen, the stress carrying capacity of the material is still non zero because the fibres are bridging the two crack faces. An intermediate model can be devised in which the contribution of the concrete and the fibres are clearly distinguished: the strains in concrete and in the fibres are the same and the total stress applied on the composite material is expressed as:

$$\sigma = (1 - c)\sigma_c + c\sigma_f \tag{14}$$

where $\sigma_c$ is the stress carried by the concrete, $\sigma_f$ the stress carried by the fibres, and c is the mechanical concentration of fibres. This coefficient is related of the proportion of fibres in the concrete and must be fitted from the response of the FRC (La Borderie, 1991). The FRC we have used, is made of a standard concrete (30 MPa in compression) and Dramix hooked fibres 08/60. The percentage of fibres is 1.2% of the weight of cement. The different material constants have been fitted from uniaxial tests:

Concrete

$$\kappa_0 = 0.7\ 10^{-4},\quad E = 20000 \text{ MPa},\ \nu = 0.2,\ A_c = 1.2,\ B_c = 2381,$$
$$A_t = 0.95,\ B_t = 10000,\ l_c = 60mm$$

Fibres and interface

$$E = 210000 \text{ MPa},\ \sigma_{max} = 85 \text{ MPa, softening modulus} = \text{-200 MPa}$$

The concentration coefficient c is equal to 2% which is higher than the percentage of fibres in FRC.

Bending tests where the load and support locations are shifted after the onset of an initial crack (Fig. 4) were performed in order to determine the residual strength of these cracked beams (La Borderie et al., 1994) and check the present model.

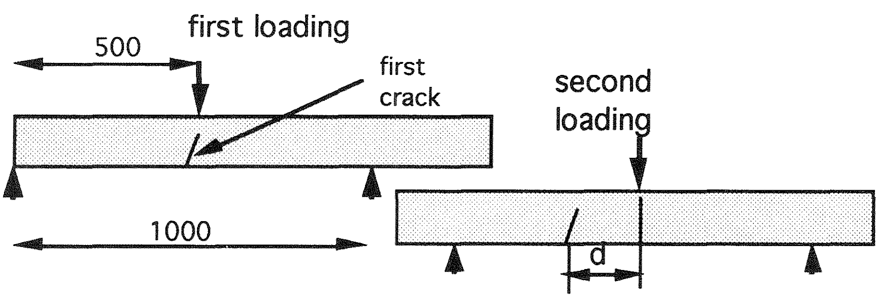


Figure 4: Test on pre-damaged specimens.

For the sake of simplicity in the finite element implementation, the contributions of fibres and of concrete is separated at the element level. Bar elements connecting each couple of adjacent nodes represent the contribution of the fibres. The area of the bar elements is computed in order to respect exactly the value of the mechanical concentration of fibres. Figure 5 shows the experiment versus the numerical prediction corresponding to a shift of d=150 mm and an initial crack length of 150 mm (3/4 of the beam depth). The numerical results give a good prediction of the maximum carrying capacity. The error is of the order of 20 percent. But, note that its impossible to obtain the experimental stiffness of the pre-damaged beams, because we have note represented, in the numerical model, the distribution of the plastic strain in the fibres within the damage zone.

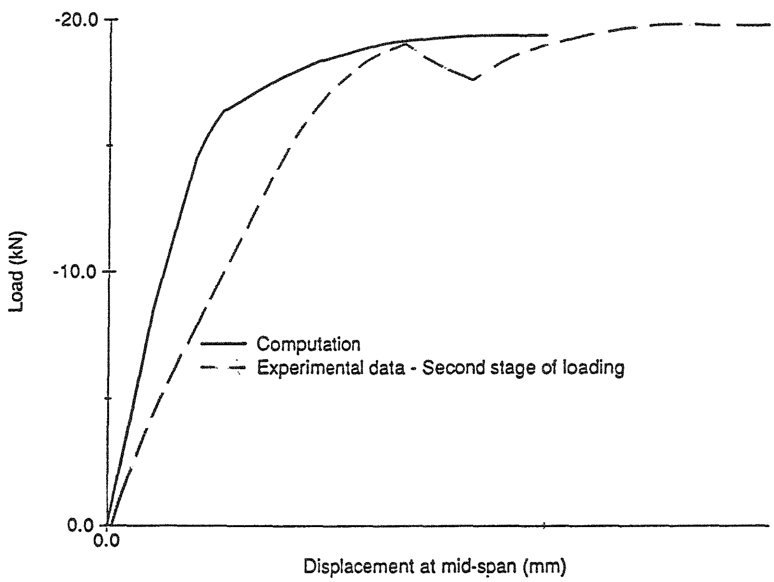


Figure 5 :Test on pre-damaged specimens: load vs. displacement under the load.

## 5 Closure

The prediction of the response of structural components can be performed using continuum-based models. The equivalence between the crack, its process zone and the distribution of damage, is based on the analysis of strain and damage localisation. The distribution of damage equivalent to a mode I crack is used in this paper. The implementation of this method leads to a good agreement for plain concrete structures. The method has been extended to fibre reinforced concrete.

The method proposed here is restricted to existing cracks created under mode I conditions, which seems to be a fairly wide range of applications. In future studies, it should be necessary to check the method for mixed mode crack propagation or on cases were crack branching is observed.

## Acknowledgements

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## References

Bazant, Z.P. and Ozbolt, J. (1990) Nonlocal microplane model for fracture, damage and size effect in structures. **J. of Engrg. Mech. ASCE**, 116, 2485-2505.

Elices, M. and Planas, J. (1989) Materials models, Chapt. 3 in **Fracture mechanics of concrete - from theory to application**. (L. Elfgren Ed.), Chapman and Hall Pubs., 16-66.

Ingraffea, A.R. (1990) Case studies of simulation of fracture in concrete dams. **Engineering Fracture Mech.**, 35, 553-564.

La Borderie, C. (1991) **Phénomènes unilatéraux dans un matériau endommageable**. Thèse de Doctorat de l'université Paris VI, Paris France.

La Borderie, C., Pijaudier-Cabot, G. and Tailhan, J.L. (1994) Verification of the residual strength of pre-damaged structures. Task report 7, Brite-Euram Project P. 3275, Laboratoire de Mécanique et Technologie, Cachan, France.

Mazars, J., Perdikaris P. and Pijaudier-Cabot G. (1994) Size effect predictions with damage modelling. **Size effect in concrete structures**, (eds. H. Mihashi et al.), E&FN Spon Pubs.,. 207-220.

Mazars, J. and Pijaudier-Cabot, G. (1989) Continuum damage theory : application to concrete. **J. of Engrg Mechanics, ASCE**, 115, 345-365.

Pijaudier-Cabot, G. and Benallal, A. (1993) Strain localisation and bifurcation in nonlocal continuum. **Int. J. Solids Struct.**, 30, 1761-1775.

Reich, R.W, Cervenka, J., and Saouma, V.E. (1994) MERLIN: a computational environment for 2D/3D discrete fracture analysis. **Computational modelling of concrete structures** (eds. H. Mang et al.), Pineridge press Pubs., 2, 999-1008.

Rots, J.G. (1988) **Computational modelling of concrete fracture**. Dissertation, Delft University of Technology, Delft, The Neherlands.

Saouridis, C. and Mazars, J. (1992) Prediction of the failure and size effect in concrete via a biscale approach. **Engrg. Comput.**, 9, 329-344.