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A CONTINUUM THERMODYNAMICS APPROACH FOR STUDYING MICROSTRUCTURAL EFFECTS ON THE NON-LINEAR FRACTURE BEHAVIOUR OF CONCRETE SEEN AS A MULTICRACKED GRANULAR COMPOSITE MATERIAL

C. Huet
Department of Materials, Laboratory for Building Materials, Swiss Federal Institute of Technology, Lausanne, Switzerland

Abstract
Further developments of a general dissipative thermodynamics formalism recently elaborated by the author for the study of the non-linear fracture mechanics of concrete are presented. Viscoelasticity and kinetic energy effects are considered, with application to computation and test examples.

1 Introduction

This paper complements a general theoretical formalism, recently elaborated by the author, Huet (1994), (1995). This formalism makes no prejudicial assumptions about the behaviour of the constituents of the material nor its microstructure, except the existence of a distribution of pre-existing microcracks of the kind that can be seen in concrete at various microstructural levels using appropriate microscopy techniques, see for instance Sunderland et al. (1993), (1995). Viscoelastic and dynamic effects are more specially considered, including upper and lower bounds for rapid crack extension. Applications to the interpretation of micromechanical simulations, acoustic emission measurements and dynamic crack propagation to unloaded regions are then provided as illustrating examples. Because of the limited space, we simply refer -

among many others - to the books edited by Elfgren and Shah (1991), Bazant (1992), Wittmann (1993), Baker and Karihaloo (1995) for some of the most recent accounts about fracture mechanics of concrete. We refer also to the works of Griffith (1920), Orowan, Irwin, Berry, Yoffé, Rice, N'Guyen and others including the book series by Liebowitz for the fundamentals of classical fracture mechanics and fracture thermodynamics as quoted in Huet (1971a), (1971b), (1973), (1974), (1982), (1994), (1995) and also more recent advances like those given in Kanninen and Popelar (1985).

2 Summary of previous results

We showed in Huet (1994), (1995) that, when appropriately handled through the consideration of the first and second principles, dissipative continuum thermodynamics - which makes use of the entropy balance equation and the Clausius-Duhem inequality together with the conservation equations for mass, momentum and energy - provides efficient tools and realistic predictions even for complicated multicracked heterogeneous and dissipative materials like concrete. It also turned out that the generalized criteria thus obtained, with possible applications not limited to the linear elastic case, are nevertheless very similar to the classical criterion obtained by Griffith (1920) in his pioneering work. Then it was shown that, as previously stated in Huet (1994), (1995), this extended Griffith theory captures the most important features of fracture phenomena in concrete without stating that the material constituting the body is homogeneous and/or purely elastic or more specifically, linearly elastic. For most existing models of dissipative mechanisms, with the notable exception of dry friction, it was also shown to be possible to express the dissipation in a form involving a global Rayleigh dissipative function, with the crack tips velocities as the argument. In the general case, the latter provides relationships that are nonlinear extensions of Onsager symmetry relationships and involve a normality rule. In particular, dissipative and possibly nonlinear mechanisms exhibited by viscoelasticity, climatic changes, chemical aging and the granular composite nature of concrete were shown to be automatically incorporated in the obtained formalism through the use of appropriate internal variables. For this, it is not required for the crack resistances to be independent of the crack lengths nor their rate and/or their previous histories. Thus, the corresponding formalism accommodates delayed fracture even when the material itself can be considered as purely elastic in the bulk. In fact, since they are dissipative effects and not reversible surface energy, they should be functions of the rates, possibly through a singular relationship providing a threshold, just as for plasticity. It was also pointed out that no assumption is needed - nor contradiction involved - about the possible existence of singularities at crack tips, since - starting from external loading relationships - the associated forces or energies are assigned to remain finite on finite surfaces or volume elements. In fact as well known, the singularity obtained in the small elastic deformation case disappears when the more realistic finite strain theory is used. The starting point for these results - and the reason for which they could apply to every complexity of microstructure and behaviour - was the universal balance equations of thermomechanics in their most primitive global form. Using the notations of Huet (1994), (1995), the main results of the obtained formalism summarize as follows:

- Current crack tip a^k, reference crack tip a^1 and associated change of variables :

$$a^k = x^k \cdot n^k \;;\; k = 1, N \quad ; \quad a^k = a^1 + \alpha^k = \underset{\sim}{a}^k(a^1, \alpha^k) \;;\; k = 2 \text{ to } N \tag{1}$$

- Explicit global free energy Φ , global potential energy Ψ, global dissipation D and Rayleigh dissipative function $\Re$:

$$\begin{aligned}\Psi &= \Phi - \int_{D_0} F^d \cdot \xi \, dV - \int_{\partial D_{0\sigma}} P^d \cdot \xi \, d\Sigma \\ &= \underset{\approx}{\Psi}\left[\wedge^{di}(t), T_0(t),\ a^k(t)\,;\ \dot{\wedge}^{di}(t), \dot{T}_0(t), \dot{a}^k(t)\,; H^-(q)\right] \\ &= \underset{\approx}{\Psi}^*\left[\wedge^{di}(t), T_0(t),\ a^1(t),\ \alpha^k(t)\,;\ \dot{\wedge}^{di}(t), \dot{T}_0(t), \dot{a}^1(t), \dot{\alpha}^k(t)\,; H^-(q)\right]\end{aligned} \tag{2}$$

$$D = \underset{\sim}{D}\left[q, \dot{q}\,; H^-(q)\right] = \frac{\partial \underset{\sim}{\Re}}{\partial \dot{q}} \cdot \dot{q} + \underset{\approx}{D}\left[H^-(q)\right] = R^k_F . \dot{a}^k + \underset{\sim}{D}_1 \tag{3}$$

$$\begin{aligned}\Re &= \underset{\sim}{\Re}\left[\ \dot{\Lambda}^d; \dot{a}^1, \dot{a}^2 \ldots, \dot{a}^k, \ldots, \dot{a}^N; H^-(q)\ \right] \\ &= \underset{\sim}{\Re}^*\left[\ \dot{\Lambda}^d; \dot{a}^1, \dot{\alpha}^2 \ldots, \dot{\alpha}^k, \ldots, \dot{\alpha}^N; H^-(q)\ \right]\end{aligned} \tag{4}$$

with q the set of involved variables - including loading parameters $\wedge^{di}(t)$, surrounding temperature $T_0(t)$, crack tip position a^k and other possible internal variables - and $H^-(q)$ its past history ; ξ^d, P^d and F^d denote displacements, densities of surface forces and of volume forces imposed on the external boundaries $\partial D_{0\xi}$, $\partial D_{0\sigma}$ and in the body D_o respectively.

- Global Clausius-Duhem inequality and dissipative identity in initial and reduced forms :

$$D = -(\dot{\Psi} + \int_{D_0} \xi \cdot \dot{F}^d dV + \int_{\partial D_{0\sigma}} \xi \cdot \dot{P}^d \, d\Sigma - \int_{\partial D_{0\xi}} P \cdot \dot{\xi}^d d\Sigma + \dot{C}) \geq 0 \tag{5}$$

$$\underset{\sim}{D} + \dot{\Psi} + X_F \cdot \dot{\Lambda}_F^1 + X_P \cdot \dot{\Lambda}_P^d - X_\xi \cdot \dot{\Lambda}_\xi^d + S\dot{T}_0 + \dot{C} \equiv 0 \tag{6}$$

$$\underset{\approx}{D}^* \left[H^-(q) \right] + \underset{\approx}{\dot{\Psi}}_{\Lambda Ta} + \dot{C} \quad \equiv \quad 0 \tag{7}$$

with $\underset{\approx}{\dot{\Theta}}_q$ the time continuation functional at constant q of the variable Θ.

- First order criteria for onset of real crack or microcrack growth:

$$R_F^k + \frac{\partial \underset{\sim}{\Psi}}{\partial a^k} = 0 \quad \Leftrightarrow \quad G^k \equiv - \frac{\partial \underset{\sim}{\Psi}}{\partial a^k} = R_F^k \equiv \frac{\partial \underset{\sim}{\Re}}{\partial \dot{a}^k} \quad ; \quad k = 1, \ldots, N \tag{8}$$

- First order criteria for onset of main crack and microcrack growth:

$$G^{*1} \equiv - \frac{\partial \underset{\sim}{\Psi}^*}{\partial a^1} = R_F^{*1} \; ; \; G^{*m} \equiv - \frac{\partial \underset{\sim}{\Psi}^*}{\partial \alpha^m} \equiv - \frac{\partial \underset{\sim}{\Psi}}{\partial a^m} = R_F^{*m} \; ; \; m = 2, \ldots, \rho \tag{9}$$

ρ being the number of simultaneously moving crack tips and with :

$$R_F^{*1} = \sum_{k=1}^{N} \frac{\partial \underset{\sim}{\Re}}{\partial \dot{a}^k} = \sum_{k=1}^{N} R_F^k \quad ; \quad R_F^{*m} = \frac{\partial \underset{\sim}{\Re}^*}{\partial \dot{\alpha}^m} = \frac{\partial \underset{\sim}{\Re}}{\partial \dot{a}^m} = R^m \; ; \; m = 2, \ldots, \rho \tag{10}$$

- Arrested and non activated cracks:

$$G^{*m} = - \frac{\partial \underset{\sim}{\Psi}^*}{\partial \alpha^m} = - \frac{\partial \underset{\sim}{\Psi}}{\partial a^m} < R_F^m \quad ; \quad m = \rho + 1, \ldots, N \tag{11}$$

- Second order criteria for initial stability at constant loading :

$$[\; R_F^k(a + \delta a) - G^k(a + \delta a) \;] \, \delta a^l \delta a^k \;=\; [\; \frac{\partial}{\partial a^l}(R_F^k - G^k) \;] \; \delta a^l \delta a^k > 0 \tag{12}$$

$$\Leftrightarrow \quad \frac{\partial G^k}{\partial a^l} = - \frac{\partial^2 \Psi}{\partial a^l \partial a^k} < \frac{\partial R_F^k}{\partial a^l} \tag{13}$$

As can be seen, the above results restitute, for dissipative heterogeneous materials, formulae most often considered valid for linearly elastic homogeneous materials only.

3 Example of computation of the quasi-static energy release rates for multicracked dissipative bodies : the viscoelastic case

It is worth showing through a specific example of dissipative behaviour how the potential energy and its release rate may effectively be calculated for structures involving inelastic materials even in the uncracked state. We choose here the viscoelastic case, since creep and related delayed effects correspond to a major feature of concrete behaviour. In fact, as already emphasized by Bazant (1993), mastering the interactions between viscoelasticity effects and fracture mechanics effects has become an objective of prime importance for concrete material and concrete structures. For viscoelastic materials such as bituminous and cement concretes which exhibit single or multiple power-law creep corresponding to appropriate continuous spectra, the global free energy will be obtained - when assuming linear non-aging viscoelastic behaviour with relaxation tensor function $r(t)$ for the solid part of the old enough material - from the tri-dimensional extensions of the Staverman and Schwarzl (1952) formulae, see Huet (1992), (1993b) as:

$$\Phi(t) = \frac{1}{2}\int_{D_o} \int_{o^-}^{t} \int_{o^-}^{t} r(2t-u_1-u_2) : d\varepsilon\,(u_2) : d\varepsilon\,(u_1)$$

$$\equiv \frac{1}{2}\int_{D_o} \left(\int_{o^-}^{t} - \int_{t}^{2t^+}\right) \sigma\,(2t-u) : d\varepsilon\,(u) \tag{14}$$

The bi-linear form in the above equation corresponds to the Brun-Clapeyron formula, derived by Brun in 1965-1969 and extensively discussed, with new derivation, in Huet (1992). In the pseudo-convolutive notation introduced there and used also in Huet (1993b), this is written, when denoting $\circ$ the Stieltjès convolution and $\square$ our pseudo-convolution :

$$\Phi(t) = \frac{1}{2}\int_{D_o} (r \circ \varepsilon) \,\square\, \varepsilon \, dV = \frac{1}{2}\int_{D_o} \sigma \,\square\, \varepsilon \, dV \tag{15}$$

Using the pseudo-convolutive virtual work theorem, this gives also in the quasi-static case, for negligible volume forces and unloaded crack surfaces:

$$\Phi(t) = \frac{1}{2}\int_{\partial D_{0\sigma}} P^d \,\Box\, \xi \, d\Sigma + \frac{1}{2}\int_{\partial D_{0\xi}} P \,\Box\, \xi^d \, d\Sigma$$
$$= \int_{\partial D_{0\sigma}} P^d . \, \xi \, d\Sigma - \frac{1}{2}\int_{\partial D_{0\sigma}} \xi \,\Box\, P^d \, d\Sigma + \frac{1}{2}\int_{\partial D_{0\xi}} P \,\Box\, \xi^d \, d\Sigma \qquad (16)$$

Thus, from Eqn (2) and for every time value t, Ψ reduces here to :

$$\Psi(t) = \frac{1}{2}\int_{\partial D_{0\xi}} P \,\Box\, \xi^d \, d\Sigma - \frac{1}{2}\int_{\partial D_{0\sigma}} \xi \,\Box\, P^d \, d\Sigma \qquad (17)$$

Since by definition, ξ^d and P^d are independent of any crack growth, we get thus for $G^{k\,stat}(t)$:

$$G^{k\,stat}(t) = -\frac{\partial \Psi}{\partial a^k} = \frac{1}{2}\int_{\partial D_{0\sigma}} \frac{\partial \xi}{\partial a^k} \,\Box\, P^d \, d\Sigma - \frac{1}{2}\int_{\partial D_{0\xi}} \frac{\partial P}{\partial a^k} \,\Box\, \xi^d \, d\Sigma \qquad (18)$$

This latter equation is, in pseudo-convolutive form, the viscoelastic transpose of a well known formula of elastic fracture thermodynamics. Since the pseudo-convolution is non-commutative, attention should be paid, when reading this formula, to the order of the factors. It can be used for evaluating $G^{k\,stat}(t)$, for instance numerically, or from experimental results. On the other hand, since a pseudo-convolutive version of the Maxwell-Betti reciprocal theorem does not exist in the general case, see Huet (1992), this formula does not provide us - at least directly - with a contour independent integral, contrarily to the quasi-static elastic case. As well known, this is also the case for the dynamic elastic case. Due to the linearity of the viscoelasticity problem at constant micro-crack pattern configuration, we obtain from this, for vanishing P^d, meaning $\partial D_{0\sigma}$ being a free surface:

$$G^{k\,stat}(t) = -\frac{1}{2}\int_{\partial D_{0\xi}} \int_{\partial D_{0\xi}} \frac{\partial k}{\partial a^k} \circ \xi^d(y) \,\Box\, \xi^d(x) \, d\Sigma_y \, d\Sigma_x \qquad (19)$$

and for vanishing ξ^d, meaning $\partial D_{0\xi}$ being a fixed boundary:

$$G^{k\,stat}(t) = \frac{1}{2}\int_{\partial D_{0\sigma}} \int_{\partial D_{0\sigma}} \frac{\partial c}{\partial a^k} \circ P^d(y) \,\Box\, P^d(x) \, d\Sigma_y \, d\Sigma_x \qquad (20)$$

In these equations, $k(t;x,y)$ and $c(t;x,y)$ are, at observation point x , time-dependent influence tensors of the stiffness and compliance types associated to the boundary densities ξ^d or P^d imposed at point y

respectively. The more general mixed cases include in addition cross-terms with transfer tensor kernels that should not be forgotten.

The main advantages - by comparison with other viscoelastic calculations performed in the literature, see for instance Popelar and Kanninen (1985) - of the above formulae is that no oversimplifying assumption about the specific viscoelastic behaviour of the material is made for their derivation and that they can be directly used in purely mechanical experiments. From the second principle of thermodynamics it can be shown easily that, for isothermal relaxation and for viscoelastic materials with the Staverman and Schwarzl form of the free energy, the energy release rate $G^{k\,stat}(t)$ at constant crack pattern is decreasing in time for all finite times at imposed constant displacement and increasing in time at imposed constant force. For constant crack tip resistance R^k, this means that delayed crack growth cannot occur during relaxation while it may occur during creep. This might provide a means to experimentally separate the influence of the bulk viscoelasticity of the solid part of the material from the delayed behaviour of the crack tip resistance studied for instance by Bazant (1993), (1994) and that we pointed out in Huet et al. (1982). For small viscoelastic structures or specimens, the time dependent potential energy for a cracked specimen at constant crack tip position can be calculated through the 3 D finite element code developed in our laboratory by Guidoum (1994), Huet et al. (1995) and which takes into account the granular character of the material.

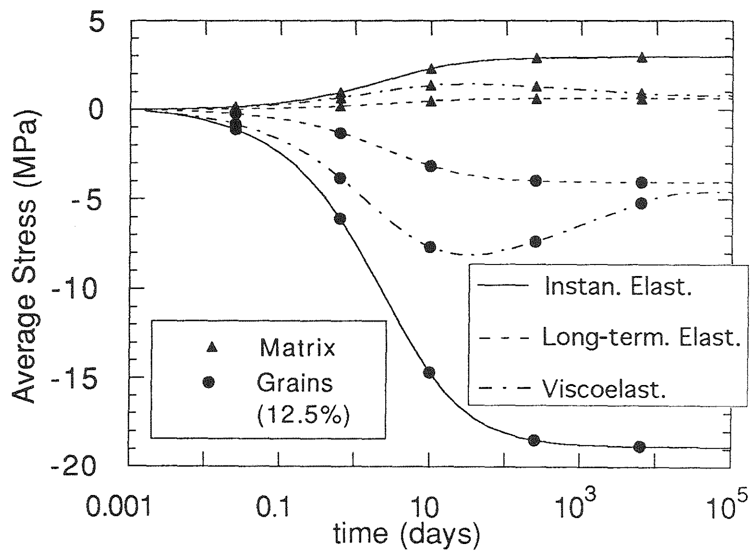


Fig. 1. Tri-dimensional simulation of time evolutions of the viscoelastic shrinkage stress averages in matrix phase (tension) and grain phase (compression), from Guidoum (1994).

For instance, computed time redistributions, between one day and thirty years, of the isotropic part of the average stress tensors in each phase for a given three year time evolution of the imposed free shrinkage have shown - due to viscoelastic relaxation by comparison with the initial elastic case - a high stress reduction and the apparition of a maximum in the first year which are important for microcracking and fracture prognostication, Fig. 1. By contrast, the maximum is not obtained in the elastic calculations. For further details and applications see Guidoum (1994), Huet (1995a), Huet et al. (1995). More generally, the bounding procedure derived in Huet (1995b) can also be used.

4 Dynamic crack propagation, crack arrest and crack restarting

4.1 General relationships

One of the most important questions when dealing with brittle or semi-brittle fracture is to know whether, once initiated, the crack growth will remain controlled by the increase of the loading imposed on the structure, or if unstable - uncontrolled and thus catastrophic - crack propagation that might yield a sudden breakdown of the structure without previous warning will occur. At least, as we shall see later on in this paper, it may yield breaking or damage in regions far beyond that which could be judged in danger simply on the basis of the usual quasistatic computations. Then, the most important information is to know how far the crack tip will run during its rapid propagation. On the other hand, since the loading conditions imposed in situ on a structure have features and magnitudes that are random to a large extent and since the macroscopic properties of the material are also random, the exact magnitude of the strength and shape of the load-deflection curve is less important. In the form given by Eq 6, the dissipative identity involves the rate of change of the kinetic energy C , the latter being a time-dependent body functional given by :

$$C = \frac{1}{2}\int_{D_0} \rho \dot{\xi} \cdot \dot{\xi} \, dV \qquad (21)$$

Although the total kinetic energy C is always non-negative, the sign of $\dot{C}$ cannot be pre-assigned in general. But if a single crack starts to grow in a body initially at rest, $\dot{C}$ is necessarily positive or - in the limit case of the controlled crack growth - zero up to the second order. This corresponds to the quasi-static conditions generally considered in the structural computations. It will decrease - at least after a time - if a running crack stops. When several cracks are in the process of starting, running and stopping independently or randomly, the sign of $\dot{C}$ is undetermined. But, at the onset of the first crack propagation, C - as a function of $\dot{\xi}^2$ - is a stationary function of $\dot{\xi}$ making $\dot{C}$ negligible. Also, after stopping of the cracks and return to equilibrium or slow evolution due to inelastic response, C and $\dot{C}$ are zero or can be neglected. On the other hand, the values of C and $\dot{C}$ can no more be omitted or neglected when dealing with problems in which the cracks are in the process of dynamic propagation. In terms of the parameters of the problem, C should be expressed in the most general case - just as Ψ^{dyn} in Eqn (2) of Section 2 - as a function of the loading parameters and the crack tips positions, their rates at time t and their previous histories:

$$C(t) \;=\; \underset{\approx}{C}\left[\wedge^{di}(t), T_0(t),\; a^k(t)\,;\; \dot{\wedge}^{di}(t)\,, \dot{T}_0(t), \dot{a}^k(t)\,; H^{-}(q)\right] \tag{22}$$

When taking the time derivatives of Ψ^{dyn} and C , the effect of the moving crack boundary should not be omitted. This introduces terms proportional to $\dot{a}^k$. At constant external temperature and constant loading conditions (F_0^d, P_0^d, ξ_0^d) , $\dot{C}$ can thus be expressed as:

$$\dot{C}(t) \;=\; \frac{\partial \tilde{C}}{\partial a^k}\dot{a}^k + \frac{\partial \tilde{C}}{\partial \dot{a}_k}\,\ddot{a}_k + \underset{\approx}{\dot{C}}_{\Lambda Ta} \tag{23}$$

The dissipative identity thus becomes, for every evolution:

$$\begin{aligned}(R_F^k + \frac{\partial \tilde{C}}{\partial a^k} + \frac{\partial \tilde{\Psi}}{\partial a^k})\dot{a}^k + (\frac{\partial \tilde{C}}{\partial \dot{a}^k} + \frac{\partial \tilde{\Psi}}{\partial \dot{a}_k})\,\ddot{a}_k \\ + \underset{\approx}{D}^*\left[H^{-}(q)\right] + \underset{\approx}{\dot{\Psi}}_{\Lambda Ta} + \underset{\approx}{\dot{C}}_{\Lambda Ta} \;\equiv\; 0\end{aligned} \tag{24}$$

giving separately:

$$\begin{aligned}&(R_F^k + \frac{\partial \tilde{C}}{\partial a^k} + \frac{\partial \tilde{\Psi}}{\partial a^k}) = 0 \;\;;\;\; \frac{\partial \tilde{\Psi}}{\partial \dot{a}_k} = -\frac{\partial \tilde{C}}{\partial \dot{a}^k} \neq 0 \;\; in\ general \;\;; \\ &\underset{\approx}{D}^*\left[H^{-}(q)\right] + \underset{\approx}{\dot{\Psi}}_{\Lambda Ta} + \underset{\approx}{\dot{C}}_{\Lambda Ta} \;\equiv\; 0\end{aligned} \tag{25}$$

where the summation convention on the superscript k is used again. Denoting $G_\Psi^{k\,dyn}$ the dynamic potential energy release rate for the k-th crack tip, this provides the rapid crack growth criterion in the form :

$$G_\Psi^{k\,dyn}(t) \equiv -\frac{\partial \tilde{\Psi}}{\partial a^k} = R_F^k + \frac{\partial \tilde{C}}{\partial a^k} \equiv R_F^k + R_C^k \tag{26}$$

Another - although equivalent - presentation of the criterion may make use of the total energy release rate $G_E^{k\,dyn}$ associated to the total energy E :

$$E = \Psi + C \quad \Rightarrow \quad G_E^{k\,dyn} \equiv - \frac{\partial E}{\partial \tilde{a}^k} \equiv - \frac{\partial \Psi}{\partial \tilde{a}^k} - \frac{\partial C}{\partial \tilde{a}^k} = R_F^k \tag{27}$$

that should not be confused with the previous one. The advantage of $G_E^{k\,dyn}$ is that it can be directly compared to appropriate assumptions made about the - possibly crack speed-dependent - crack tip resistance force R_F^k with validity of this assumption checked by use of an inverse method if the change in the kinetic energy can be measured together with the potential energy one.

4.2 Practical consequences

It is not possible to state a general order relationship between the dynamic potential energy release rates $G_\Psi^{k\,dyn}$ and the static one $G^{k\,stat}$. This occurs despite the fact that - as provided by the classical minimum theorem of elasticity - the dynamic potential energy Ψ^{dyn} is always, in that case of an elastic matrix that can be an instantaneous one, larger than the static potential energy Ψ^{stat} for the same configuration and lengths of the microcracks and the same loading conditions. But $G_\Psi^{k\,dyn}$ can be larger than $R^k(t)$ by periods due to quasi-periodic exchange of potential and kinetic energy in the vibration field due to the crack propagation. The same holds for $G_E^{k\,dyn}$, especially when $\dot{C}$ is negative. Nevertheless, since quasi-static calculations are much simpler, it is worthwhile to examine what can be gained from the knowledge of $G^{k\,stat}$. On the other hand, the possible dependence of $R^k(t)$ on the crack growth, its rate and its history should not be forgotten. Because of the lack of space, all possible cases cannot be studied here. In some instances or periods, $\dot{C}$ is negative and the crack tip may stop ; but for $G^{k\,stat}$ larger than R^k, it will restart just after stopping since the value of the former is recovered once C has been reduced to zero ; this is a case corresponding to the well known slip-stick phenomena encountered when pealing an adhesive tape. Because of these dynamic effects, the crack tip may also propagate in zones for which $G^{k\,stat} < R^k$. In these regions, $\dot{C}$ is negative and the crack tip will eventually stop. After stopping of the crack, C will decrease till zero at constant crack length due to the other causes of dissipation, that provide damping. Since the static criterion is no more fulfilled at constant loading, it will not restart after stopping unless the loading is again increased, the needed increase being generally of a finite value. But it may happen that the ligament is completely broken before stopping of the main crack.

4.3 Lower and upper bounds for rapid crack extension

From the above results, it can be easily shown that - when appropriately interpreted in the light of the dissipative dynamic theory - the knowledge of $G^{k\,stat}(a^k)$ provides lower and upper bounds for the extension of crack propagation - or distance of arrest of the k th crack tip. In fact, for the cases - more frequently obtained under constant displacement - where $G^{k\,stat}$ is first increasing with the crack length and then is decreasing, the considered crack will at least propagate up to the length given by the static - possibly non linear - criterion. Moreover it will propagate at most up to this length a^{k*} which satisfies the condition:

$$\Psi^{dyn}(a^{k*}) \;=\; \Psi^{stat}(a_0^k) \;-\; W_d(a^{k*}) \tag{28}$$

with a_0^k the initial crack length and $W_d(a^{k*})$ the statically computed total dissipated energy. For constant loading conditions, the lower the additional dissipative phenomena will be, the further the unstable crack will possibly run, the further from the critical state the crack will be in its final position and thus the greater will be the increase in the further loading or stretching necessary in order to make the crack growth restart. It should be also emphasized that - just as in the case of a single crack and in the absence of crack arrestor mechanisms - the shorter the first microcrack that will reach the critical state, the further will be the final extension of the unstably running crack. This confirms that, from the point of view of brittleness, small microcracks may be considered as more dangerous than large ones. This explains why materials with fine microstructure are at the same time more resistant and more brittle than materials with coarse microstructure. But in the case of concrete, another factor has to be taken into consideration, namely the crack arrestor effect provided by the presence of stiffer and/or tougher granules. This can be seen more easily through numerical simulations as illustrated below.

5 Numerical simulations on models of concrete microstructure

Numerical simulations performed on bi-dimensional models of concrete considered as a random multicracked granular composite have shown load deflection curves that are indented although globally softening, see Fig. 1, taken from the work of our past doctoral student Wang (1994), see also Wang et al. (1992), (1995). Since the matrix is considered perfectly brittle in these simulations, the obtained softening and indented behaviour

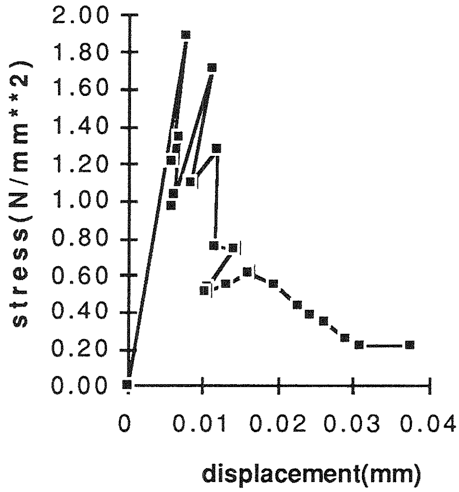


Fig. 2 Indented computed stress-displacement curve for a specimen in tension, from Wang (1994).

is fully due to the crack arrestor effects of the granules when taken to be stiffer than the matrix. The observed indentations are in fact a succession of limited snap-backs providing local micro-instabilities even under constant displacement conditions. In fact, this indented behaviour can be observed very often as can be seen on most real time records published in the literature. They are related to acoustic emission phenomena, like the ones observed on concrete specimens under load as reported many times.

The presence of the granules may also strongly modify the crack pattern, for instance by yielding branching of the main crack even in quasi-static conditions as shown from Wang (1994) by Fig. 3 for the anchor-bolt problem. The computed crack pattern - and also the load deflection curve, that exhibits two maxima - is highly realistic as shown by the comparison with experimental results, see for instance Ohlsson and Elfgren (1991) and the preliminary report by Elfgren (1992) in the Proceedings of the Breckenridge Framcos-1 Conference. It can be seen also that one of the obtained branches is the - in fact unstable - one we obtained in the homogeneous case through LEFM computations, see Elfgren (1992). In fact all these kinds of paths, with and without branching, were found in the real experiments performed on wall shape specimens in several laboratories.

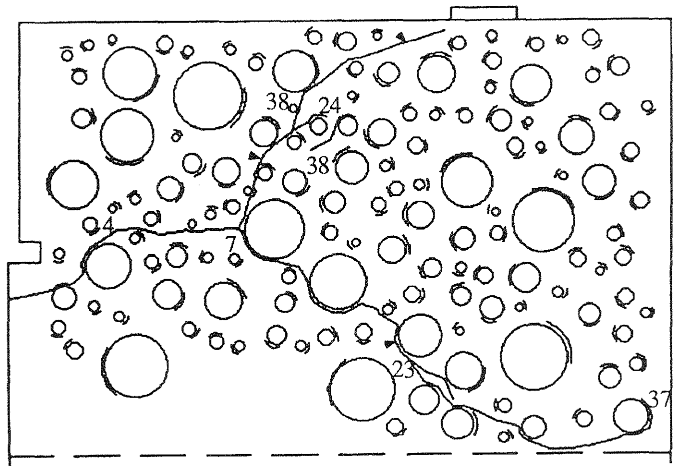
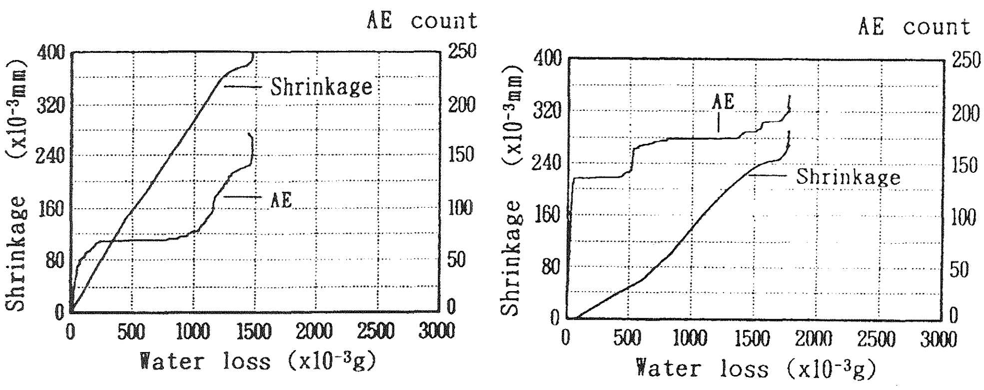


Fig. 3 Computed crack patterns for the anchor bolt problem, from Wang (1994).

6 Application to the interpretation of acoustic emission measurements

Among many others, very interesting acoustic emission measurements on tubular and solid cylindrical specimens of cement paste and mortar in the process of drying under controlled atmosphere conditions have been reported in a recent paper by Mihashi and Numao (1995). While the shrinkage is regularly and continuously - in fact almost linearly - increasing in terms of the water loss, the cumulated acoustic emission (A.E.) count shows a stepwise discontinuous behaviour. At the very beginning of the water loss on thin wall specimens, the A.E. count increases regularly and rapidly. But at some water content, it stops almost completely while the water loss continues to increase regularly. When a sufficient water loss - that can be several times higher than the one for which A.E. stopped - has been reached, acoustic emission starts again and may reach very high rates. For some specimens, the process may repeat one or several times, leading to a multistep discontinuous behaviour of the A.E. count. The interpretation we propose here is related to the local microinstabilities that have been shown by numerical simulation associated with the extended Griffith theory described above. It can be considered that, during the process of hardening, the endogeneous shrinkage due to hydration is enough to create microcracks in the pure cement paste as well as in the cement matrix of the mortar since the not yet hydrated clinker grains act as inert inclusions hindering the hydration product shrinkage, and thus developing tensile stresses concentrated in the vicinity of grains or crystals surfaces and/or interfaces. Experimental evidence of such shrinkage microcracks in cement paste has been obtained in our laboratory using a confocal microscope, see Sunderland et al. (1993), (1995). Because the behaviour of the material during the primary hardening process is highly dissipative - involving a large amount of viscous and plastic deformations to the detriment of kinetic energy - it may be assumed that, just before the immersion of the specimen in a drying atmosphere, a significant number of the shrinkage microcracks are in the critical state, or very near it. Thus, when drying is beginning, these critical microcracks are able to grow immediately. This explains why acoustic emission begins at the very beginning of the water loss. In the meantime, other microcracks are still at rest. During this rest, the stress around the still resting microcracks is continuously increasing because of the increase of the - partially hindered - shrinkage. When the stress is high enough, some more microcracks start to grow. Because these microcracks are very short, and because the potential energy is not fed by any external loading, we are - at least for some of these microcracks - in the local semi-stability conditions, meaning sudden unstable crack propagation on a short distance almost immediately followed by crack arrest.



(a) (b)

Fig. 4. Water loss vs. shrinkage and AE, from Mihashi and Numao (1995), with permission,
(a) thin walled HCP; W/C = 0.4 ; (b) thin walled mortar; S/C = 0.4

This process being very fast, it occurs at constant water content. As discussed in Section 4.2 above, unstable crack propagation means excess of released potential energy by comparison with the dissipated one, involving quasi-periodic exchanges of kinetic and potential energy at frequencies corresponding to the spectrum of natural frequencies of the specimen, meaning that vibratory and acoustic wave propagation phenomena are occurring in the specimen. These phenomena are the very ones that are detected by the A.E. transducer. But when the crack tip has arrested after rapid motion, it has - as also explained in Section 4.2 - overshot the position that could be expected from the purely static dependency of the static energy release rate $G^{k\,stat}$ on the crack tip position. The considered crack tips are thus, at least for some of these microcracks, in mechanical states that are hypocritical, and possibly far from the critical state. Thus, during a period that may be long, the shrinkage may go on re-opening the microcracks at constant length without further increase in the acoustic emission count, or with a much smaller increase, as seen in the experimental curves for the thin wall specimens. When the shrinkage stresses have increased enough, some of the previously resting microcracks reach the critical state again and the preceding process is repeated. This interpretation needs further work for confirmation, namely through the use of the Guidoum's computer code mentioned at the end of Section 3 above.

7 Application to dynamic main crack propagation in non-loaded regions

It is well known that - for perfectly or almost perfectly brittle materials - the classical Griffith theory explains very well why cracks may run deep in regions that are completely unloaded - and unstressed - as long the crack tip has not reached these. This occurs for instance in a glass plate submitted to local punching in static conditions and even in temporary ones in the form of a local shock. The dissipative heterogeneous extension of the Griffith theory presented above shows that, depending on the particular conditions of the problem, similar behaviour may be obtained even for semi-brittle materials. This has been confirmed by preliminary experiments performed in our laboratory on normal concrete square horizontal plates supported by a centered small support acting as an anvil and submitted to the shock of a falling ball impacting the plate just above the central support location. The thicknesses of the plates - ranging from 5 to 15 millimetres - are about ten times to twenty times lower than their in plane side dimension, which is about 20 centimeters, see Sunderland et al. (1995). Thus, even during the punching, there is no static stress on the whole volume of the plate except the very small region interested by the local Hertz problems of the falling ball. The diameter of the largest grains of the concrete is about 20 to 30 millimeters, allowing thus the bridging and friction fracture mechanisms to occur, specially for those grains that have an end in the thickness of the plate and yield a dilatancy effect involving a secondary mode III process which contributes to the potential energy. It was possible to adjust the falling height in order to obtain at will complete propagation or arrest of the cracks before their reaching the plate sides. The shape and morphological features of these cracks were also studied using the confocal microscope and the image analysis computer code, see Sunderland et al. (1995). These experiments show that the self-stress field generated by a crack in the course of its dynamic propagation in normally unstressed regions may occur in semi-brittle materials like concrete as well as in almost perfectly brittle materials like glass. In fact, the same experiments, using the same macroscopic geometry and the same loading device were performed also on glass plates with results qualitatively very similar on the macroscopic scale. Moreover, since the specimens used were of rather limited laboratory dimensions, these experiments also show that such brittleness phenomena are not limited to the case of concrete structures of very large sizes. Thus it may happen that the size effect laws, as derived from purely static experiments, may not apply to dynamic crack propagation. It may also happen, as previously mentioned in Huet (1994), that the danger of unstable crack propagation at very long distances may be highly underestimated by quasistatic calculations when an appropriate procedure - which can be the bounding one mentioned in Section 4.3 above - is not used. This can be particularly dangerous when estimating the security of very large size structures like concrete dams, for which the R curve - together with the Fracture Process Zone dimension - will exhibit a plateau after a few grain diameters while non monotonic $G^{k\,stat}$ curves with a maximum may be found by numerical simulations, see Huet (1994a), Wang (1994).

8 Concluding remarks

As shown in this paper, the obtained general Fracture Thermodynamics formalism provides first and second order fracture criteria that are safe and rigorous. Because of their similarity with the pioneering - but here re-interpreted - Griffith criterion, it is unnecessary to consider that, in the dissipative case, the use of the latter is only an approximation or a heuristic trick. The corresponding results turn out to allow good predictions and interpretations of the numerous and various features exhibited by crack slow growth and rapid propagation phenomena observed on concrete structures or specimens whatever their sizes, shapes and internal constitution might be. This holds for both the quasi-static and dynamic regimes and includes the influence of concrete microstructure, namely with the help of computerized micromechanical simulations, allowing thus behaviour predictions and composition optimizations. Of particular importance is the initial geometry and distribution of the largest cracks or largest micro-cracks prior to any external loading, since their length governs not only the onset of the crack growth, but also the somewhat antagonist crack propagation stability, and since their magnitude relative to the size of the structure cross section can be of much more significant influence than the value of the dissipative specific energy of fracture - or crack tip resistance - and even the tensile strength itself. On real structures, they should be determined by in situ non-destructive observations for the further development of which there is thus a strong motivation. It is confirmed in particular that there is no intrinsically brittle material, as demonstrated for instance by the controlled fracture experiments - with obtention of perfectly stable softening curves - that we performed on glass as early as the late sixties, see Huet (1971b), but only more or less brittle structures for a given material. For a given structure, the decrease of brittleness may generally be obtained indifferently by the increase of the initial crack to ligament lengths ratio, by a crack resistance increasing in the process of crack growth - the case for a crack reaching a tougher granule or exhibiting fractal tortuosity and/or branching - or by the decrease of the potential energy release rate - the case for a crack approaching a stiffer granule às shown by Wang (1993). The latter effect should be more effective since it begins to act when the crack tip is still at a distance from the interface. As confirmed by experiment, decreasing the stiffness of the granules - lightweight aggregates, large pores - or increasing the stiffness of the matrix - high performance concrete - generally makes the structure more brittle. On the other hand, even with very high specific fracture energy, a homogeneous structure with a fracture energy independent of the crack length will behave in a perfectly brittle fashion if the appropriate geometrical and loading conditions are met. As shown in Huet (1994a) and also emphasized by Bazant (1994), this is the case for very large concrete structures like dams for which the FPZ has completed its development long before the main crack tip has deeply penetrated the wall thickness, yielding thus a constant - although comparatively high - main crack tip resistance force (or fracture energy). Moreover, it is the instantaneous parameters at each stage of the process, and not the final averaged ones, that are relevant for the predictions even if the averaged values are easier to determine and provide useful comparisons. It can be also stated that, when appropriately handled, quasi-static computations provide enough information in order that the risk of unstable dynamic crack propagation may be evaluated together with lower and upper bounds on the possible crack extensions. For the case of concrete, the influence on the potential energy of the self-equilibrated shrinkage stresses - which should not be confused with plasticity effects since they have the opposite influence of reinforcing the crack driving force - should not be forgotten and the same for the viscoelasticity effects for which a rigorous extension of an important elasticity formula has been - seemingly for the first time -

provided here in this paper. Since the viscoelastic response times for concrete are distributed on a wide continuous spectrum that include times much shorter than the characteristic times of quasi-static and even dynamic or shock experiments, these viscoelasticity effects cannot be avoided even in the short term. Moreover, they govern - at least in part - the delayed fracture behaviour. The other part comes from the crack tip behaviour itself, for which the classical Eyring model - for which we gave in Huet (1993b) an expression for the Rayleigh dissipative function - seems most appropriate, as already considered in the literature.

It should be also pointed out that some care should be taken when incorporating friction effects due to the sterical hindrance between the partially loosened aggregates and their previous matrix, as obtained in monotonic experiments. In fact, it seems that the corresponding residual strength cannot be considered as structurally reliable since - in particular - it can be more or less progressively destroyed by cyclic loading, for instance through local attrition phenomena. It seems that this aspect deserves special attention and probably further research. It may be specially important for high risk structures - like dams, large bridges, large towers and nuclear plants - since they experience variable seasonal and/or service loading and may be exposed to earthquakes or other accidental loadings. Other consequences of the above results may be found in the quoted papers by ourselves and our coworkers.

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