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# FICTITIOUS CRACK MODELLING OF KILN FURNITURE CERAMICS

M.A.J. van Gils, L.J.M.G. Dortmans and G. de With,
Centre for Technical Ceramics, Eindhoven, The Netherlands

## Abstract
The ability of the Fictitious Crack (FC) model to describe the room-temperature mode I fracture behaviour of five different refractory ceramics is investigated. An efficient implementation of the FC model in the Finite Element Method is introduced. With the experimental results of the Wedge Opening Loaded test specimens the parameters of the FC model are estimated. With these parameters the results of Single Edge Notched Beam tests are predicted. Comparison with the measured values indicates that the results of a single geometry can well be simulated with the FC model but that extrapolation to different geometries gives erroneous results. From the measured force-displacement curves it is made clear that the apparent fracture energy is strongly dependent upon the ligament length which is in disagreement with the basic FC model assumption that the fracture energy is a material constant.

## 1 Introduction

Kiln furniture made of coarse grained, porous refractory ceramic in general shows a good thermal shock resistance. For predicting the thermal shock resistance the theory originally developed by Hasselman (1969) can be used. Although this theory is applicable for purely brittle materials, for the prediction of the actual lifetime of structural components a more sophisticated tool is required. The fracture behaviour of the coarse grained material cannot be described with the Linear Elastic Fracture Mechanics theory due to the existence of a process zone in front of the crack tip. This process zone results in a nonlinear fracture behaviour which should be incorporated in the constitutive behaviour. Then failure predictions can be made for an arbitrary geometry using the Finite Element Method (FEM).

To model crack formation and crack growth in coarse grained, heterogeneous materials like concrete, rock and ceramics the Fictitious Crack (FC) model is often used (Carpinteri *et al.*, 1987; Llorca and Steinbrech, 1991). With this FC model the effect of nonlinear phenomena in the process zone on the mechanical behaviour of the structure can be simulated. The phenomenon which is most likely to occur in coarse grained materials is the interlocking of grains in the developing crack.

In the FEM the FC model is usually introduced with the aid of interface elements. In this paper a different implementation will be introduced which largely reduces this required CPU time for a calculation.

Two types of mode I experiments have been set up, the Wedge Opening Loaded geometry and the Single Edge Notched Beam geometry. The parameters of the FC model will be estimated by comparing the experimental and numerical force-displacement curves for the Wedge Opening Loaded geometry. These parameters will then be used to predict the results for the other geometries. This procedure will give information on the validity of using the FC model for describing the fracture behaviour of the present materials.

## 2 Experiments

In order to evaluate the mechanical behaviour of the refractory ceramics two types of mode I experiments have been set up. These experiments use the well-known Single Edge Notched Beam (SENB) and the Wedge Opening Loaded (WOL) configuration (Brühwiler

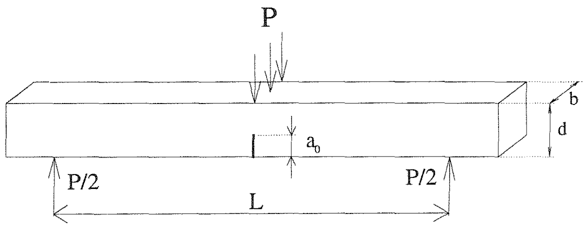


Fig. 1. SENB configuration

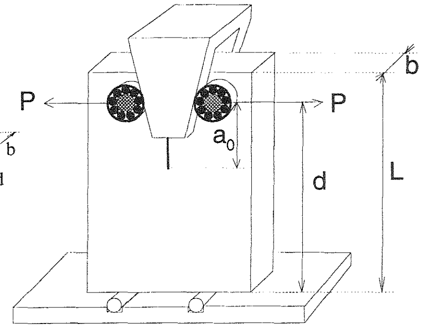


Fig. 2. WOL configuration

and Wittmann, 1990) respectively. The two configurations are shown in figures 1 and 2. With these two geometries a large range in specimen size can be tested. The specimen sizes used are listed in table I. For the WOL specimens and the larger SENB specimens the opening

Table 1. Specimen sizes for SENB and WOL

| | d [mm] | b [mm] | L [mm] | $\alpha_0 = a_0/d$ [-] |
|---|---|---|---|---|
| SENB | 15-30 | 15-50 | 4 d | 0.0, 0.25, 0.5, 0.75 |
| WOL | 120 | 30 | 151 | 0.25, 0.5 |

of the crack is simultaneously measured with the applied force. This Crack Opening Displacement (COD) is measured with the use of a Linear Voltage Displacement Transducer (LVDT) which is attached to the specimen.

Five different refractory ceramics, all made by Sphinx Technical Ceramics, Maastricht, the Netherlands, are tested. These materials are the alumina and mullite based materials named Teoxit and Leoxit, the cordierite and alumina based materials Alcorit and Ticorit and the SiC based material Sicorit. All the materials have a large grain size ( 0.2-1.0 mm) and are highly porous (20-30%). The micro-structure of these materials is comparable with the micro-structure of concrete. The test specimens are all made from the same batch of material and have identical notch directions.

## 3 Fictitious Crack model

### 3.1 Introduction

With the Fictitious Crack (FC) model (Hillerborg *et al.*, 1976; Rots, 1988) a crack is initiated when in some point of the structure the maximum principal stress exceeds the tensile stress $f_t$. A crack is formed in that particular point by separating the material. The direction of the crack is perpendicular to the direction of the maximum principal stress. The developing crack is, however, still able to transfer stresses between the crack surfaces. Figure 3 is a graphical illustration of the resulting situation. The constitutive behaviour of a point along the crack path is defined by the stress-crack opening displacement relation or softening curve $\sigma_N(u_N)$. The shape of this $\sigma_N - u_N$ relation is a material property. Some possible choices for the softening curve are illustrated in figure 4. The area under this curve equals the fracture energy $G_f$ of the material:

$$G_f = \int_0^{u_{cr}} \sigma_N du_N \tag{1}$$

### 3.2 Implementation of the FC Model

In order to be able to apply the Fictitious Crack model for various geometries, the combination of the FC model with the Finite Element Method (FEM) is required. A possible implementation makes use of special interface elements (Rots, 1988; Schellekens, 1992) to simulate the crack path. Disadvantages of this method are the relatively

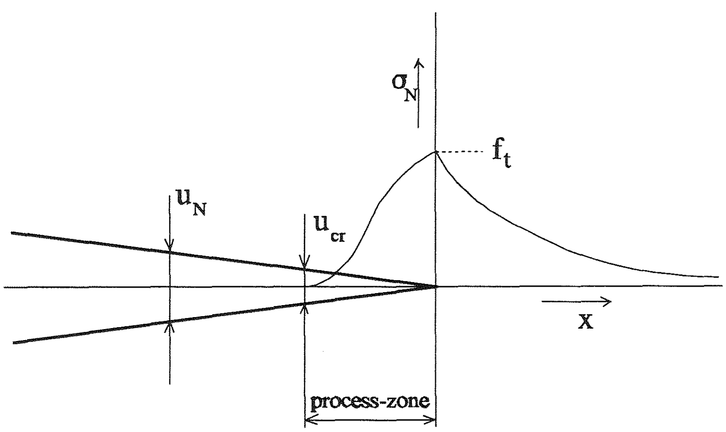


Fig. 3. FC model

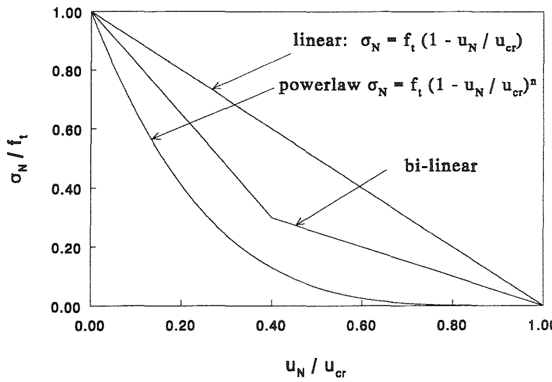


Fig. 4. $\sigma_N - u_N$ constitutive relations

large amount of required CPU time and the occurrence of numerical instabilities.

These problems can be avoided by using an alternative approach called the 'inverse flexibility method' as introduced by several researchers (Carpinteri *et al.*, 1987; Lemaitre and Chaboche, 1990) which makes use of the superposition principle. This method is applicable for small deformations and rotations (geometrically linear problems). The material behaviour outside the crack path is assumed to be linear elastic.

With these conditions the structure with the Fictitious Crack model can be solved by using the superposition principle on the (linear elastic) parts of the structure that are normally attached to the interface elements. The problem to be solved can then be formulated as:

$$\boldsymbol{f} = \boldsymbol{K}\boldsymbol{u} + \boldsymbol{H}\boldsymbol{p} \tag{2}$$

where $\boldsymbol{f}$ is a column with forces acting on the nodes along the crack with displacements given in column $\boldsymbol{u}$ and $\boldsymbol{p}$ is a column with external forces acting on the structure. The matrices $\boldsymbol{K}$ and $\boldsymbol{H}$ are obtained from a single linear elastic finite element calculation. With the constitutive model $\boldsymbol{\sigma}(\boldsymbol{u})$ associated with the FC model, equation 2 can be solved. The stress values $\boldsymbol{\sigma}$ at the crack-path nodes are translated to the required forces $\boldsymbol{f}$ at those nodes by nodal lumping.

With the constitutive relation $\boldsymbol{\sigma}(\boldsymbol{u})$ two unknown properties remain, the displacements $\boldsymbol{u}$ and the external force(s) $\boldsymbol{p}$. For the additional constraints needed to solve the equation several choices can be made. For the traditional modelling with interface elements the external force $\boldsymbol{p}$ is prescribed. This approach is difficult to use because the sizes of the successive load-steps in the incremental procedure are unknown. This often leads to undesired divergence of the iterative process when the step sizes are not chosen correctly.

In the present method the position of the fictitious crack tip is prescribed. This procedure turns out to be numerical stable (Carpinteri *et al.*, 1987; Lemaitre and Chaboche, 1990).

# 4 Estimation of the FC model parameters

## 4.1 Determination of the response surfaces

For the estimation of the values of the parameters required for the FC model, comparison of experimental and numerical results is necessary. The calculated P-COD curve using the parameters of the chosen FC model must resemble the measured P-COD curves. Because evaluation of the complete P-COD curve for each specimen is cumbersome, a discrete number of characteristic results is chosen. The parameters will be estimated on the basis of these results. The characteristic experimental results c.q. the response variables chosen for the parameter estimation are illustrated in figure 5. These four response variables are the maximum force $P_{max}$, the COD at maximum force $COD_{P_{max}}$, the COD at 10 percent of the maximum force $COD_{0.1P_{max}}$ and the area $A$ under the P-COD curve from COD = 0 to COD = $COD_{0.1P_{max}}$.

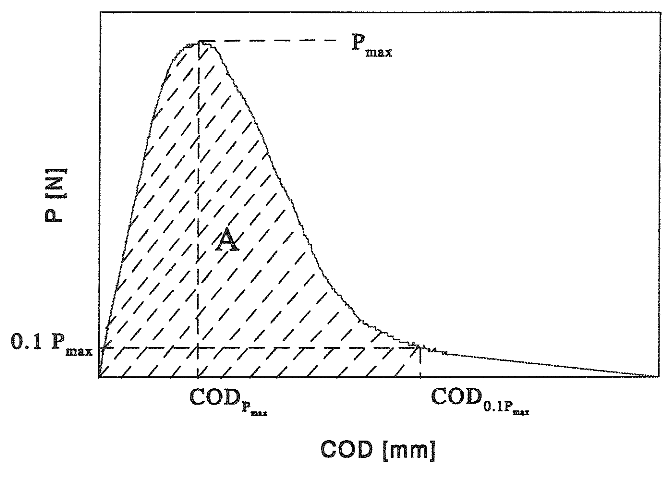


Fig. 5. Characteristic experimental results

In order to estimate the parameters it is necessary to know the response ($P_{max}$, $COD_{P_{max}}$, ...) of the FC model with variation of the parameters ($f_t$, $u_{cr}$, ...). This response must be determined from calculations with the FC model. For a specific geometry (e.g. a WOL specimen) the response must be calculated for a certain range of the FC model parameters. The result of these calculations are four response surfaces, one for each response variable.

### 4.2 Determination of the FC model parameters

With the descriptions of the response surfaces for the various geometries, the model parameters associated with the different materials can be determined from the measured responses. The least-squares estimates for the model parameters can be calculated using a $\chi^2$-minimization. With the use of the $\chi^2$-minimization the minimum value of $\chi^2$ should, for a correct model, be approximately equal to the number of degrees of freedom of the minimization problem. In that case the residual sum is caused by the scatter in the measurements and not by a 'wrong' model. From the distribution of the $\Delta\chi^2$ contours the confidence limits of the fitted parameters can be determined.

## 5 Experimental and numerical results

The experiments and the estimation of the FC model parameters have been carried out for the five materials. The results of one material, Teoxit, will be used as example. The value of the Young's modulus $E$ is determined from the initial curve in the P-COD curves of the WOL specimens with $\nu$=0.25. This value of the Young's modulus ($E$ = 23.7 GPa) is kept constant. The power-law equation ($\sigma_N = f_t\,(1 - u_N/u_{cr})^n$) is chosen for the constitutive relation for the softening curve. The parameter estimations indicated that the power-law curve gave a substantial improvement over the linear curve ($\chi^2_{pow} < \chi^2_{lin}$) The use of a bilinear curve gave, however, no additional improvement in comparison with the use of the power-law curve. Three model parameters, the tensile stress $f_t$, the critical crack opening $u_{cr}$ and the exponent $n$, will have to be determined from the results of the measurements. For the power-law model the response surfaces are calculated and the least-squares ($\chi^2$) estimates of the parameters based on the WOL results are calculated. The following values for the parameters and confidence limits are the result for the material Teoxit:

$$\begin{aligned} f_t &= 5.243 \pm 0.188 \text{ MPa} \\ u_{cr} &= 0.5014 \pm 0.188 \text{ mm} \\ n &= 14.0 \pm 0.647 \end{aligned}$$

The estimated P-COD curve for the WOL geometry with a relative notch length of $\alpha_0 = 0.25$ is shown together with the experimental results in figure 6.

With the FC model parameters fitted on the experimental WOL results, predictions can be made for the other geometries tested. For the largest SENB's the COD's are also measured so a comparison of the predicted P-COD curves and the measured P-COD curves is possible for these geometries. The results for this SENB geometry with a relative crack length $\alpha_0$=0.25 is shown in figure 7. For the whole range of the tested geometries the predicted maximum forces and the average measured maximum forces can be compared. From these maximum forces of the differently sized specimens a so-called Bažant size effect graph (Bažant and Kazemi, 1990) can be determined. This size effect graph for the material Teoxit is illustrated in figure 8. The straight line in this graph is the size effect according to Linear Elastic Fracture Mechanics (LEFM) with the parameters determined from the results of the WOL measurements. It is clear that the size effect of the investigated materials cannot be described with the LEFM theory. From figure 7 it appears that the measured response is more brittle then the predicted response. This indicates

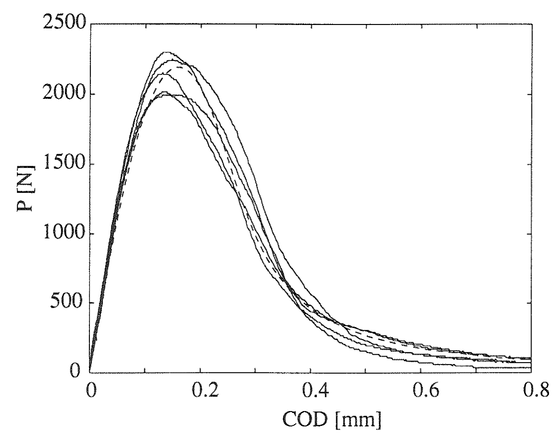


Fig. 6. Calculated P-COD curve (- - -) for a WOL specimen with $\alpha$=0.25 based on the power-law FC model together with the experimental curves

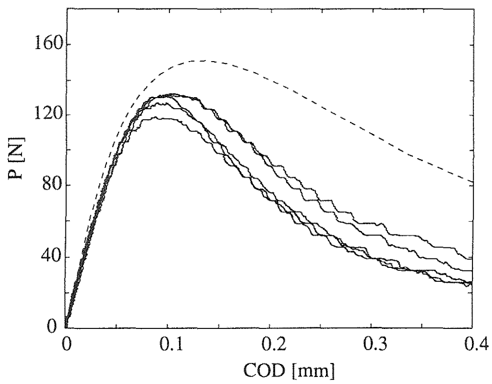


Fig. 7. Calculated P-COD curve (– – –) for a SENB specimen (L=200 mm, d=50 mm) with $\alpha_0$=0.75 based on the power-law FC model together with the experimental curves

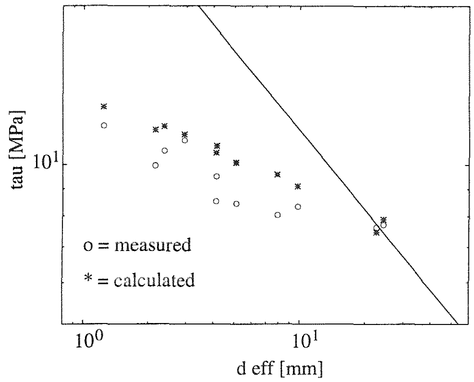


Fig. 8. Bažant size effect graph

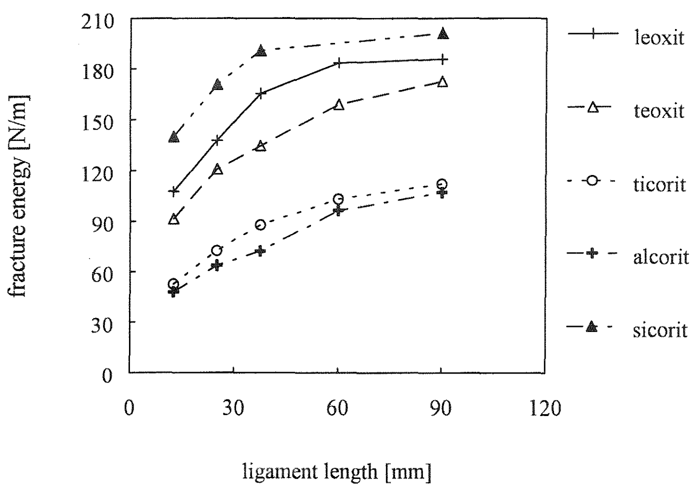


Fig. 9. Apparent fracture energy as function of the ligament length

a decrease in apparent fracture energy for a smaller ligament length. This dependence of the apparent fracture energy upon the ligament length can be further investigated by estimating the FC model parameters for the individual geometries. The fracture energy associated with these FC model parameters can be calculated according to equation 1. In figure 9 these values of the apparent fracture energy as a function of the ligament length are illustrated for the five tested materials. From figure 9 it is clear that the apparent fracture energy is strongly dependent upon the ligament length. In the present FC model the fracture energy is, however, a material property.

## 6 Conclusions

The ability of modelling (mode I) crack propagation in five different refractory ceramics with the Fictitious Crack model has been investigated. A numerically efficient, stable and easy to use implementation of the FC model in the FEM has been introduced.

Different geometries with a large range in sizes are tested. With the FC model a very good resemblance of the measured and calculated P-COD curves of the WOL specimen was obtained. The predicted P-COD curves for the SENB geometry based on the estimated parameters show deviations from the measured responses. These deviations become larger when the ligament length decreases. The apparent fracture energy decreases with decreasing ligament length. This dependency is not incorporated in the present FC model. For a correct description of the fracture behaviour of refractory ceramics the FC model needs therefore to be adjusted which is topic of present research.

## References


Bažant, Z.P. and Kazemi M.T. (1990) Size effect in fracture of ceramics and its use to determine fracture energy and effective process zone length. **J. Am. Ceram. Soc.**, 73, 1841-1853.

Brühwiler, E. and Wittmann, F.H. (1990) The wedge splitting test, a new method of performing stable fracture mechanics tests. **Eng. Frac. Mech.**, 35(1/2/3), 117-125.

Carpinteri, A., Colombo, G., Ferrara, G. and Giuseppetti, G. (1987) Numerical simulation of concrete fracture through a bilinear softening stress-crack opening displacement law, in **Fracture of Concrete and Rock: SEM/RILEM International Conference** (eds S.P. Shah and S.E. Swartz), 131-141.

Hasselman, D.P.H. (1969) Unified theory of thermal shock fracture initiation and crack propagation in brittle materials. **J. Am. Cer. Soc.**, 52(11), 600-604.

Hillerborg, A., Modeer, M. and Petersson, P.E. (1976) Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. **Cem. & Concr. Res.**, 6, 773-782.

Lemaitre, J. and Chaboche, J.-L. (1990) **Mechanics of solid materials**. Cambridge University Press, Cambridge.

Llorca, J. and Steinbrech, R.W. (1991) Fracture of alumina: an experimental and numerical study. **J. Mater. Sci.**, 26, 6383-6390.

Rots, J.G. (1988) Computational modelling of concrete fracture. Dissertation. Delft University of Technology, Delft.

Schellekens, J.C.J. (1992) Computational strategies for composite structures. Dissertation. Delft University of Technology. Delft.