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A STRATEGY FOR ANALYSING THE MECHANICAL BEHAVIOUR OF CONCRETE STRUCTURES UNDER VARIOUS LOADING: THE LCPC EXPERIENCE

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Abstract

Continuous models (whether standard or enriched) and discrete crack models are in general considered as alternatives for analysing concrete structures under various loading. This paper discusses these models from the standpoint of material observation, and proposes a strategy based upon the complementarity of both approaches: global information as loading capacity, global displacements, zones where irreversible evolution in the structure occur etc., can be accessed by continuous models, which account for the evolution of the material rigidity during the cracking process and irreversible deformations, both associated to the same physical phenomenon, i.e. cracking. More precise information as crack-length, crack-spacing, crack-width etc. need to be accessed by a discrete crack model which abandons the concept of a representative elementary volume for the modelling of concrete cracking in the finite element analysis of material and structural failure. The proposed strategy is compared with a one stage strategy of enriched non-linear continuum models. An example for each type of model is presented.

1 Introduction

Modelling of cracking of concrete structures aims to account for a certain number of physical mechanisms which occur over a wide range of scales of material description. They can be summarised as follows:

1.1 Structures submitted to quasi-static loading

The cracking process includes three principal stages:

- the creation of microcracks randomly distributed throughout the structure which is governed by the heterogeneity of concrete,
- the percolation of these microcracks to form one or more macrocracks; this stage of localization of cracking is governed by the boundary conditions of the structure,
- the propagation of the macrocracks until the appearance of a failure mechanism.

These stages can be related to the following physical mechanisms (Acker et al., 1987, Rossi,1991):

- *irreversible evolution in the matrix on initial stress release*: initial stresses induced by non-uniform shrinkage during hydration of the cement paste at the scale of the heterogeneous material, are released when cracking occurs;
- *friction in the crack-lips*: due to the great heterogeneity of concrete and due to the release of initial stresses, the intensity of these friction forces depends on crack pattern and boundary conditions of the structure;
- *viscous phenomena coupled with the cracking process*, which delay the crack propagation; these viscous phenomena are due to the presence of free water in the concrete pores.

1.2 Structures submitted to thermal and hygral gradients

Thermal and/or hygral exchanges with the exterior induce gradients in concrete structures, which lead to a severe state of stresses, which may be beyond the strength developed: the concrete cracks. Due to the time scale in which these thermo-hygro-mechanical couplings occur, some delayed behaviour of concrete (due to presence of free water), as relaxation phenomena, are also involved.

1.3 Structures submitted to dynamic loading

In addition to the classical phenomena involved in the dynamic behaviour of structures (waves propagation, inertia effect, etc.), apparent rate effects influence strongly the material behaviour of concrete under dynamic loading:

- *Rate effects* related to the presence of water in the pores at the nano-level of material observation (Rossi et al., 1994a), lead to an apparent increase of (nominal) material strength with loading rate increasing, and should be taken into account in the modelling.

2 What kind of model for what kind of information needed ?

To take into account the above physical mechanisms in the framework of finite element analysis, two levels of answer corresponding to two modelling levels are in general distinguished:

- *Continuous approach to cracking:* the physical mechanisms related to concrete cracking are not explicitly evoked in the modelling, but encountered by a set of macroscopic variables (plastic, damage variables, smeared crack strains, etc.), which define the irreversible evolution of the material relative to a macroscopic scale adopted for the observation and quantitative analysis of the physical phenomena involved. A continuous model is defined by the scale of observation (scale of laboratory test specimen) which coincides with the scale of modelling (scale of constitutive modelling).
- *Discrete approach to cracking:* principal "crack-items", as for instance position, length and width of the cracks during the entire cracking process (including the transition from diffuse cracking to localized cracking) are used explicitly in the modelling. Owing to the random nature of crack creation and propagation, this explicit modelling of concrete cracking needs to account necessarily for the heterogeneity of the matter, i.e. for probabilistic aspects in the modelling. In other words, the scale of observation with respect to the modelling scale is not fixed, but plays an important role within the modelling.

These two modelling levels are not opposed, but complementary when using them with respect to the information required in the analysis:

- a continuous model gives access to global information: structural load bearing capacity, the global displacements (deflection...), the parts of the structure where irreversible evolution may occur, etc.
- the second modelling level gives direct access to crack-patterns, crack-width, crack-spacing and failure mechanisms (for instance rigid body motions). Furthermore, with respect to the stochastic approach required (see above) a fiabilist analysis of the structural behaviour can be performed from the statistical results obtained concerning load capacity, global displacements etc. (for instance by using a Monte Carlo technique).

The choice of using the one or the other modelling depends thus on the type (and the quality) of information required in the performed analysis. In return, when the analysis focus on crack-patterns, crack-width, failure mechanisms etc., i.e. on information relevant to the second modelling level, a continuous model will be used in a preliminary phase to detect the non-linear zones in the structure, and will allow to optimize the mesh for the discrete modelling. In this logic, any analysis of the non-linear behaviour will start by linear elastic simulations, first to study stress gradients in the structure, which imposes the meshing for the non-linear simulations, and secondly to evaluate the degree of pertinence of performing 2D non-linear simulations with respect to a 3D simulation (which gives the "real" stress fields).

3 Discussion of the pertinence of the strategy proposed

The two modelling levels presented here above are rather standard, but are generally considered as alternatives ("either - or") for analysing the failure behaviour of brittle materials. Moreover, as well known, standard continuum models reach their limits on the onset of strain localization. This has prompted a surge of research activities over the last decade aiming at enriching the standard continuum model by adding higher-order deformation gradients, non-local concepts, Cosserat continua, rate dependent continua, or probabilistic aspects (for a review see de Borst et al., 1993). We think that this one-fold strategy which limits itself to the continuous approach sets some questions:

1. The continuous approach (whether standard or enriched by localization limiters) is based upon the concept of a representative elementary volume, which -in turn- implies the existence of a scale defining the dimensions of the elementary volume. This scale is not intrinsic to the matter. It is the application which fixes it.
2. It has been shown experimentally and numerically (Rossi et al., 1994b, 1994c, Rossi and Guerrier, 1994, Rossi and Ulm, 1995) that the representative elementary volume for concrete, is never small compared with the dimensions of standard concrete structures. Hence, the scale of material modelling may be greater than the scale at which the model is applied (i.e. the finite element mesh size).
3. In the enriched approaches, an internal length scale is defined, considered as an intrinsic material parameter. But, its physical significance keeps a matter of intensive discussion.

Clearly, non-linear continuum models for concrete cracking are the more relevant, the more the information used (input) and requested (output) are at equal basis, i.e. having the same degree of pertinence. In other words, preference should be given to robust and non-sophisticated models with variables of clear physical significance and accessible by standard material tests. In this light, a continuous model, which accounts for the evolution of the material rigidity and irreversible deformations, both associated to the same physical phenomenon, i.e. cracking, seems sufficient with respect to the information required. At this stage of the analysis, the strain-softening behaviour can be neglected, avoiding thus the loss of well-posedness of the rate boundary value problem and related numerical problems. Furthermore, from a more physical standpoint, the strain-softening behaviour, i.e. a descending relation between stress and strain experimentally obtained, is governed by the boundary conditions, thus in form and shape not necessarily intrinsic to the material. If more information is required, another type of model should be used, which abandons the concept of a representative elementary volume for the modelling of concrete cracking. Continuous and discrete approach -*together*- then form a powerful tool for analysing the non-linear behaviour of concrete structures.

In what follows, two models used or under development in the LCPC are presented, that respect the previous requirements with respect to different loading cases mentioned in the introduction.

3 Continuous approach: Plastic degrading model

This model has been proposed by Ulm (1994) for quasi-static loading. The extension to chemo-mechanical couplings (for instance hydration reaction, creep etc.) can be found in Coussy and Ulm (1995). The latter can also be used for the (continuous modelling) of concrete in high rate dynamics. Here, we limit the presentation to the main features of this continuous model.

3.1 Quasi-static loading conditions

The purpose of this plastic degrading model is the modelling of two apparent phenomena related to cracking: the permanent (or plastic) deformations and the irreversible degradation of elastic material properties (also designed as damage in the literature). With respect to standard coupled plastic-damage models (see for instance Lemaitre and Chaboche, 1990), where damage and plastic mechanisms are assumed to occur independent of each other, the plastic degrading model associates both phenomena to the same physical origin, i.e. cracking. More precisely, since the plastic variables, namely the plastic strain ε^p and the hardening variables χ model the irreversible evolution associated to concrete cracking in a continuous manner, the damage associated to cracking, necessarily coincides with the plastic evolution. For quasi-static evaluations and isotropic material behaviour the following state equation is adopted:

$$\sigma = 2G(\chi)\left(\mathbf{e} - \mathbf{e}^p\right) + 3K(\chi)\left(\varepsilon - \varepsilon^p\right)\mathbf{1} \qquad (1)$$

where $G(\chi)$ and $K(\chi)$ are the shear modulus and the bulk modulus, the evolution of which depend upon hardening variables χ. Furthermore, $\varepsilon = \mathbf{e} + \varepsilon\mathbf{1}$ and $\varepsilon^p = \mathbf{e}^p + \varepsilon^p\mathbf{1}$ are the strain tensor and the plastic strain tensor (with $\varepsilon = \mathrm{tr}\varepsilon / 3$ and $\varepsilon^p = \mathrm{tr}\varepsilon^p / 3$). Like in the standard plastic model, plastic evolution occur, when a loading point σ is at the boundary of the elasticity domain D_E defined by loading function $f(\sigma,\zeta)$, thus

$$\sigma \in D_E \qquad \Leftrightarrow \qquad f(\sigma,\zeta) \le 0 \qquad (2)$$

where ζ is the hardening force (current material threshold), which depends upon hardening variable χ (i.e. $\zeta=\zeta(\chi)$). The evolution of hardening force ζ can be associated to the evolution of an energy *frozen* at the level of the heterogeneous material which is not recovered as useful mechanical work upon unloading. More precisely: with respect to the heterogeneity of the matter, plastic evolution that occur during loading correspond at the level of the heterogeneous material to a release of initial stresses (see introduction), associated to an elastic energy *frozen* in the constituents. A part of this energy is dissipated by friction at the crack lips, while altering the initial stress state, and with this the frozen energy. At the macroscopic level, the dissipation associated to friction phenomena is modelled in terms of plastic dissipation (σ:$d\varepsilon^p$ in time interval dt) reduced/increased by hardening/softening effects ($-dU = \zeta d\chi$). When neglecting second order strain terms in the state equations of a plastic degrading model, hardening force ζ derives from frozen energy U like in the standard plastic model

$$\zeta = -\frac{dU(\chi)}{d\chi} \tag{3}$$

A detailed discussion of this energy can be found in Coussy (1995), and applied to concrete in Ulm (1994). Here, we only note, that with respect to its origin, i.e. the material heterogeneity, frozen energy U depends on the scale of observation of the heterogeneity of the matter, as well as hardening force ζ, and thus the threshold, i.e. when plastic evolution occur. The evolution of plastic variables ε^p and χ are given by the flow and hardening rule:

$$d\varepsilon^p = d\lambda \frac{\partial g(\sigma,\zeta)}{\partial \sigma} \quad \text{and} \quad d\chi = d\lambda \frac{\partial h(\sigma,\zeta)}{\partial \zeta} \tag{4}$$

where $d\lambda$ is the plastic multiplier, and $g(\sigma,\zeta)$ and $h(\sigma,\zeta)$ the plastic and hardening potential defining the directions taken by plastic strain increments $d\varepsilon^p$ and hardening increments $d\chi$. Note clearly, that hardening variable χ is a plastic variable (for instance the plastic dilatation $\chi = \varepsilon^p$ or plastic distortion $\chi = \gamma^p_{eq} = \int 2\mathbf{de}^p{:}\mathbf{de}^p$), modelling the evolution of elasticity domain D_E through state equation (3), as well as the evolution of elastic material properties $G(\chi)$ and $K(\chi)$ in state equation (1). Plastic variable χ can thus be accessed -experimentally and numerically- independent of the degradation of elastic material properties. In other words, the information used (input) and requested (output) are at equal basis, since they are measured/calculated independent of each other. Note however, that plastic and hardening potential $g(\sigma,\zeta)$ and $h(\sigma,\zeta)$ as well as plastic criterion $f(\sigma,\zeta)$ determined experimentally for concrete are defined with respect to the scale of observation and will change, when passing from the scale that defines the representative elementary volume, where the model is designed, to a modelling scale below (Rossi and Ulm, 1995). To the knowledge of the authors, little attention has been paid to this fact in adaptive mesh strategies for elastoplastic problems.

The plastic degrading model is implemented in the finite element code CESAR-LCPC, and is accessible with different loading functions, plastic and hardening potentials. For concrete, we generally apply a 3 or 4 parameter Willam-Warnke criterion, with an associated flow rule ($g(\sigma,\zeta)=f(\sigma,\zeta)$). Strain-softening in the yield criterion is not considered (i.e. $dU / d\chi \geq 0$).

3.2 Concrete at early ages

The plastic degrading model can easily be extended to the continuous modelling of concrete cracking at early ages, where strains of thermal and chemical origin (autogeneous shrinkage) due to the hydration of the cement paste may induce a severe state of stresses beyond the strength developed. Furthermore, the elastic strength of the material increases with the hydration progressing (ageing). The modelling of these thermo-chemo-mechanical couplings can be found in Coussy (1995), Ulm and Coussy (1995), Coussy and Ulm (1995), where it is shown how a chemical reaction (here the hydration reaction) can be integrated in the constitutive modelling. State equation (1) now reads

$$\sigma = 2G(\xi,\chi)\left(\mathbf{e} - \mathbf{e}^p\right) + 3K(\xi,\chi)\left(\varepsilon - \varepsilon^p - \alpha\theta - \beta\xi\right)\mathbf{1} \tag{5}$$

where ε^p and χ are still the plastic variables, modelling the irreversible skeleton evolution associated to cracking. Furthermore, $\theta = T - T_0$ is the temperature variation, ξ the hydration degree, α the thermal dilatation coefficient and β the chemical dilatation coefficient (negative for shrinkage). Furthermore, the ageing phenomenon is taken into account by a dependence of elastic properties on the hydration degree ξ. The evolution of hydration degree is given by a kinetic law of the hydration reaction, reading:

$$\dot{\xi} = \frac{1}{\eta(\xi)}\left(\xi(\infty) - \xi(t)\right)\exp\left(-\frac{E_a}{RT}\right) \tag{6}$$

where $\eta(\xi)$ is an apparent viscosity of the macroscopic hydration-reaction, $\xi(\infty)$ the asymptotic hydration degree, and where the Arrhenius term $\exp(-E_a / RT)$ accounts for the thermo-activation in the kinetic of the hydration reaction. For the experimental determination see Ulm and Coussy (1995).

The evolution of the plastic variables is still given by flow and hardening rule (4), i.e. once a loading point σ reaches the boundary of elasticity domain D_E (i.e. $f(\sigma,\zeta)=0$). Due to the strength-growth associated with the hydration reaction the elasticity domain evolves independent of load application. This is the phenomenon of chemical hardening, which can be taken into account in the plastic model by a dependence of hardening force ζ on hydration degree ξ, respectively with a dependence of frozen energy U on hydration degree ξ:

$$\zeta(\xi,\chi) = -\frac{\partial U(\xi,\chi)}{\partial \chi} \tag{7}$$

This is consistent with the origin of frozen energy as evoked here before, which represents the (elastic) energy related to initial stresses induced by non-uniform shrinkage during hydration of the cement paste at the scale of the heterogeneous material and released/altered when cracking occurs. Note however clearly the difference in time scales of chemical and plastic evolution. The evolution of hydration degree ξ is related to the time scale of the hydration kinetics (i.e. (6)), while plastic increments $d\varepsilon^p$ and $d\chi$, modelling the cracking, occur simultaneously with any variation in loading with respect to a chemical hardening state $\zeta(\xi)$, i.e. for a given material strength.

Finally, the modelling of early age concrete as presented here before does not account for creep effects. In fact, as already mentioned in the introduction, any modelling needs to identify the physical mechanisms at the basis of an apparent phenomenon. This applies equally to the modelling of creep. In other words, to account for creep in our modelling strategy its physical origin needs still to be explored. The above example only sought to show, how a continuous modelling without (nearly) any sophistication concerning the modelling of concrete cracking, can be extended to the modelling of phenomena, where a coupling of the strain with a physical phenomenon is involved, of which the kinetics cannot be considered as instantaneous with respect to the time scale of observation. This remark applies to concrete creep and drying as well as to viscous phenomena at the origin of apparent rate effects in high rate dynamics.

4 Discrete approach: Probabilistic modelling of concrete cracking

4.1 Quasi-static loading conditions

Cracking of concrete is strongly influenced by the heterogeneity of the matter: the tensile strength of concrete is mainly related to that of the cement paste, which -in turn- is governed by the presence of voids, microcracks etc. created during the concrete hardening by non-uniform shrinkage during hydration at the scale of the heterogeneous material, i.e. at the scale of the concrete aggregates. This heterogeneity of the matter constituting concrete can be considered to be at the basis of apparent size effects, governing the overall cracking behaviour at the macroscopic scale of material observation (i.e., scale of laboratory test-specimen). This has led to the development of the *probabilistic modelling of concrete cracking* over the last decade (Rossi and Wu, 1992). It belongs to the family of statistically based deterministic models using the finite-element method. It accounts for cracks as geometrical discontinuities (discrete crack approach), and for the heterogeneity of the matter by random distribution functions with experimentally determined mean values, $m(f_t)$ and $m(E)$, and standard deviations, $s(f_t)$ and $s(E)$, of tensile strength f_t and Young's modulus E, respectively. For their experimental determination, a major experimental research has been performed at the LCPC, which has led to the proposing of analytical expressions of the distribution functions, (Rossi et al., 1994b).

In the finite element analysis, these functions are used by replacing in the experimental determined distribution function the volume of the test specimen V_t by the volume of each singular solid finite element. This is consistent with physical evidence: the smaller the scale of observation (respectively the modelling scale) with respect to that of the structure, the larger the fluctuation of the local mechanical characteristics, and thus the (modelled) heterogeneity of the matter. In other word, the probabilistic model abandons the concept of a representative elementary volume by considering each finite element volume as a material volume. This renders the numerical results mesh-independent (Rossi and Guerrier, 1994). With respect to the local and probabilistic character of the approach, the volume of the solid mesh elements must be sufficiently small with respect to the volume of the modelled structure, so that the probabilistic analysis performed on the scale of the mesh element is representative with respect to the structure.

The cracks are modelled using special contact elements that interface the solid elements. A crack (i.e. a contact element) "opens" (appearance of a geometrical discontinuity) when the stress normal to a fracture plane

$\sigma_N = \mathbf{n}.\sigma.\mathbf{n}$ reaches the local tensile strength f_t randomly distributed, thus:

$$\sigma_N - f_t \leq 0 \tag{8}$$

where

$$\sigma_N = \mathbf{n}.\sigma.\mathbf{n} = 2G(\lambda - \varepsilon) + 3K\varepsilon \tag{9}$$

with the elastic properties G and K randomly distributed. Furthermore, $\lambda = \mathbf{n}.\varepsilon.\mathbf{n}$ is the principal extension normal to a possible fracture plane oriented by unit normal $\mathbf{n}$, and $\varepsilon = \mathrm{tr}\varepsilon / 3$.

Crack criterion (8) refers to the fact that concrete cracking corresponds to a mode-I mechanism, in tension as well as in compression - independent of the scale of observation. In compression failure occurs due to the appearance of oblique cracks (Torrenti et al., 1993). The oblique cracks are created locally by tensile stresses. In order to capture this failure mechanism in terms of modelled heterogeneity, the modelling scale has to be small with respect to the "structural" scale at which the failure occurs (small columns created by vertical cracks within the sample), much smaller than that of the sample. At a scale above, this oblique cracking appears as a shear failure and in the limit of a homogeneous material, no tensile stresses occur in the material under compression. Hence, relative to the modelled heterogeneity in the analysis, a shear-crack criterion may be necessary, reading in its simplest form:

$$|\tau| - c \leq 0 \tag{10}$$

where

$$\tau = \mathbf{t}.\sigma.\mathbf{n} = 2G\gamma \tag{11}$$

with $2\gamma = 2\mathbf{t}.\varepsilon.\mathbf{n}$ the angle variation in the $\mathbf{t} - \mathbf{n}$ fracture plane. Furthermore, c is the local cohesion of the material. Since this crack-criterion reflects a mode-I criterion at a modelling scale below, cohesion c cannot be regarded as an independent material characteristic - in contrast to the tensile strength f_t. In fact, since the oblique cracks open once the tensile stress reaches the local tensile strength, cohesion c is related to the tensile strength, such that:

$$c = \delta f_t \tag{12}$$

where δ is a coefficient of proportionality, assumed constant. This assumption implies that the coefficient of proportionality is independent of size effects and allows for its determination from experimental data, ($\delta = c / f_t \approx 5$, see Rossi et al., 1994b). With crack opening local tensile strength f_t and cohesion c are set to zero, and remain zero throughout the calculation (local irreversible fragile tensile behaviour). In other words, the strength is not recovered when the crack recloses, and only normal compression stresses are admissible. The friction between the two edges of the geometrical discontinuity created (edges of the crack) is taken into account by a cohesionless Mohr-Coulomb criterion reading:

$$|\tau| - \sigma_N \mathrm{tg}\varphi \leq 0 \tag{13}$$

with $\varphi = 45°$ the angle of friction. With respect to the local irreversible fragile tensile behaviour, friction is only activated when the element re-closes after opening. .

4.2 Concrete at early ages

Recently, some new developments of the model were realised to treat the problems of cracking due to thermo-chemo-mecanical and hydro-mechanical couplings (Ulm et al., 1995). In the first, crack criterion (8) reads

$$\sigma_N - f_t(\xi) \leq 0 \tag{14}$$

where $f_t(\xi)$ is the randomly distributed tensile resistance as a function of hydration degree ξ, modelling the chemical hardening phenomenon, as described in section 3.2. Furthermore, the normal stress reads now

$$\sigma_N = \mathbf{n}.\boldsymbol{\sigma}.\mathbf{n} = 2G(\xi)(\lambda - \varepsilon) + 3K(\xi)(\varepsilon - \alpha\theta - \beta\xi) \tag{15}$$

where $G(\xi)$ and $K(\xi)$ are the randomly distributed elastic material properties as a function of hydration degree ξ. More details about this extension of this probabilistic modelling of early age concrete cracking can be found in Ulm et al. (1995). Here, we only sought to show how a probabilistic modelling can be extended to account for coupling phenomena. In fact, with respect to the two-stage strategy proposed, the extensions need necessarily be done on both modelling levels, with respect to the information used (input) and requested (output). At present, some works are also done to account at both modelling levels for rate effects, in order to extent the analysis to high rate dynamics.

5 Conclusion

This paper re-opens the discussion on modelling strategies and proposes a two-stage modelling strategy for analysing the behaviour of concrete under various loading: continuous and discrete approach -*together*- form a powerful tool for analysing the non-linear behaviour of concrete structures, as far as the information used (input) and that requested (output) are at equal basis, i.e. having the same degree of pertinence. In this light, a continuous model based upon the concept of an representative elementary volume is used when global information are requested, while detailed information about crack pattern, crack-spacing, crack width etc. is obtained with a discrete probabilistic crack approach, which abandons the concept of a representative elementary volume. An example for each type of modelling is presented for quasi-static loading and concrete at early ages.

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