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MESH SENSITIVITY EFFECTS IN SMEARED FINITE ELEMENT ANALYSIS OF CONCRETE FRACTURE

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Abstract

Smeared crack finite element constitutive model based on the fracture energy concept is an accepted numerical tool for the finite element fracture analysis of concrete structures. Discrete cohesive cracks are simulated by means of material softening in continuum mechanics. Localization limiters must be introduced in order to avoid sensitivity to the finite element size and orientation. The influence of irregularities in discretization, using various localization limiters, is discussed in this paper and demonstrated on few examples. Practical recommendations for the numerical simulation of concrete fracture when using smeared crack models are drawn.

1 Introduction

Fracture is an important mode of failure in both plain and reinforced concrete structures. Two dominant techniques used in the finite element modelling of fracture are discrete and smeared approaches, respectively (ACI, 1995). The smeared crack model introduced by Rashid (Rashid, 1968) and Červenka (Červenka, 1970) has become the most widely used approach in practice, and will be discussed in this paper. The smeared crack model is computationally convenient and can handle single, multiple and distributed cracks in a unified manner. Thus, it can be used for both, plain and reinforced concrete structures.

Concrete fracture is modeled through changing material properties. In this process, an important phenomena is the development of a fracture process zone with subsequent crack localization, which is modeled by tensile strain softening of concrete. The fracture energy of concrete in combination with localization limiters must be employed in order to model properly this behavior. The strain localization may be accompanied by some unfavorable effects, when applied to finite elements. They are caused by the element size and shape, as well as by the orientation of the mesh with respect to cracks.

2 Localization limiters in smeared crack approach

After crack intialization strain softening of the material leads to a locally unstable stress-strain field in the finite element model, known as a crack band. The crack band represents a discrete crack within the continuum displacement field. In order to prevent the localization of strains into a zero volume a localization limiter must be introduced. In the present study, two types of localization limiters are investigated: (*a*) a refined crack band model and (*b*) a nonlocal continuum based on the microcrack interaction approach.

2.1 Crack band model

The crack band model was introduced by Bažant (Bažant and Oh, 1983) and is widely and succesfully applied. The localization limiter in this model is the crack band size L, which is used as a reference relation between the strain ϵ within the crack band (normal to the crack surface) and the crack opening w ($\epsilon = w/L$). The fracture energy G_f required for a unit crack propagation is the same in the crack band model and in the discrete crack approach.

The most important feature of the crack band model is that it can effectively handle the problem of mesh size sensitivity, provided the crack localizes in one element. However, the localization into one element is ensured only when the elements are rectangular in shape and the crack direction is parallel to the element side. In the general case of inclined cracks the crack band can extend over more than one element row, e.g. in diagonal shear failure of reinforced concrete beams. Therefore, some computer codes, for example DIANA, allow the specification of the crack band width as an input parameter.

However, if only one crack band width is considered for the whole structure, some problems may occure if the inclination of the crack varies.

The authors have refined the crack band formulation in the computer code SBETA (Červenka and Pukl, 1994). In this formulation the crack band size is derived as the projection L_b of the element dimension in direction of the crack. The method gives a smooth transition between cracks, which are paralell or inclined to elements. It is also valid for elements with irregular shapes.

It was found that elements with inclined cracks consume more energy corresponding to the projected size than elements with a crack perpendicular to the element sides. To consider this effect an orientation factor γ was introduced as $L = \gamma L_b$, where L_b is the projected size of the element and γ is a factor with a linear variation between 1.0 for cracks running perpendicular to the element sides and 1.5 for 45° inclined cracks. In case of irregular elements with nonparalell sides an average side direction is considered. This model was used in the present study.

2.2 Nonlocal continuum with microcrack interaction approach

A more general localization limiter offers the nonlocal continuum concept by spatial averaging of material state variables. It has been demonstrated recently (Bažant, 1994), that the nonlocal damage in fracture process has the physical meaning of interaction of growing microcracks. Therefore, a nonlocal approach based on microcrack interaction was introduced (Ožbolt and Bažant, 1995). In this model the opening of the microcrack at a certain place influences the microcrack development in a neighborhood called normalizing volume.

The normalizing volume is defined by the characteristic length L_{ch}, which is related to the concrete maximum aggregate size, and from this viewpoint is treated as a material property. In the finite element analysis of the continuum the characteristic length is also related to the finite element size, the dimensions of the structure and the strain field gradients. In particular the representative volume must be large enough for the finite elements size and small enough for the strain gradient field size. The experience shows, that the size of the normalizing volume should be on an average at least three times the element size. With a smaller element size the model is reduced to the local analysis (crack band approach).

The relation between the size of the normalizing volume and the strain gradients can be approximated by the characteristic dimension of the structure, such as the beam depth. According to the numerical experience, the size of the normalizing volume should not be larger than approximately 1/4 of the characteristic dimension of the structure.

3 Numerical simulations

The localization limiters described in Section 2 are based on quite different approaches. However, they serve the same purpose, namely, to reproduce the crack propagation in the continuum. The aim of this study was to evaluate and compare the effectivity of these limiters and to investigate their performance.

The subject is very important for the application of nonlinear finite element analyses in practice, which is shown in numerous international bench mark comparative projects (Elfgren, 1992; Bonnard & Gardel, 1994; Shirai, 1994). Therefore, the authors have performed an investigation to compare the effect of the finite element size, shape and orientation on the crack propagation in the concrete structure for the above described localization limiters.

Two computer codes were used in the study: the commercially available program SBETA (SBETA, 1992) based on the smeared crack model with the refined crack band approach described in Section 2.1, and the research program MPM by Ožbolt based on the nonlocal microplane material model with the microcrack interaction approach (Ožbolt and Bažant, 1995). They are applied to two cases of crack propagation in a plane stress situation: three-point bending and direct tension.

3.1 Finite element model cases

It was important to choose simple loading cases with clearly defined stress fields and with generaly high effects of mesh irregularieties. Such cases should be free of significant singularities in the uncracked state. Two loading cases were considered: three-point bending without notch and direct tension with a small imperfection.

A plain concrete prism with relative dimensions 100:20:12 was modeled in the numerical study. Similar specimens are often investigated by other authors (RILEM, 1989; Karihaloo, Carpinteri and Elices, 1993). For three-point bending specimens with dimensions 1000x200x120 mm, and for direct tension specimens with dimensions 333x66.7x40 mm were used.

The material properties in all cases were identical: initial modulus of elasticity $E_c = 34000$ MPa, Poisson's ratio $\nu = 0.2$, compressive strength $f_c = 35$ MPa, tensile strength $f_t = 3.0$ MPa and fracture energy $G_f = 100$ N/m.

Several types of finite element meshes were used with variable element shape and orientation. The basic regular mesh (A) was with 50x10 elements, i.e. the element side was 20 mm long. Double mesh density (mesh D) was also considered for three-point bending. The irregular mesh patterns (coarse B and C, fine E and F) were derived from the basic mesh patterns (see Fig.2).

3.2 Results

The peak loads obtained in the analyses are summarized in Table 1. The peak loads F_u are related to the peak loads obtained for the regular mesh ($F_{u,r}$). In the three-point bending analysis the deviation of the results shown in Table 1 is less than 5% for coarse meshes and less than 1.5% for fine meshes for both localization limiters. In the tension analysis the observed deviation is less than 1.5%.

Table 1. Summary of relative peak loads

localization limiter	finite element mesh		three-point bending		tension	
	density	pattern	$F_u/F_{u,r}$ [%]	deviat. [%]	$F_u/F_{u,r}$ [%]	deviat. [%]
crack band	coarse	A	100.0	–	100.0	–
		B	95.1	-4.9	100.9	+0.9
		C	98.1	-1.9	100.2	+0.2
	fine	D	100.0	–	–	–
		E	100.7	+0.7	–	–
		F	101.2	+1.2	–	–
non-local continuum	coarse	A	100.0	–	100.0	–
		B	96.7	-3.3	100.0	+0.0
		C	104.7	+4.7	101.4	+1.4
	fine	D	100.0	–	–	–
		E	99.7	-0.3	–	–
		F	99.9	-0.1	–	–

The load-displacement curves from the analyses are compared in Fig.1. The deviations at descending branch, caused by the irregular finite element mesh patterns, are larger in the case of crack band analysis than observed in the nonlocal analysis. In both approaches the mesh refinement reduces the scatter.

The crack patterns at termination of the analysis using the crack band model are shown in Fig.2. The cracks are depicted by lines oriented in crack direction. The line thickness is related to the value of the principal tensile strain, which represents the crack width. In

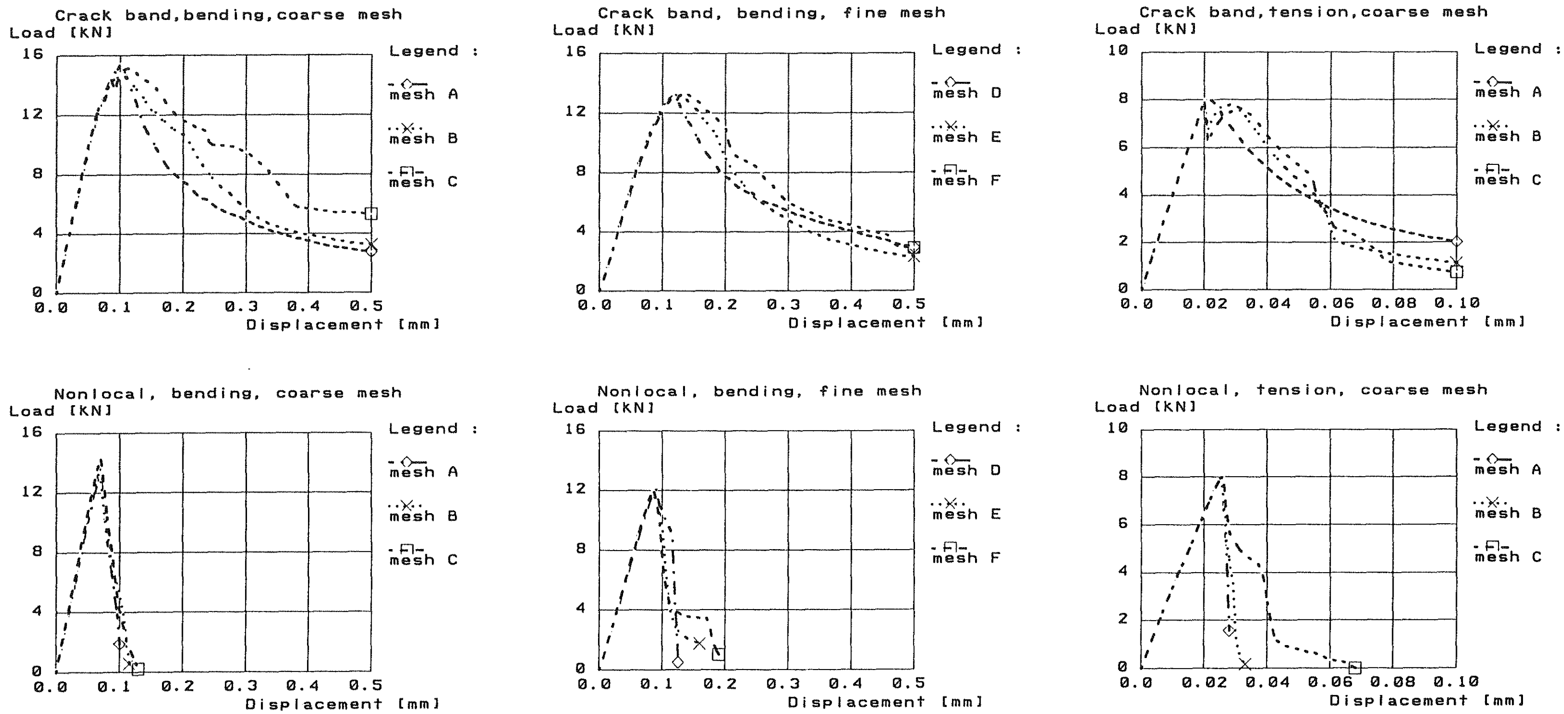

Fig.1 Comparison of load-displacement diagrams

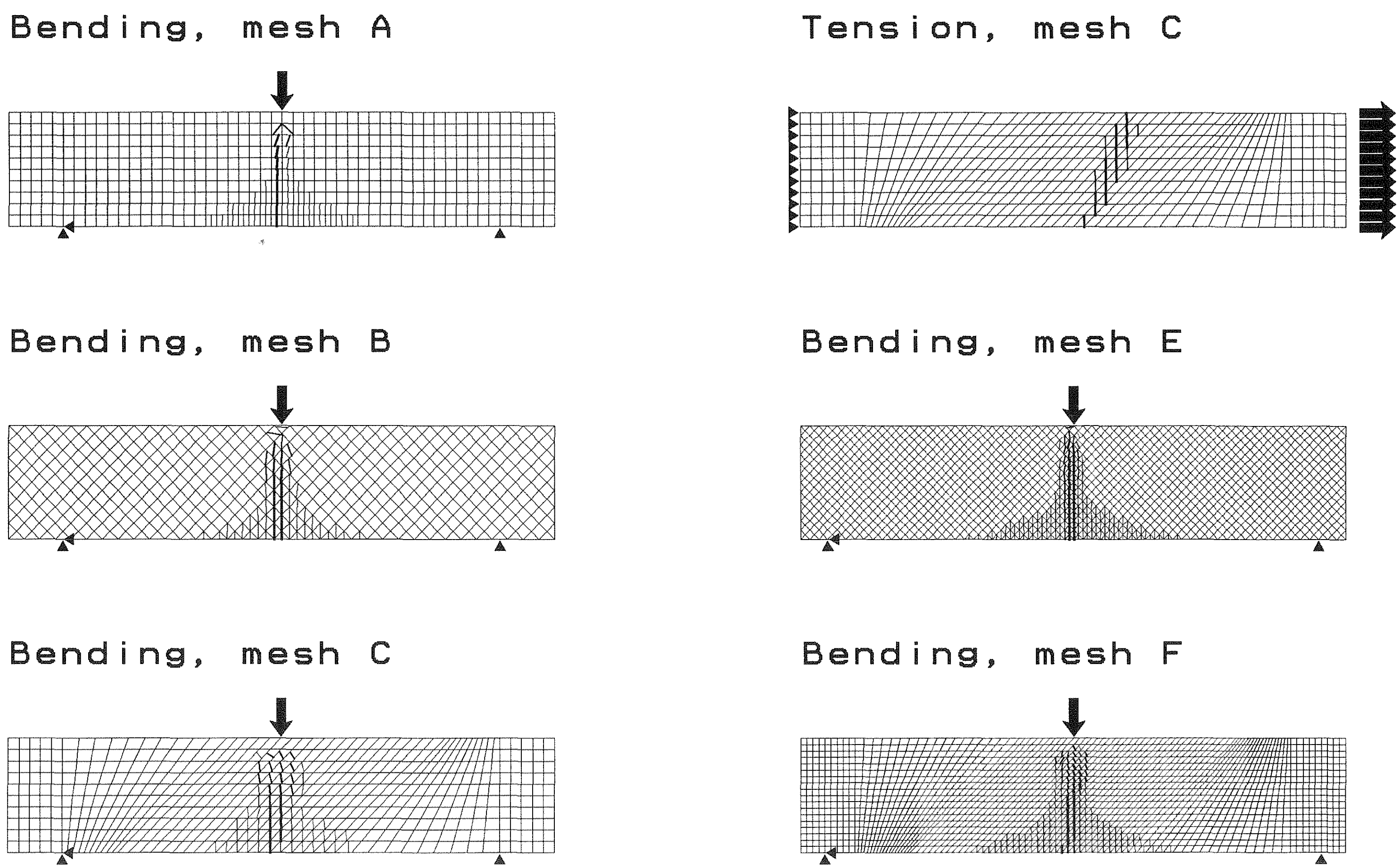

Fig.2 Crack patterns, refined crack band model

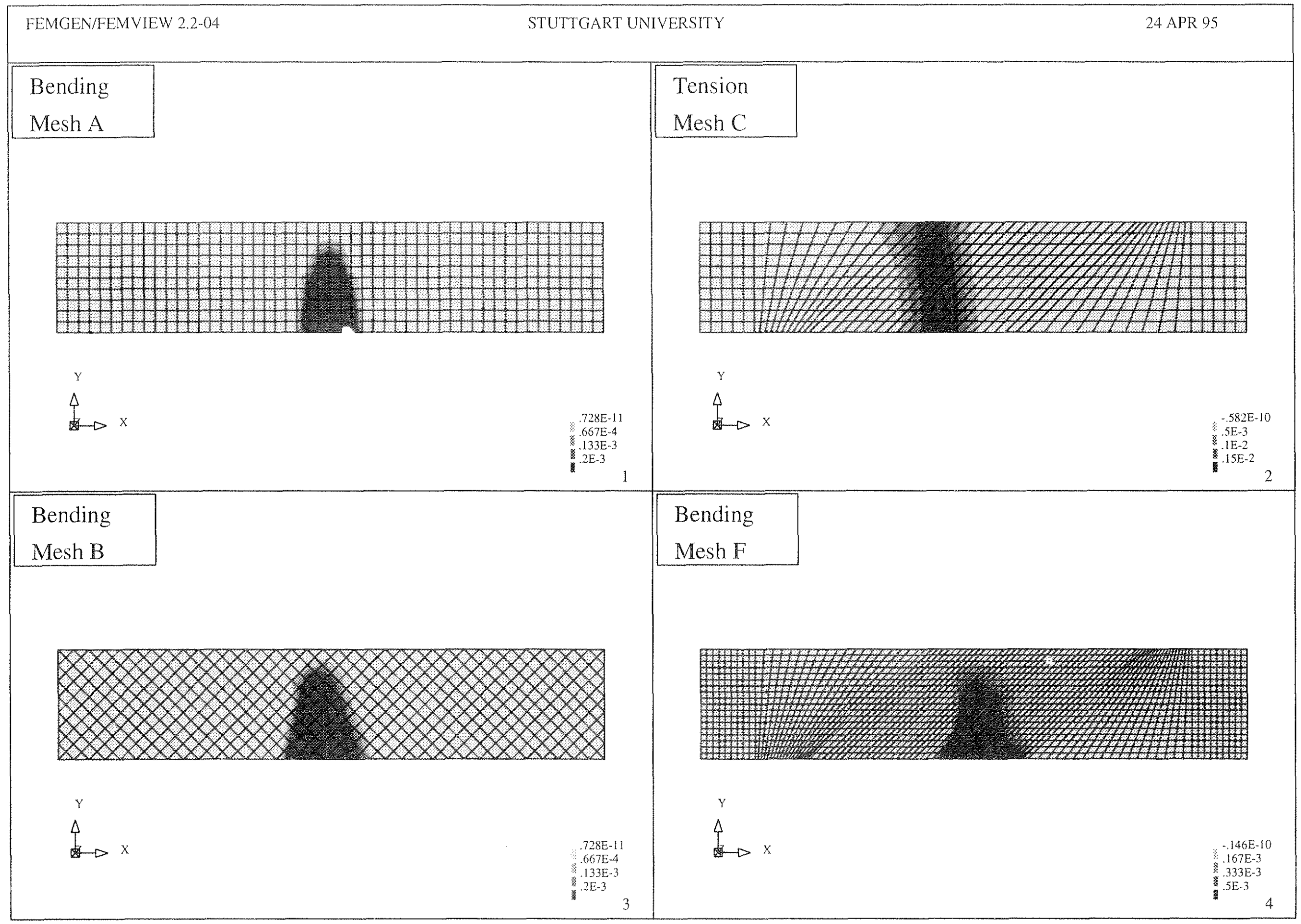

Fig.3 Crack patterns, nonlocal model with microcrack interaction

the irregular meshes, the crack propagates in the same direction as observed for the regular mesh, however, due to the irregularities it localizes in a wider region. Consequently, more energy for the crack propagation is consumed in the case of irregular meshes. Therefore, the descending branch of the load-displacement curves do not coincide with those obtained when using regular mesh, especially for the coarse meshes (Fig.1 - upper part).

The crack patterns obtained from the nonlocal analysis are shown in Fig.3. The cracks are plotted in terms of principal strains and are depicted with dark color. The mesh irregularities influence the crack development only negligibly. Therefore, the descending branch of the load-displacement curves obtained using the nonlocal concept are almost identical (Fig.1 - lower part).

4 Conclusions

- The effect of shape and orientation of finite elements on the results of a numerical analysis using a smeared crack approach can be significantly reduced if an appropriate localization limiter is utilized.
- The refined crack band model gives results which can be accepted in most practical cases. The advantage is an easy and transparent implementation.
- The nonlocal continuum model with microcrack interaction approach practically eliminates mesh sensitivity in the studied cases with fine meshes. Due to an averaging technique it reduces the size and orientation effects better than the crack band model on both, peak load and post-peak response.
- In practical applications the nonlocal concept with microcrack interaction approach can be used for relatively coarse meshes (similar to those in the crack band model) with acceptable results. The finer the mesh, the smaller the influence of the mesh orientation on the numerical results.

References

ACI (1995) Finite element analysis of fracture in concrete structures. **Report of ACI Committee 446 - Fracture Mechanics** (eds W. Gerstle and D. Darwin), Detroit, U.S.A.

Bažant, Z.P.(1994) Nonlocal damage theory based on micromechanics of crack interactions. **Journal of Engineering Mechanics**

ASCE 120 (3), 593-617; with Addendum, 120 (3), 1401-1402.

Bažant, Z.P. and Oh, B.H. (1983) Crack band theory for fracture of concrete. **Materials and Structures RILEM** 16, 155-177.

Bonnard & Gardel (1994) Benchmark on numerical analysis of concrete structures - Schweizerische Interessengemeinschaft zur Wahrnehmung gemeinsamer Interessen an der Entwicklung nuklearer Technologien, Neuchatel.

Červenka, V. (1970) Inelastic finite element analysis of reinforced concrete panels. Ph.D. Dissertation, University of Colorado, Boulder, U.S.A.

Červenka, V. and Pukl, R. (1994) SBETA analysis of size effect in concrete structures, in **Size Effect in Concrete Structures** (eds H. Mihashi, H. Okamura and Z.P. Bažant), E&FN Spon, London, 323-333.

Elfgren, L. (1992) Round robin analysis and tests of anchor bolts, in **Fracture Mechanics of Concrete Structures** (ed Z.P. Bažant), Elsevier Applied Science, London, 865-869.

Karihaloo, B.L., Carpinteri, A. and Elices, M. (1993) Fracture mechanics of cement mortar and plain concrete, in **Advanced Cement Based Materials** 1, Elsevier Science Publishing, 92-105.

Ožbolt, J. and Bažant, Z.P. (1995) Numerical smeared fracture analysis: Nonlocal microcrack interaction approach. Accepted for publication in International Journal for Numerical Methods in Engineering.

Rashid, Y.R. (1968) Ultimate strength analysis of prestressed concrete pressure vessels. **Nuclear Engineering and Design** 7, 334-344.

RILEM (1989) **Fracture mechanics of concrete structures - From theory to applications** (ed L. Elfgren). Chapman and Hall, London.

SBETA (1992) Computer program for nonlinear finite element analysis of reinforced concrete structures in plane stress state. Program documentation, Peekel Instruments, B.V., Rotterdam.

Shirai, N. (1994) JCI round robin analysis of size effect in concrete structures, in **Size Effect in Concrete Structures** (eds H. Mihashi, H. Okamura and Z.P. Bažant), E&FN Spon, London, 295-322.