FEM ANALYSIS OF TENSILE FRACTURE PHENOMENA IN CONCRETE STRUCTURES

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Abstract
In this research, formulation of the tensile fracture phenomena is conducted, and Finite Element Method (FEM) for a body containing displacement discontinuity is considered as a suitable method for the investigation of tensile fracture behavior in concrete structures. With the aid of the formulation and FEM, several factors that govern tensile fracture are examined, such as, the unloading path in the tension-softening behavior, and the transmission of shear stresses across crack surfaces. Special attention is paid to whether or not this model can reproduce the process whereby smaller cracks, which are initially generated uniformly in concrete, can join to form a single large crack, which keeps growing and opening. A plain concrete beam without a notch is analyzed to demonstrate the phenomena of cracking localization in Mode-I crack growth. Pullout test specimen of practical significance is considered to study the crack growth phenomena under mixed-mode loading conditions. The effect of shear transfer across crack surfaces on cracking localization is also examined.

1 Introduction

Research work in recent years has led to a better understanding of the mechanism of tensile fracture in concrete and identified the type of mechanical modeling capable of reproducing and predicting fracture phenomena (Hillerborg et al., 1976; Bažant and Oh, 1983; Nirmalendran and Horii 1992). Research on concrete fracture mechanics has now moved from the above initial and pioneering epoch to the second stage, which is aimed at the application of the above work into various engineering problems. The main tasks of current concrete fracture mechanics include: (a) the development of a rational model suitable for the analysis of tensile fracture under arbitrary loading and boundary conditions (see, for example, Rots, 1988; Hillerborg, 1989), (b) the implementation of the obtained knowledge into practical design codes (see, for example, CEB/FIP Model Code 1990, 1993; Walraven, 1994), and (c) the application of fracture mechanics to the development of new cementitious materials (see, for example, Li, 1993).

In this work, a rational method suitable for the analysis of tensile fracture phenomena in concrete structures under general loading and boundary conditions is established. Special attention is given to reproducing the process whereby small cracks, which are initially generated uniformly in concrete, can join to form a single large crack, which keeps growing and opening under increasing load. First, a plain concrete beam without a notch is analyzed by neglecting and taking into account the unloading path in the tension-softening behavior in order to investigate the phenomena of cracking localization in mode-I crack growth. Finally, pullout test specimen with lateral confinement (Lun, 1990) of practical significance subjected to mixed-mode loading conditions is analyzed to study the crack growth phenomena which is encountered in real structures, and to examine the effect of shear transfer during cracking localization. The numerical results obtained from the present analysis are compared with the available experimental results.

2 Model

The solid is assumed homogeneous and the behavior at any point is generally assumed to be isotropic linear elastic as long as the major principal stress is less than the tensile strength. The displacement discontinuity (crack) is formed when the maximum principal tensile stress level at the element center reaches the tensile strength in the direction normal to the maximum tensile stress. The tension-softening relation is employed upon the formation of crack with increasing crack opening displacement. The transferred stress is assumed to be zero for crack opening displacements exceeding a critical crack opening displacement. Secant unloading is assumed for crack closure phenomenon (see Fig. 1).

3 Method of analysis

3.1 Formulation

Mechanical behavior of tensile fracture phenomena (displacement discontinuities) is formulated by deriving an incremental formulation for a solid body containing an internal discontinuous surface. A detailed description of this formulation can be found elsewhere (see Ali, 1993). Considering incremental deformations small enough and assuming linear material behavior, an incremental formulation (similar to those of Dvorkin, 1990, and Wan, 1990) for the equilibrium of the cracked body in its final form can be written as

$$\int_{\Omega} \delta w_{i,j} \Delta\sigma_{ij} d\Omega = \int_{\Omega} \delta w_i \Delta f_i d\Omega + \int_{\Gamma_h} \delta w_i \Delta h_i d\Gamma + \int_{\Gamma^C} \delta w_i^C \Delta p_i^C d\Gamma \quad (1)$$

where $\delta w_{i,j}$ are associated incremental virtual strains, $\Delta\sigma_{ij}$ are incremental Cauchy stresses, δw_i are associated incremental virtual displacements, Δf_i represent incremental body forces, Δh_i are prescribed incremental body tractions, δw_i^C are associated incremental virtual displacement jumps, Δp_i^C are incremental surface tractions at the internal discontinuity surface.

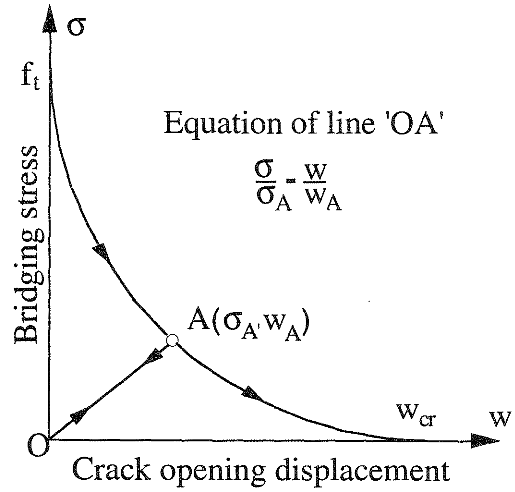


Fig. 1. Tension-softening relation with unloading path

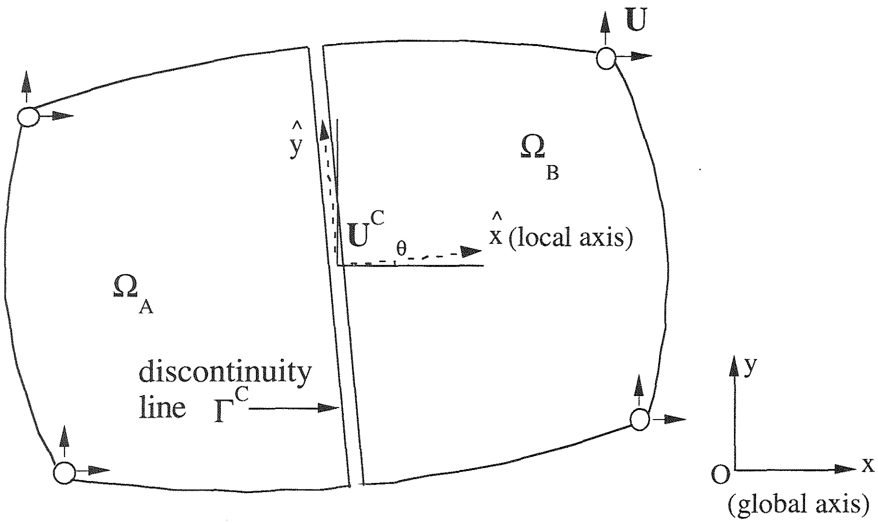


Fig. 2. Finite element with discontinuous line

3.2 Finite element equations

Following piecewise polynomial basis functions (Dvorkin et al., 1990) for a cracked element (see Fig. 2), the resulting displacement interpolation in compact symbolic notation can be written as

$$\Delta\mathbf{u} = \mathbf{N}\Delta\mathbf{U} + \mathbf{N}^{C}\Delta\mathbf{U}^{C} \tag{2}$$

where $\Delta\mathbf{U}$ is the ordinary vector of incremental nodal displacements, $\Delta\mathbf{U}^{C}$ represents additional degrees of freedom corresponding to the displacement discontinuity (crack), $\mathbf{N}$ and $\mathbf{N}^{C}$ are the associated matrices of interpolation polynomials respectively.

Making use of equation (1) and equation (2), after some algebra, the finite element equations for the 4-node cracked element (see, Ali, 1993), in general, can be written as

$$\begin{bmatrix} \int_{\Omega} \mathbf{B}^{T}\mathbf{D}\mathbf{B}d\Omega & \int_{\Omega} \mathbf{B}^{T}\mathbf{D}\mathbf{B}^{C}d\Omega \\ \int_{\Omega} \mathbf{B}^{CT}\mathbf{D}\mathbf{B}d\Omega & \int_{\Omega} \mathbf{B}^{C^{T}}\mathbf{D}\mathbf{B}^{C}d\Omega + \int_{\Gamma^{C}} \mathbf{E}d\Gamma \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{U} \\ \Delta\mathbf{U}^{C} \end{Bmatrix} = \begin{Bmatrix} \int_{\Omega} \mathbf{N}^{T}\Delta\mathbf{f}d\Omega + \int_{\Gamma_h} \mathbf{N}^{T}\Delta\mathbf{h}d\Gamma \\ \int_{\Omega} \mathbf{N}^{CT}\Delta\mathbf{f}d\Omega + \int_{\Gamma_h} \mathbf{N}^{CT}\Delta\mathbf{h}d\Gamma \end{Bmatrix} \tag{3}$$

where $\mathbf{B}$ and $\mathbf{B}^{C}$ are the associated strain-displacement matrices respectively, $\mathbf{D}$ is material stiffness matrix, $\Delta\mathbf{f}$ is the body force, $\Delta\mathbf{h}$ represents tractions on the cracked surface, and $\mathbf{E}$ is crack stiffness matrix. Additional degrees of freedom corresponding to $\Delta\mathbf{U}^{C}$ are condensed at the element level, and are assumed to be constant within the element.

3.3 Analysis procedure

The load increment analysis is conducted by controlling the load increment so that one element cracks or a change in slope of tension-softening relation occurs in one step. In this method, at every step, a unit load is applied as the testing load, and for every uncracked element, the exact amount of load increment necessary for cracking the element, or, for change in slope of tension-softening curve for the cracked element is established in advance (Ali, 1993). The minimum of the values is adopted as the load factor for the analysis. This process is continued until piecewise linear complete load-displacement response is obtained.

4 Analysis of results

4.1 Unnotched beam specimen

Cracking localization is the process whereby small cracks, which are initially generated uniformly in concrete, can join to form a single large crack, which keeps growing and opening under increasing load. In reality, the localization of crack propagation is observed and the beam fails due to one large crack, though such localization cannot be reproduced in conventional analysis. Whether many cracks are initiated at the initial stages or not is not yet clear. In fracture mechanics analysis, however, it is expected that cracking is first distributed in the beam, and is then localized into one dominant crack, which continues to grow and open with other smaller cracks being unloaded and closed. To investigate this phenomena, an unreinforced concrete beam without a notch as shown in Fig. 3 is considered. A uniform mesh is used all over

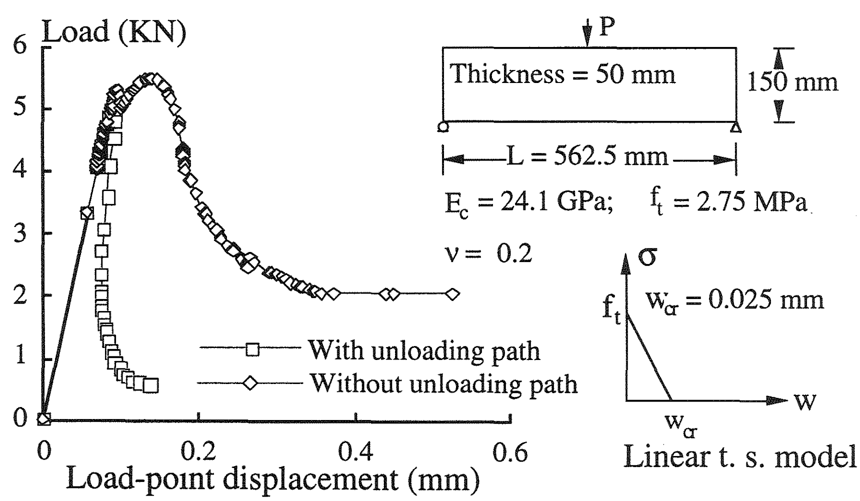


Fig. 3. Load-displacement response

the specimen. The finite element model comprises square four-node cracked plane stress elements (see Fig. 4). This research assumes that the existence of an unloading path in the tension-softening behavior is of essential importance (see Fig. 1), and compares the two cases where the unloading path is either allowed for or neglected. In the case, when the unloading path is neglected, the tension-softening relation is satisfied irrespective of whether the crack opens or closes. By incorporating this model in the program the analysis yields the response as shown in Fig. 3. The localization of cracking cannot be reproduced as depicted in Fig. 4. In the case, when the unloading path is taken into account (see Fig. 1), all cracks, except one at the center, enter unloading paths and start to close (see Fig. 5). The crack at the center continues to grow, and load-displacement curve shown in Fig. 3 is obtained. The difference of these two cases shows the importance of the unloading path in the tension-softening behavior. It is clear that the unloading path in the tension-softening behavior must be introduced in order to obtain a realistic behavior from the analysis, which leads to localization of microcracking and distinct softening. Hence, it might be concluded that one major reason why the localization of cracking was not reproduced in conventional analysis is the complete neglect of the unloading path or the poor judgment of the bifurcation condition at the initiation of unloading. Recently, Shirai (1994) demonstrated the importance of cracking localization by comparing the results from various investigations, and he

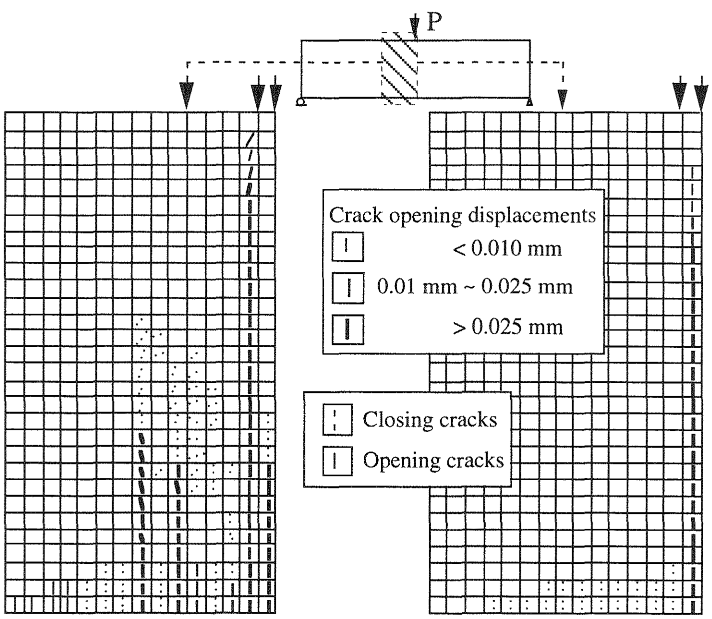


Fig. 4. Crack Pattern at final stage without unloading path

Fig. 5. Crack pattern at final stage with unloading path

showed that localization of microcracking exerts a significant influence on the response of beam specimens.

4.2 Pullout test specimen

To examine whether or not the proposed model for the mode-I fracture can reproduce crack growth phenomena under mixed-mode loading conditions, the pullout test specimen with lateral confinement tested by Lun (1990) was selected for the investigation (see Fig. 6). The finite element mesh represents only half the specimen, utilizing the symmetry of the structure and the stress state. The load-displacement response established from the present analysis along with experimental data is shown in Fig. 6. In the present analysis, emphasis is not placed into predicting the correct value of the peak load since a material model describing the non-linear response of concrete in compression is not included in the program. Shirai (1994) compares the experimental load-displacement response with the numerical results from many investigations, and it is shown that most of the predicted responses from numerical analysis do not agree with the test results. There is a significant variation even in the experimental results from different sources.

Crack pattern obtained from the present analysis at the final loading step is shown in Fig. 7. Many cracks form and open, eventually close and open again. At this moment, however, it cannot be concluded that the tensile fracture phenomenon under mixed mode loading conditions is exactly reproduced from these results since the localization of microcracking which occurs in mode-I loading condition is not obtained. In actual experiments, such localization of microcracking which is initially distributed is not identified. More accurate measurements are required for the observation of cracking.

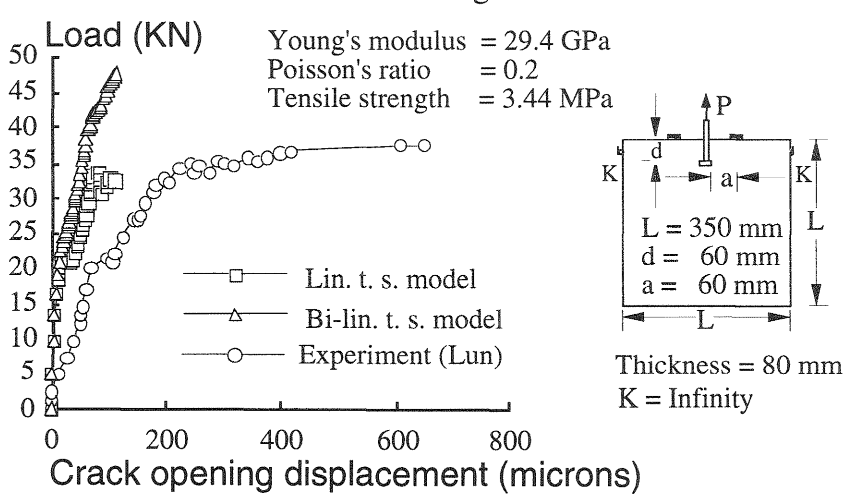


Fig. 6. Load-displacement response

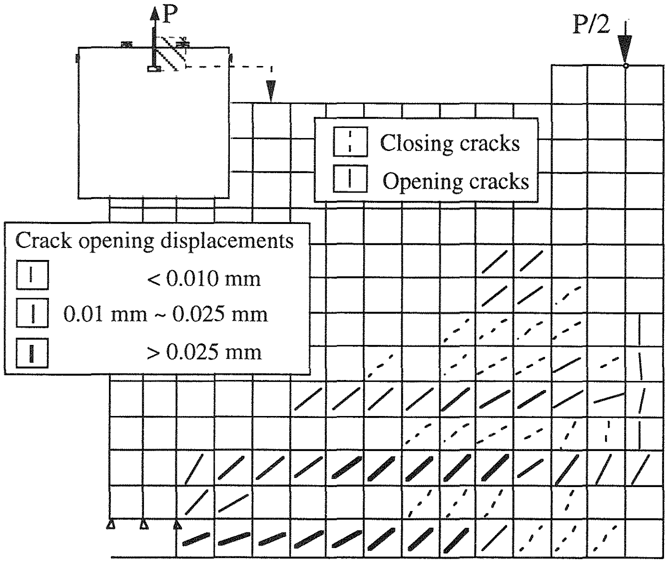
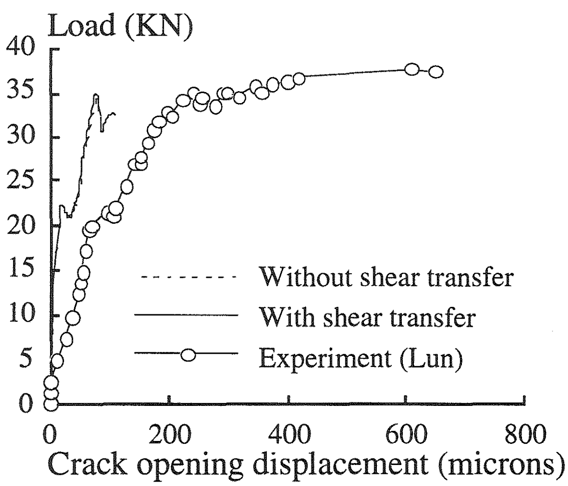
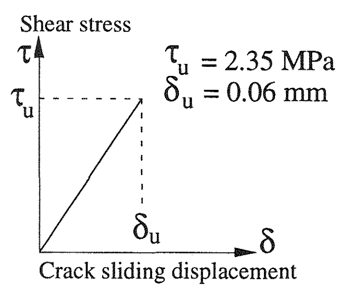


Fig. 7. Crack pattern at final loading step

By inspection of the numerical results obtained from the analysis of the pullout test specimen (Fig. 6), it is noted that crack sliding displacements begun to exist after the formation and opening of cracks. However, the size of such displacements is not significant in this analysis. The displacement paths obtained from numerical analysis were compared with those used by Hassenzadeh (1989) in his experiments, and a simple model for shear transfer across the crack surfaces is established as depicted in Fig. 8. By incorporating this model in the program the analysis yields the predictions as shown in Fig. 8. The load-displacement





Model for shear transfer

Fig. 8 Load-displacement response

response and the cracking pattern in this case are almost similar to those without considering shear transfer across the crack surfaces. It is concluded, therefore, that the transmission of shearing stresses across crack surfaces does not appear to affect cracking localization in this analysis.

5 Conclusions

It is shown that unloading path in the tension-softening behavior of concrete plays a key role in the localization of microcracking in mode-I loading condition. The use of an unloading path in the tension-softening behavior was insufficient to describe cracking localization under mixed-mode loading conditions. Further investigation is required to clarify whether or not the proposed model for the mode-I fracture can reproduce tensile fracture phenomena under mixed-mode loading conditions. The present work also established that the influence of shear transfer during crack growth under mixed-mode loading conditions is not significant. However, in order to arrive at a general conclusion, further investigation is required using different practical problems.

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