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F. E. ANALYSIS OF SFRC T-GIRDERS SUBMITTED TO SHEAR: A COMPARISON WITH EXPERIMENT

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Abstract

Two types of FE analysis are used in this paper and compared with experiments from a practical point of view, i.e. in terms of information obtained by each method. The first one is based on a continuous approach. Considering SFRC, a plastic model allows to model the tensile behaviour after cracking, with a constant plateau of the level of the 'residual' tensile strength. The second one is based on a discrete model, taking into account the heterogeneity of the matter by a probabilistic approach. The model describes cracks as geometrical discontinuities using special interface elements. This approach is adapted to SFRC using the results of uniaxial tensile tests: a force depending on the normal displacement acts between the interface elements. The two approaches appear to be complementary.

1 Introduction

It is important for the engineers, in the case of complex loading, to know performance and limit of finite elements (FE) methods available. Load capacity of a structure is the first result that a model is expected to give, but the big advantage of FE methods is their ability to predict global deformations as well as local state of strain. A lot of continuous approaches (elastoplasticity, damage, smeared crack ...) can lead to good results concerning the global behaviour of reinforced concrete (RC) structures. Nevertheless, these modelling are not sure to be able to answer the question of the life span of structures. In fact life span of structures if closely linked to its cracking state and it is difficult to get the crack opening from a smeared state of irreversible strains. A discrete modelling, that accounts for cracks as geometrical discontinuities, has been developed to analyse the local behaviour of concrete structures. To predict the cracking pattern - length, width, orientation - it treats the heterogeneity of the matter by a probabilistic approach concerning elastic properties and cracking criteria respectively.

A lot of researches and developments have been carried out on steel fiber reinforced concrete (SFRC) during the last thirty years, but fibers are still rarely used in truly structural applications. This fact is mainly due to the lack of accurate analysis methods and some steps have been made concerning design procedures - Casanova and Rossi (1995a). One of the big field of applications of SFRC should be the control of crack opening, in conjunction with conventional steel reinforcement, in order to limit penetration of aggressive agents.

Starting from an accurate mechanical characterization of SFRC, the two types of FE methods - continuous and discrete - are used in this paper to quantify the structural contribution of fibers. Comparing with large scale experiment, the discrete probabilistic approach leads to realistic results concerning the cracking pattern. These results are quite promising in optimizing the design of a structure considering both global and local behaviours.

2 Mechanical behaviour of SFRC

It is shown that steel fibres, in lengths greater than 20 mm and diameters greater than 0.4 mm, and in currently use proportions, less than 1.5 % in volume, mainly act after the cracking of concrete - Rossi et al. (1987). For the modelling it is then the non linear domain which is important for the analysis of SFRC structures. A crack appears as soon as the main tensile stress reaches the tensile strength of the matrix. Then the fibers act by bridging the crack. The capacity of fibers for carry loads is directly related to the crack opening. To characterize this post crack behaviour uniaxial tensile tests can be performed - Casanova and Rossi (1995a).

3 Continuous modelling : elastoplasticity

It is assumed that a continuous approach may be used if thin cracks are considered. SFRC is modelled as an elasto-perfectly plastic material in tension, with a 'post-cracking' plateau of the same level as the tensile strength, σ_f. The three parameters Willam-Warnke criterion (fig. 1) - Willam and Warnke (1975)- is used in the FE-program CESAR-LCPC.

The main difficulty is the definition of σ_f, because the post-cracking stress is a function of the crack opening, w. An iterative procedure has been used. It consists, first, in defining a 'limit' value of the crack opening, w_m. This value should agree with the hypothesis of continuity and thus be small with respect to the structure. σ_f is then calculated as the mean value of the stress for $w \in]0 ; w_m]$ (fig. 2). A FE calculation is carried out using this value. It leads to permanent strains which are interpreted as cracking. The difficulty is now to convert these strains into crack opening. This is achieved by multiplying the maximum permanent strain by a 'reference length'. It has been shown - Casanova 1995b - that, considering bending and shear behaviour of a beam, this length is related to the height of the structure. The calculated crack opening is compared to the limit value, w_m, and the FE calculations end when both value are roughly of the same order.

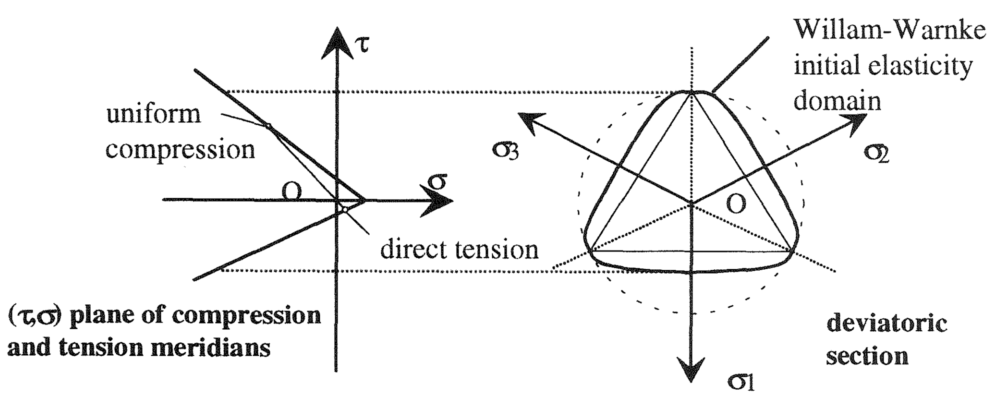

Fig. 1. Willam - Warnke criterion

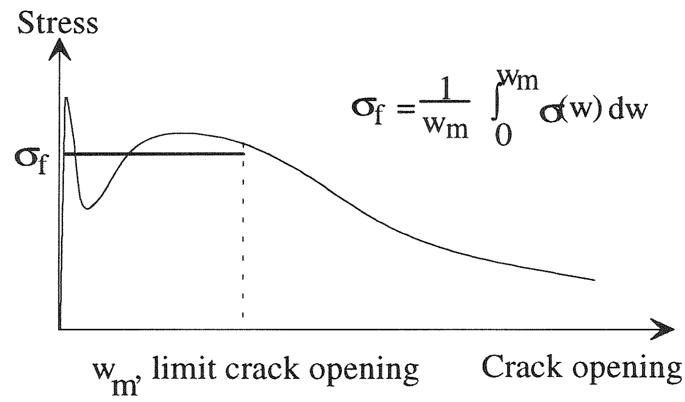

Fig.2. Definition of the tensile post-cracking strength

4 The LCPC's probabilistic model

4.1 Cracking of plain concrete

Concrete, because of its composite structure and the phenomena taking place during hardening, is a highly heterogeneous material. Its local characteristics -Young modulus and tensile strength- are randomly distributed and depend on the scale of observation. As a result, cracking of concrete takes place randomly and discontinuously. Based on this physical reality, a numerical model using special contact finite elements has been developed - Rossi and Wu (1992). In a structure, modelled with massive elements, Young moduli are randomly distributed using a distribution function depending on the elements' volume. Contact elements are placed at each interface of the massive elements. The cracking criterion is reached when the normal tension in the element is equal to a critical value randomly distributed (fig. 3). The random distribution is based on the distribution, the parameters of which -mean value and standard deviation- depend, through a power law, on the compressive strength of concrete and the ratio of the volume of the biggest aggregate by the volume of the specimen - Rossi and Wu (1994).

4.2 Modelling of SFRC

Modelling of SFRC is achieved by introducing, at each Gauss's point of an opened special contact element, a stress which is a function of the normal displacement calculated at each point (fig. 3). The numerical scheme is then an 'initial stress' one in which the residual is the difference between the elastic stress and the imposed stress due to the fibers.

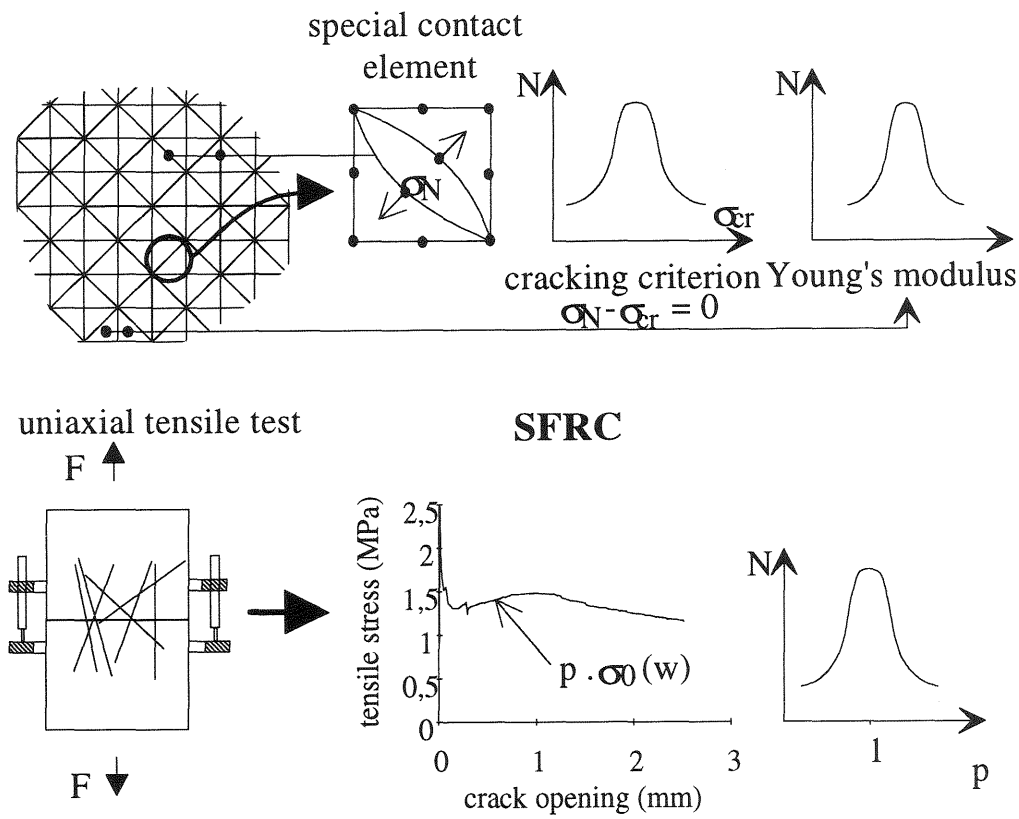

Fig. 3. The LCPC's probabilistic model

In order to take into account the heterogeneity of the fiber distribution, the mean tensile behaviour (σ_0)is weighted by p. This factor follows a truncated gaussian law the mean of which is 1 and the relative scattering of which is equal to the relative scattering of the area below the experimental tensile curve.

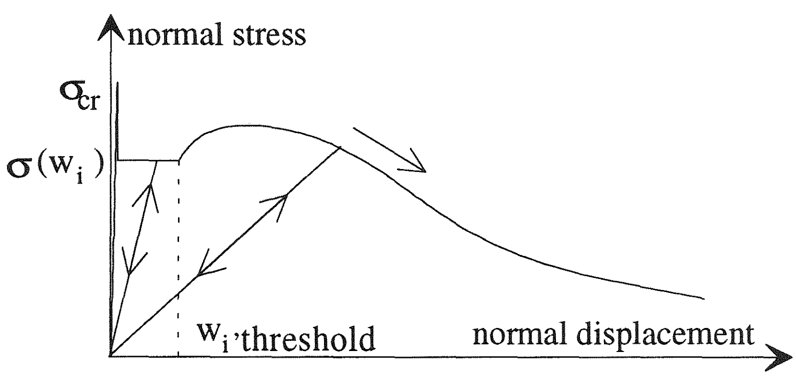

Fig. 4. Evolution of the stress as a function of the normal displacement

Fig. 4 illustrates the evolution of the post cracking stress with the variation of the crack opening. Crack opening w_i is the threshold which corresponds to a crack completely propagated in the section of the specimen tested in tension. Before this crack opening, the propagation of the crack leads to bending in the specimen and the measured force does not truly corresponds to uniaxial tension.

Each increment is considered as a physical equilibrium state. Thus, if, during an ulterior increment, the normal displacement of the element decreases, the normal stress decreases linearly. If the normal displacement increases, the normal stress is calculated following the evolution law of the element (fig. 4).

5 Experimental program

Predictions of the presented FE modelling are compared with centred flexion tests carried out on large scale double-T girders -6.2-m long, 0.8-m high and 0.15-m web thick- designed to fail in shear. The goal of this experimental program is to compare steel fibres with classical stirrups [4]. The SFRC used contains 100 kg/m^3 of hooked-end steel fibers - 60-mm long, dia. 0.8 mm. The longitudinal reinforcement is made of 8 rebars -dia. 40 mm, yielding stress equals 580 MPa. The mean 28-days compressive strength on cylinders is 40 MPa. Uniaxial tension tests have been performed (fig. 5) on specimens cored in a piece of girder cast in the same conditions as the SFRC girder.

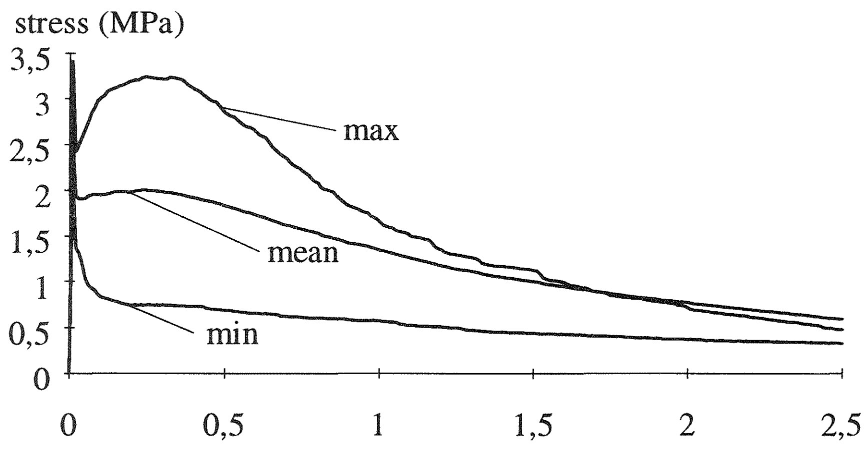

Fig. 5. Mean diagram and scattering (9 uniaxial tension tests)

The girder is asymmetric in order to concentrate the investigation tools in the area where the failure is supposed to occur. A global investigation tool, stereophotogrammetry [6], is used to measure the whole cracking pattern. This method gives a 50 μm precision on normal and transverse displacement. It shows that normal displacements are roughly 5 times higher than transverse displacements when the crack opening is greater than 0.1 mm.

6 Modelling vs experiment

6.1 Plastic model

Fig. 6 shows the mesh and limit conditions used for the computation. It is assymetric because of the geometry of the girder. The tensile strength σ_f (1.9 Mpa) is calculated using the mean diagram of fig. 5 according to fig. 2. Fig. 7 shows the distribution of permanent strains. Some areas of strain concentration appear inclined at 30° in the thinnest web of the girder -0.15 mm- and may be interpreted as macrocracks. The maximum permanent strain in these areas is 1 ‰ that, multiplied by the 0.8h, corresponds to a crack opening of 0.5 mm which agrees with the value of the plateau. The global rigidity of the girder is well predicted (fig. 8) and other comparisons show that this model give good results as far as 80 % of the ultimate load.

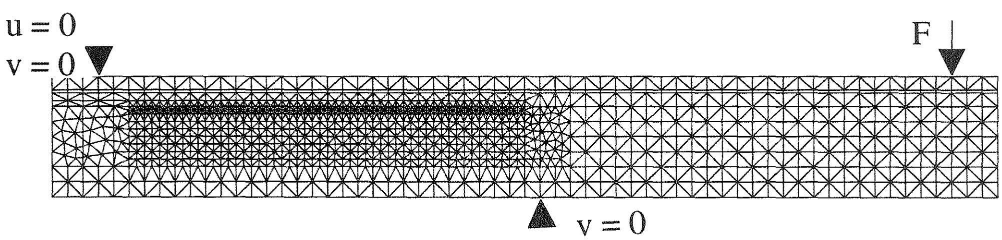

Fig. 6. Mesh and limit conditions

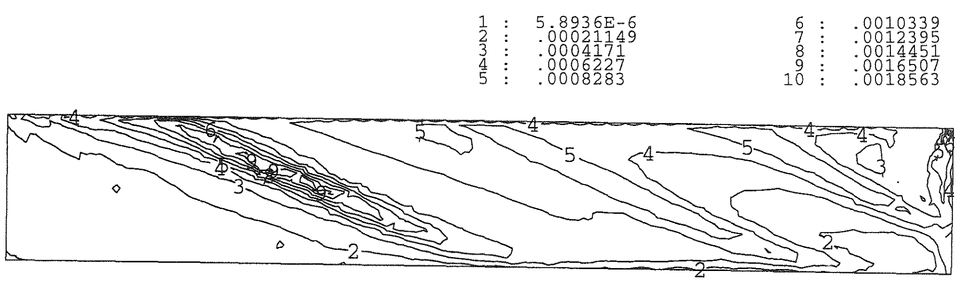

Fig. 7. Permanent strains in the web (central-left part of the mesh)

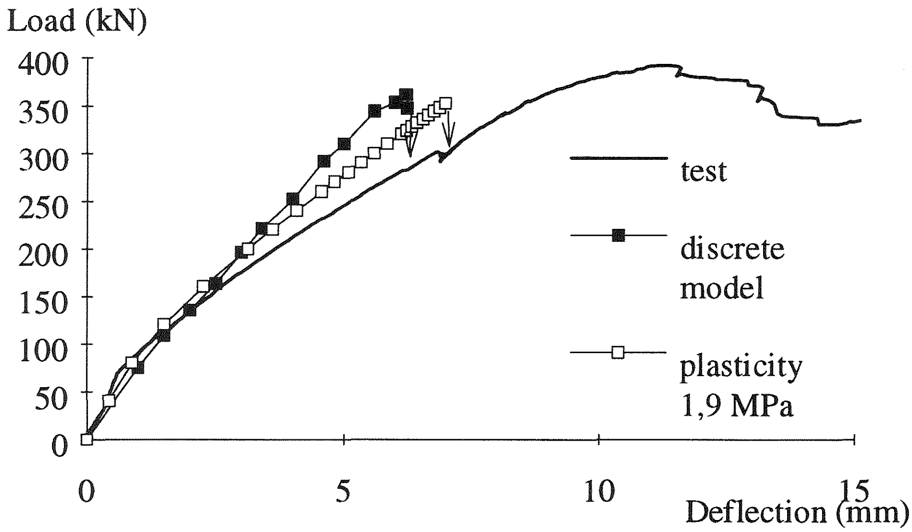

Fig. 8. Load - deflection diagram : modelling vs experiment (CPU : plasticity 6 h. SUN SPARC 10 station; discrete model 48 h. IBM SP2)

6.2 Discrete probabilistic model

Only the left-hand part of the girder is modelled using the special contact elements. The other side is modelled using elastic elements. It explains that the rigidity of the girder is overestimated (fig. 8). Figure 9 shows that the cracking pattern is well predicted in terms of number of cracks, spacing and opening.

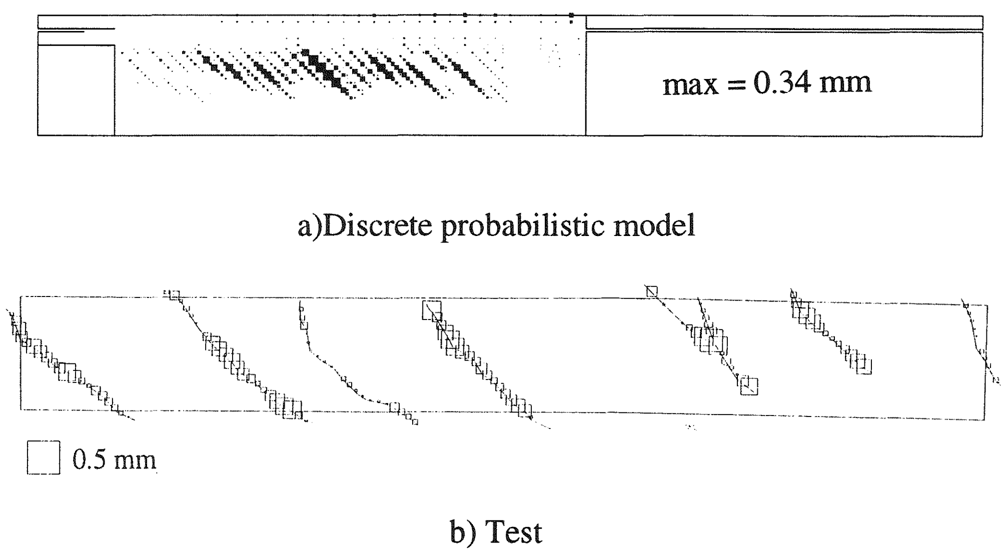

Fig. 9. Cracking pattern of the SFRC girder at a load of 360 kN

The load peak corresponds to the onset of horizontal cracks along the flanges. In spite of an imposed displacement scheme, it was not possible to calculate the post peak behaviour. It is partly due to the resolution scheme chosen for SFRC. An 'initial stresses' scheme is used and lead to numerical oscillations. Another part corresponds to a limit of the 2D modelling because the experiment shows 3D cracks which lead to aggregate interlocking.

7 Conclusion

Continuous modelling leads to a correct evaluation of the maximum load, the deflection and the damage areas in relatively short computation time. Nevertheless, it does not give the cracking pattern that is precisely of major concern considering the serviceability of structures. The use of the probabilistic discrete model leads to satisfactory results considering this cracking pattern as well as the load bearing capacity. This model gives complementary and new important information to the engineer in order to optimize the design of structures. The cost is a higher computation time.

8 References

Casanova, P. and Rossi, P. (1995a) Analysis of SFRC structures, in **Fiber Reinforced Concrete Modern Developments** (eds N. Banthia and S. Mindess), UBC, Vancouver, Canada, 335 - 349.

Casanova, P. (1995b) Bétons renforcés de fibres métalliques : du matériau à la structure. PhD thesis, ENPC, France.

Rossi, P., Acker, P. and Malier Y. (1987) Effect of steel fibres at two different stages: the material and the structure. **Materials and Structures**, 20, 436 - 439.

Rossi, P. and Wu, X. (1992) Probabilistic model for material behaviour analysis and appraisement of concrete structures. **Magazine of Concrete Research**, 44 (161), 271-280.

Rossi, P., Wu, X., Le Maou, F. and Belloc, A. (1994) Scale effect on concrete in tension. **Materials and Structures**, 27, 437 - 444.

Torrenti, J. M., Desrues, J., Benaija, E. H. and Boulay, C. (1991) Stereophotogrammetry and localization in concrete under compression. **Journal of Engineering Mechanics**, 117, 1455 - 1465.

Willam, K.J. and Warnke, E.P. (1975) Constitutive model for the triaxial behaviour of concrete. **IABSE Proc.**, 19, Bergamo, Italy, paper III-1.