Fracture Mechanics of Concrete Structures,
Proceedings FRAMCOS-2, edited by Folker H. Wittmann,
AEDIFICATIO Publishers, D-79104 Freiburg (1995)

APPLICATIONS OF LINEAR ELASTIC FRACTURE MECHANICS TO REINFORCED AND PRESTRESSED CONCRETE STRUCTURES

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Abstract
Starting from previous studies, the report deals with the application of Linear Elastic Fracture Mechanics to reinforced and prestressed concrete structures. A particular correlation is established between the critical bending moments of a beam in plain, reinforced and prestressed concrete. For these last structures, are considered both the cases of unbonded and bonded reinforcement.

1 Introduction

The author had published a paper on the application of Fracture Mechanics to reinforced concrete structures submitted to bending moments, in the hypothesis of High Strenght Concrete (HSC) -C70/80, whose high brittleness makes it suitable to be treated, at least in first approximation, within LEFM.

Starting from the papers quoted in bibliography (Okamura, Watanabe, Carpinteri and others), we consider a rectangular cross section (b x d see fig. 1) of a bent beam as composed by a concrete uncracked section (delimited by the tip of a crack of depth a and submitted to a compression force) and by a steel section (reinforcement submitted to a tensile force).

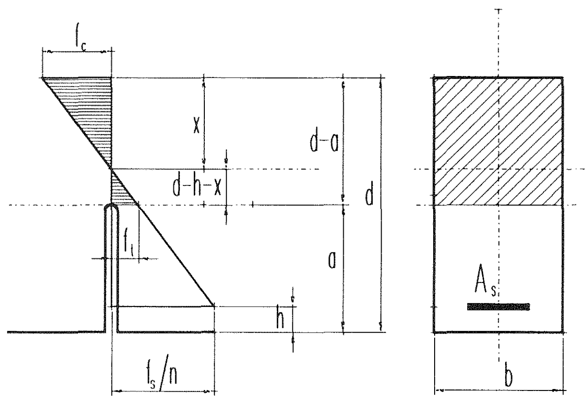

Fig. 1. Stress diagram on a cracked cross section of a reinforced concrete beam

The total external moment, M, was considered from the superposition of a couple formed by the tensile force F in the reinforcement and by a compression force, always F, in the concrete cross section located at the half depth of the beam and equal to tensile force in reinforcement, plus a bending moment M_1 (fig.2) according to:

$$M = F\,(d/2 - h) + M_1. \tag{1}$$

At the crack tip, K_I (the stress intensity factor) is given by superposition of the effects of M_1 and F:

$$K_I = \frac{M_1 Y_m}{bd^{3/2}} - \frac{F Y_p}{bd^{1/2}} \tag{2}$$

where Y_m and Y_p are known functions of ratio a/d (depth of the crack/ total depth of the beam).

Another information is requested in order to know the distribution of the total moment in the two shares according to (1).

The adopted hypothesis resulted from the planarity of cross sections and the linear elastic distribution of stresses in steel reinforcement and in the uncracked concrete section (delimited by the tip of the crack, where the critical value K_{IC} is reached).

In conclusion, with reference to the fig. 1, a dimensionless formula for the total critical moment was established:

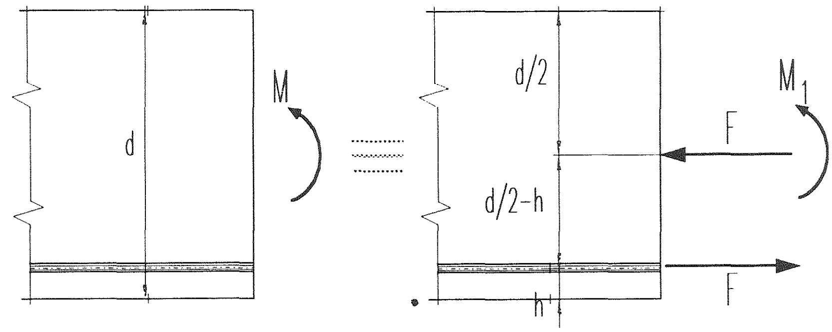

Fig. 2. Distribution of moments in cracked cross section of a reinforced concrete beam

$$\overline{M} = \frac{M_{cr}}{K_{IC} b d^{3/2}} = \frac{1}{Y_m - \frac{n\mu(1-h/d-x/d)}{J/(bd^3)}\Psi} \qquad (3)$$

where: $x = \frac{(d-a)^2 + 2n\mu d(d-h)}{2(d-a+n\mu d)}$

(distance of neutral axis from the upper edge of the cross section)

$$J = \frac{bx^3}{3} + \frac{b(d-h-x)^3}{3} + n\mu bd(d-h-x)^2$$

(moment of inertia of active section)

having assumed:

$$\Psi = Y_p + (1/2 - h/d)\Psi_m$$

depending from parameters a/d and $n\mu$ (where $n = E_s / E_c$ and $\mu = A_S / bd$, i.e. reinforcement ratio).

The main results of this research are summarized in the diagrams of fig.3, where for any crack depth and for any value of the product $n\mu$, the values of corresponding critical moments are drawn.

It was possible to establish the minimun reinforcement ratio (horizontal branch of the diagram), showing a steady cracking for $n\mu = 0.01$, i.e. $\mu = 0.15\%$ for $n = 6$. This result, in agreement with EC2, has been obtained without any hypothesis on the values of the involved parameters.

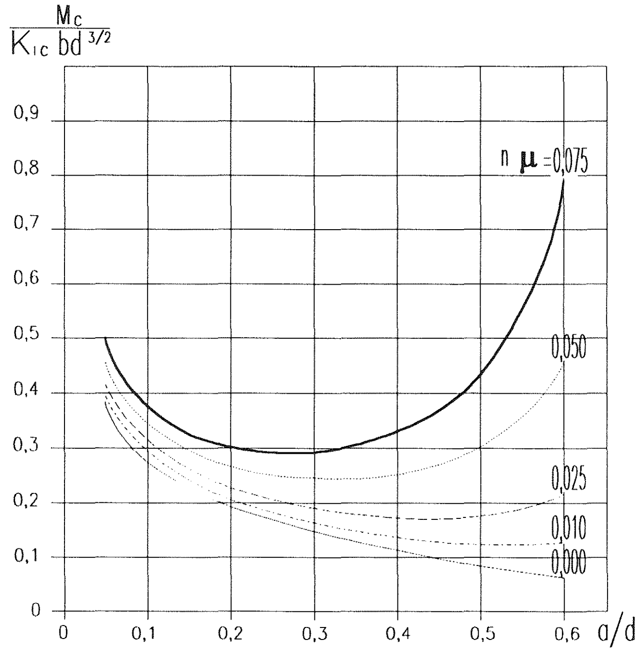

Fig. 3. Diagrams of critical moments versus crack depth for reinforced concrete beams

2 Prestressed unbonded structures

The formulations of Fracture Mechanics prove to be very simple, not requiring supplementary hypotheses about the stresses in the reinforcement, in the case of prestressed unbonded structures, where the prestressing force can be considered constant, not depending on external loads. So the arbitrary hypothesis of planarity of cross section and elasticity for the stresses in the reinforcement, as assumed in the previous study for the reinforced concrete, is no longer necessary.

Always with reference to a rectangular cross section according to fig.1, we suppose F be the prestressing load applied by unbonded tendons, located at distance h from the lower edge, and M the external moment; the effect of M and F is equivalent to a force F located at the half depth of the beam and to a moment M_1 according to (1), so that K_I is still

$$K_I = \frac{M_1 Y_m}{bd^{3/2}} - \frac{F Y_p}{bd^{1/2}}$$

where now F is known and constant.

Inversely the total (dimensionless) moment $\overline{M}$ can be written:

$$\overline{M} = \frac{M_{cr}}{K_{IC} bd^{3/2}} = \frac{1}{Y_m}\left\{1 + \frac{F\Psi}{K_{IC} bd^{1/2}}\right\} \qquad (4)$$

The total moment $\overline{M}$ can be drawn versus a/d, assuming as a parameter the quantity $F/\left(K_{IC} bd^{1/2}\right)$.

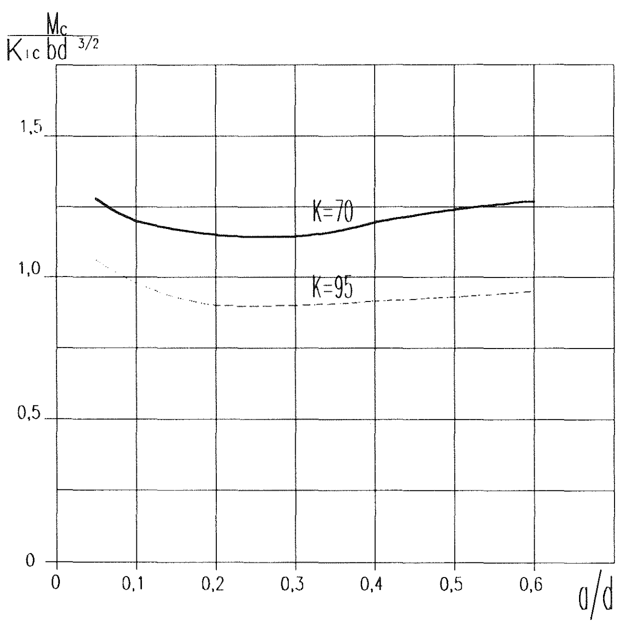

Fig. 4. Critical moments for unbonded prestressed concrete beams (2 types of concrete: K_I = 70-90 N $mm^{-3/2}$)

The diagrams of fig. 4 are drawn, with reference of actual cases, in these assumptions (using units N, mm):

- prestressing force F = 225000
- rectangular cross section with d = 300, b = 100, h = 60, (fig.2)
- two types of concrete with K_{IC} respectively 70 and 95.

The stresses induced by the only prestressing are as follows (positive value are compression):
- upper edge -6
- lower edge +21

Supposing a moment M_u (induced by a live load) of 33000 KN.mm, we obtain the total stresses as follows:
- upper edge +16
- lower edge -1.

The flat shape of the curves indicates values of M_{cr} little influenced by a/d, practically constant in the range a/d = 0.1 - 0.5. In the two examined cases the minimum values are comprised between 42000 - 44000 KN.mm, also little influenced by K_{IC}.

The ratio $M_{cr} / M_u \approx 1.3$ could assume the meaning of a cracking ratio; the value is the same as imposed by italian recommendations for prestressed concrete.

3 Prestressed bonded structures

The case of the prestressed concrete (with bonded tendons) can be treated by superposition of the results of reinforced and prestressed (unbonded) concrete, always within the hypothesis of linear elasticity for the uncracked cross section of the concrete. At the tip of the crack, K_I, due to the unbonded prestressing, is expressed by:

$$K_I = \frac{M_1 Y_m}{bd^{3/2}} - \frac{FY_p}{bd^{1/2}}$$

where is

$$M_1 = -F(d/2 - h)$$

and then

$$K_I = -\frac{F\Psi}{bd^{1/2}}$$

These values add up to the value of K_I due to the loads, according to the (2) and (3) already written in the case of reinforced concrete for an external moment M. In fact we suppose that the prestressing reinforcement, injected and bonded, could collaborate to bear the bending moment acting after prestressing and injecting of the tendons. By superposition of the two effects, we can write:

$$K_{IC} = \frac{1}{bd^{3/2}} \left\{ M \left[Y_m - \frac{n\mu(1 - h/d - x/d)\Psi}{J/(bd^3)} \right] - Fd\Psi \right\}$$

whence we obtain the value of the critical moment:

$$\frac{M_{cr}}{K_{IC} bd^{3/2}} = \left\{ 1 + \frac{F\Psi}{K_I bd^{1/2}} \right\} \frac{1}{Y_m - \frac{n\mu(1 - h/d - x/d)\Psi}{J/(bd^{3)}}}$$

This expression can be written also in the form:

$$\overline{M} = \frac{M_{cr}}{K_{IC} bd^{3/2}} = \overline{M}_{unb} Y_m \overline{M}_{ca} = \frac{\overline{M}_{unb} \overline{M}_{ca}}{\overline{M}_o}$$

where $\overline{M}_{unb}$ is the corresponding moment of the unbonded prestressed cross section, $\overline{M}_{ca}$ is the relevant moment of a reinforced concrete cross section, Y_m is the known dimensionless function which has the meaning of the inverse of the critical moment of a plain concrete cross section $\overline{M}_o$ according to the (3) with $n\mu = 0$.

A calculation has been carried out always on the same rectangular cross section, with the same reinforcement ($n\mu = 0.05$), comparing bonded and unbonded prestressed beams (see fig.5).

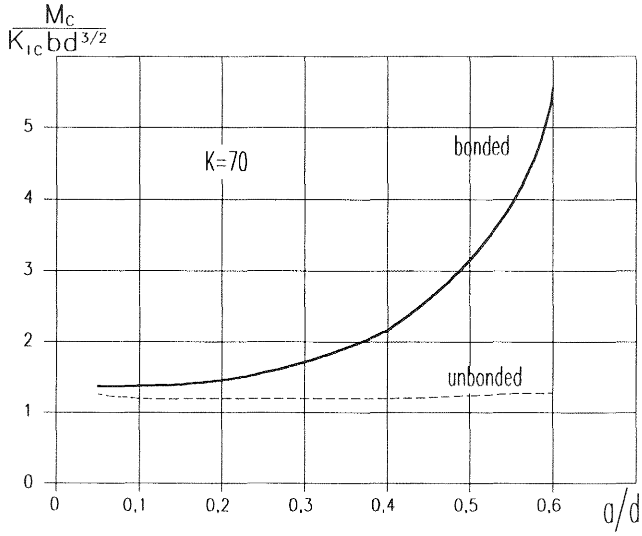

Fig.5. Critical moments: comparison between bonded and unbonded prestressed beams

The difference is remarkable for advanced cracking which gives rise, in bonded beams, to considerable increases of the stresses in reinforcement (with increases of the corresponding moments); at contrary in unbonded beams the stresses (and therefore the moments) remain pratically constant.

In the actual production of prestressed beams, during prestressing of the tendons, a share of the load (for instance, the dead load) is already acting, also in order to reduce the tensile stresses induced by prestressing.

This share of moment ΔM gives rise to a moment ΔM_1 according to:

$$\Delta M = F(d/2 - h) + \Delta M_1$$

whence

$$K_I = \frac{\Delta M_1 Y_m}{bd^{3/2}} - \frac{F Y_p}{bd^{1/2}} = \frac{\Delta M Y_m}{bd^{3/2}} - \frac{F\Psi}{bd^{1/2}} \qquad (5)$$

The residual moment M - ΔM will be supported subsequently by the bonded structure and will give a contribution to the value of K_I, according to previous results, as follows:

$$K_I = \frac{M - \Delta M}{bd^{3/2}} \left\{ Y_m - \frac{n\mu(1 - h/d - x/d)\Psi}{J/(bd^{3)}} \right\} \tag{6}$$

Adding in value and sign the (5) and (6), we obtain in total:

$$K_I = \frac{\Delta M Y_m}{bd^{3/2}} + \frac{M - \Delta M}{bd^{3/2}} \left\{ Y_m - \frac{n\mu(1 - h/d - x/d)\Psi}{J/(bd^{3)}} \right\} - \frac{F\Psi}{bd^{1/2}}$$

whence:

$$\frac{M}{K_I bd^{3/2}} = \frac{1}{Y_m - \frac{n\mu(1-h/d-x/d)\Psi}{J/(bd^{3)}}} \left\{ 1 - \frac{\Delta M}{K_I bd^{3/2}} \cdot \frac{n\mu(1-h/d-x/d)\Psi}{J/(bd^{3)}} + \frac{F\Psi}{K_I bd^{1/2}} \right\} \tag{7}$$

The final result can be written more expressively in terms of a relation between the critical (dimensionless) moments of the different above considered cases, in the following form:

$$\overline{M} = \frac{\overline{M}_{ca} \overline{M}_{unb}}{\overline{M}_o} + \Delta\overline{M}(1 - \overline{M}_{ca} / \overline{M}_o)$$

where is $\Delta\overline{M} = \Delta M / (K_{IC} bd^{3/2})$ and the expression in brackets, multiplying $\Delta\overline{M}$, is negative and, if $\Delta\overline{M}$ tends to $\overline{M}, \overline{M}$ tends to the value of $\overline{M}_{unb}$.

5 References

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