Abstract
Non-linear finite element analyses based on non-linear fracture mechanics and an experimental investigation of the behavior of reinforced concrete columns are presented. The tests consisted of long, slender, eccentrically loaded reinforced normal and high strength concrete columns. The parameters varied in the study were concrete strength, stirrup spacing, and slenderness of the columns. The experimental results indicate that closer stirrup spacing gives a more ductile behavior after the peak load and that high strength concrete columns require more stirrups to gain the same ductility as the normal strength concrete columns. Further, the results show that increased compressive strength gives an increase of the maximum bearing capacity. The results also indicate that the midheight deflection at the maximum load depends on the slenderness of the column rather than the concrete strength. Non-linear finite element models, with material models for concrete based on non-linear fracture mechanics, have been developed and the analyses show good agreement with the test results.
1 Introduction

The use of high strength concrete has accelerated in recent years. The high compressive strength is especially advantageous in compressed members such as columns. The columns may be designed more slender and, consequently, offer economic benefits. Some researchers, Cusson and Paulter (1994), Saatcioglu and Razvi (1992), Mander et al. (1988) and Ahmad and Shah (1982), have studied short columns in normal and high strength concrete under axial loading. Only a few have studied full-scale reinforced high strength concrete columns subjected to eccentrically applied axial loads, Bjerkeli et al. (1990), Lloyd and Rangan (1990). However, the behavior of high strength concrete columns is not yet fully understood. As a part of a national research project concerning the properties of high strength concrete, this project, High-performance Concrete in Compressed Members, deals with columns subjected to axial loading. The aim of the study is to gain a better understanding of the mechanical behavior of axially loaded concrete columns until final failure. To simulate the failure mechanism of the columns non-linear finite element models, based on non-linear fracture mechanics, have been developed.

2 Test program

The test series reported here consisted of twelve long slender columns, with compressive cylinder strengths varying from 33 to 99 MPa. The parameters varied in the study were concrete strength, stirrup spacing, and slenderness of the columns. Half of the long slender columns were made of high strength concrete, the other half of normal strength concrete. For each slender column, an identical short stub column was cast. In addition, four short stub columns of high strength concrete and six short stub columns of normal strength concrete were cast. These ten columns varied in different configurations and quality of longitudinal and lateral reinforcement. The purpose of these specimens was to study the confinement effect. The results of the tests of the short stub columns are reported elsewhere, Claeson (1994).

Eight of the twelve long slender columns had a cross-section of 200 x 200 mm, while the lengths were 3.0 m, group B, or 4.0 m, group C (both with reinforcement configuration b), see figure 1. The remaining four columns, group A, were of the dimensions 120 x 120 x 2400 mm (reinforcement configuration a). It should be noted that the concrete cover,
that is, the distance from the edge to the stirrup, was 15 mm for all of the columns.

Fig. 1. Geometry and details of reinforcement configurations a), and b), and c) schematic figure of load arrangement

### 2.1 Material properties
The concrete mixes, designed with target compressive cube strengths of 50 and 120 MPa, were produced at the Structural Engineering Laboratory at Chalmers University of Technology. Silica fume and plasticizer were used in the high strength concrete mixes to obtain high strength, workability, and the reduction of fine particle segregation. The detailed concrete composition and hardened concrete properties are described in detail elsewhere, Claeson (1994). The column specimens were cast horizontally in a steel form. The concrete was thoroughly vibrated by means of an internal vibrator. Deformed bars of the Swedish quality Ks40S was used as lateral reinforcement, while Ks60 was used as longitudinal reinforcement.

### 3 Test set-up
The tested columns were hinged at both ends and loaded with a compressive axial force applied with an initial end eccentricity of 20 mm, see figure 1c. All of the tests were carried out in a Losenhausen vertical hydraulic column testing machine with a capacity of 10 000 kN. The load, which was evaluated by measurements from an oil pressure gauge, was increased at a constant rate without interruption. When the load approached the cal-
culated maximum load, the oil pressure gauge was used to indicate how the
deformation should be increased in order to capture the post-peak curve.

To ensure that the failure would occur in the instrumented region of the
columns, the ends of the test specimens were further confined with stirrups
spaced apart 50 mm or less.

4 Test results

Table 1 shows the maximum load and the midheight deflection at maximum
load for each column. When comparing the results from the tests of the
slender columns, some effects may be observed. By increasing the concrete
strength the maximum bearing capacity increased. A closer stirrup
spacing did not give an increase in load bearing capacity but did give the
columns a more ductile behavior. The results also indicate that the concrete
strength did not affect the midheight deflection at maximum load for
the same cross-section and slenderness.

Table 1 Results from tests of long slender columns

<table>
<thead>
<tr>
<th>Column</th>
<th>Column group</th>
<th>Concrete</th>
<th>( f_{c,\text{cyl}} ) [MPa]</th>
<th>( f_{c,\text{cube}} ) [MPa]</th>
<th>Long. bars</th>
<th>Stirrup spacing [mm]</th>
<th>Max. load [kN]</th>
<th>Midheight deflect. [mm]</th>
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<td>N</td>
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<td>H</td>
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The column group refers to figure 1c. The compressive cylinder strength, \( f_{c,\text{cyl}} \), and
the compressive cube strength, \( f_{c,\text{cube}} \), refer to compression tests of specimens with
sizes \( \phi 150 \times 300 \text{ mm} \) and \( 150 \times 150 \times 150 \text{ mm} \), respectively. The concrete is of either
normal (N) or high (H) strength concrete.
5 Fracture mechanics analysis

5.1 General
One aim of the study was to develop non-linear finite element models that could simulate the failure mechanism of the columns and together with the experiments help to gain a better understanding of the mechanical behavior until final failure, including the post-peak behavior. These models were created in the non-linear finite element program ABAQUS, HKS (1992).

To model reinforced concrete, the program combines standard elements of plain concrete with a special option, called rebar in ABAQUS. This option strengthens the concrete in the chosen direction to simulate the behavior of a reinforcement bar. The rebar option is used with a metal plasticity model to describe the material behavior of the reinforcing steel. With this approach the material behavior of the plain concrete is considered independently of the reinforcement.

5.2 Constitutive model for concrete
The smeared crack approach has been chosen to model cracked reinforced concrete. In the smeared crack concept a cracked solid is imagined to be a continuum in the notion of stress and strain. This means that the behavior of cracked concrete can be described in terms of stress-strain relations. The model of concrete behavior is intended to be used for relatively monotonic loadings under fairly low confining pressures (less than four to five times the largest compressive stress that can be carried by the concrete in uniaxial loading). Cracking is assumed to occur when the stresses reach a failure surface called "crack detection surface", see figure 2. The failure surface is a Coulomb line written in terms of the first and second stress invariants. Once a crack has been detected its orientation is stored and a second crack at the same point may only form orthogonal to this direction. Prior to cracking, the concrete is modeled sufficiently accurately in tension as an isotropic, linear elastic material.

When the principal stress components are dominantly compressive the response of the concrete is modeled by an elastic-plastic model. The elastic stage is limited by a Drucker-Prager yield surface. Once yielding has occurred an associated flow law together with isotropic hardening are used. The uniaxial stress-strain relations in compression, used in the analyses, were determined through tests on cylinders (φ150 x 300 mm), cast with concrete from the same batch as the columns. The fracture energy was determined from RILEM beams, and were together with the tensile strength used to calculate the tensile softening relation.
5.3 Constitutive model for reinforcement

The reinforcement bars were modeled by a linear elastic, perfectly plastic material model. The modulus of elasticity and the yield strength of the reinforcement bars were determined through tension tests, and the Poisson's ratio was approximated to be 0.3.

5.4 Numerical studies

Two different analyses were carried out. The first analysis used three-node beam elements to model the columns. In the second analysis three-dimensional solid elements with 20 nodes were used. While the first analysis require less computational effort it cannot simulate the effect of transversal reinforcement. This is one reason why a three-dimensional model had to be developed. Figure 3 shows the FE-mesh and the boundary conditions for the two levels of analysis.

The Newton-Raphson iteration method is the standard method in ABAQUS and is usually sufficient for monotonic loading. Analyses of reinforced concrete structures, however, often exhibit local maximum points in the load-displacement curve with snap-through or snap-back behavior, see Crisfield (1991). To get around this situation, the load was applied as a deformation using the Newton-Raphson model together with a line search approach. Numerical convergence problems were nevertheless found in several cases for the iterative solution technique, especially when trying to obtain the descending branch of the load-displacement relation.

Fig. 2. Compression yield surface and crack detection surface a) in the $p$-$q$ plane ($p =$ hydrostatic stress, $q =$ deviatoric stress) and b) in plane stress, see ABAQUS, HKS (1992)
Fig. 3. FE-mesh and boundary conditions for the beam analysis and the analysis with solid elements

6 Comparison between results of FE-models and test results

In this paper we have focused on comparing the load-deflection relations from the FE-models with the results from tests of the long slender columns. As can be observed in the figures 4 to 7, the accuracy of the models is satisfactory up to the region around maximum load. It was observed that the beam element analysis gave a stiffer response in spite of the fact that no transversal reinforcement was included. The analysis with solid elements predicted the response accurately and took into account effects such as confinement. One advantage of developing finite element models is the possibility to perform parameter studies to identify important variables and to examine their influence on the column behavior more thoroughly compared to what is possible by only performing tests. As far as the results indicate, this will be possible with the models developed in this study.
Fig. 4. Comparison between an FE-analysis and an experiment on a 3 m long, slender normal strength concrete column (specimen 5)

Fig. 5. Comparison between an FE-analysis and an experiment on a 3 m long, slender high strength concrete column (specimen 7)

Fig. 6. Comparison between an FE-analysis and an experiment on a 4 m long, slender normal strength concrete column (specimen 9)
Fig. 7. Comparison between an FE-analysis and an experiment on a 4 m long, slender high strength concrete column (specimen 11). The experimental curve reveals creep at about 100 kN. At that load security bolts were removed and measurement equipment was adjusted. This may explain the deviation of the FE-curve and the experimental curve.

7 Conclusions

Twelve long slender reinforced concrete columns have been tested under eccentric axial loading. The main purpose of these tests was to investigate the differences and similarities in the behavior of normal versus high strength concrete columns. When comparing the results from the tests of the slender columns, two effects may be observed. By increasing the concrete strength the maximum axial load increased. A closer stirrup spacing did not give an increase in load bearing capacity but did give the columns a more ductile behavior. Further, the concrete strength did not seem to affect the midheight deflection at measured maximum load.

The FE-analyses showed satisfactory accuracy up to maximum load. The load-displacement relations corresponded well with those obtained from the tests. However, the finite element analyses will be further evaluated in the near future.

8 Acknowledgement

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9 References


