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# NUMERICAL MODELLING AND DETERMINATION OF FRACTURE MECHANICS PARAMETERS FOR CONCRETE AND ROCK: INTRODUCTION

J.G.M. van Mier

*Delft University of Technology, Faculty of Civil Engineering,  
Stevin Laboratory, Delft, The Netherlands*

## **Abstract**

The paper gives an introduction to the FraMCoS-2 conference workshop on "Numerical Modelling and Determination of Fracture Mechanics Parameters". Essential for practical applications of numerical fracture models to emerge is that the model parameters are defined in an unbiased manner. Experimental research is essential to measure the necessary material constants. However, it is not always clear whether the model parameters are indeed unbiased, and moreover, whether the experimental method adopted gives the parameters needed in a certain model. The aim of the workshop is to bring together experimentalists and numerical specialists in order to better grasp the problems encountered both in experimentation and in modelling, and hopefully to come to a better formulation of fracture mechanics parameters, their definition and the way in which they should be measured for any given type of concrete or rock.

## **1 Introduction**

Numerical models for the analysis of reinforced concrete structures have been developed since the 1960s. The idea is that by using the new numerical tools, crack growth and non-linear behaviour of the material can

easily be incorporated, and even could reduce the amount of research work needed for physical testing. But how much of this is true ? Basic to the philosophy is that material parameters needed in the numerical models can be measured in a direct and unbiased manner. The material properties should be independent of test conditions. For different materials such properties should be measured in an easy straightforward manner, i.e. without using too sophisticated measuring techniques. The following questions can then be raised. Are the numerical model used and the material properties related, i.e. are they interwoven in an unbreakable fashion ? Or is it possible to measure parameters such as fracture energy, tensile strength or brittleness in such manner that the values can be used in any of the models proposed to date ? Moreover, can this be done in a standardised manner ? All these questions seem related, and are considered of utmost importance for practical applications of fracture mechanics of concrete (and rock). In this conference workshop it is tried to bring together numerical specialists and experimentalists in order to develop and discuss possible roads to determine model parameters for fracture. A format is chosen where the numerical specialists are invited to give an outline of the parameters needed in the various (numerical) fracture models proposed to date. These models can be set up according to different philosophies. Important is to recognise that the materials concrete and rock are heterogeneous. This heterogeneity, or rather the effect caused by the heterogeneity can be introduced following different concepts. Namely, heterogeneity can be introduced directly by implementing it through a stochastic distribution of properties of the finite elements, or rather it can be introduced indirectly by choosing appropriate non-linear constitutive laws. Which method leads to the best, mesh independent, results is open to debate.

## **2 Three-level approach applied to concrete**

The three-level approach commonly used in materials science was first introduced to fracture of concrete-like materials by Wittmann (1983). Three levels of observations and modelling are distinguished as shown schematically in Figure 1. The levels are the micro-, meso- and macro-level. At each subsequent level more or less detail is recognised in the material structure. The most global approach to modelling is the macro-level, where the material is regarded as an equivalent continuum. No material structure is distinguished, and all non-linear behaviour is included in the constitutive law. The advantage from a numerical point of view

seems that the structure that is analyzed can be divided in relatively large finite elements, which tends to reduce the computational effort.

The second level of observations is the meso-level. At this scale,  $10^{-3}$  to  $10^{-2}$  m, individual aggregate particles are distinguished. The particles are assumed to be embedded in a matrix of cement mortar, i.e. a mixture of hydraulic cement and fine sand particles. The grading of the sand and aggregates is generally selected such that a dense particle skeleton is obtained. The particles are bonded together through very thin layers of cement. At the macro-level, the response of the concrete under mechanical load can be explained and computed from interactions of the aggregate particles and the cement matrix at the meso-level. The interfacial transition zone plays a major role as it is generally weaker than both the matrix and the aggregate particles.

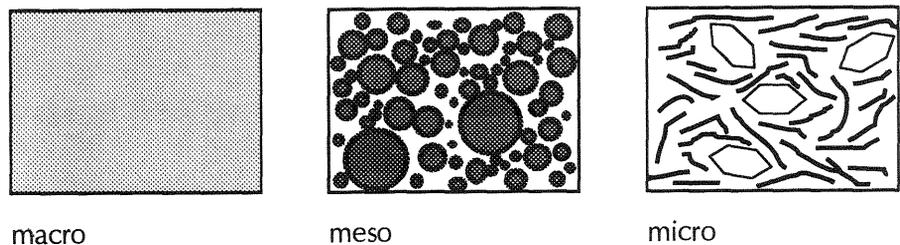


Figure 1. Macro-, meso- and micro-level for concrete fracture.

The behaviour of the cement-matrix and the interfacial zone between aggregates and matrix can be explained by considering the material structure at the level of the individual CSH particles. These particles are rolled up plates of nanometre to micrometre scale, bonded together through VanderWaals forces. Water between the particles plays an important role, in particular in relation to strength, creep and shrinkage. Of course, the meso-level behaviour of the aggregative material should be studied at the micro-level as well. In general the aggregates consist of heterogeneous rock.

It will be obvious that the structural behaviour at each of these levels requires different types of input parameters. The main question to be addressed now is the following: which parameters are important and essential to describe the fracture behaviour at each of these dimensional levels, and how can they be determined in the laboratory ? In the remainder of this paper we will limit the discussion to meso-level and macro-level modelling only.

### 3 Macroscopic fracture models for concrete

As mentioned before, at the macrolevel we are interested in describing the fracture of concrete as a continuum. Two different approaches can be distinguished here, namely a (smeared) continuum approach (crack band models) and a discrete approach. In the crack band models, the crack is smeared over a larger volume, generally extending over part of a finite element or sometimes several elements, whereas in the discrete approach the crack appears as a discrete jump between element boundaries. Elementary to these models is a continuum based fracture law, which is in general derived from Linear Elastic Fracture Mechanics (mainly in the discrete models), or from Non-Linear Elastic Fracture Models like the Fictitious Crack Model by Hillerborg et al. (1976), both for continuum and discrete approaches.

Several parameters are needed in Hillerborg type fracture models, namely, the uniaxial tensile strength  $f_t$  of the concrete (which is assumed to be the failure criterion of the material), the fracture energy  $G_f$  (defined as the area under the post-peak stress-crack opening diagram, and which can be considered as a propagation criterion), and the shape of the descending (or softening) branch. The tensile strength depends on the size of the structure (or size of the specimen that is used, e.e. Carpinteri & Ferro (1992)), but also on the boundary conditions under which the tensile test was performed (Van Mier et al. (1994)). Similar observations are made for the fracture energy, which increases with increasing specimen size. In addition, the shape of the descending branch also depends on the boundary conditions adopted in the test: the fracture energy decreases when the uniaxial tensile test is carried out between rotating rather than fixed loading platens. Thus, the parameters needed in the smeared continuum models are dependent on boundary conditions and geometry. A direct way to circumvent this dilemma has not been found to date, but seems essential when we would like the numerical models to have some predictive capabilities: in the end it would be highly desirable to have the option to compute the behaviour of hitherto un-tested structures.

Smeared crack models seem to give mesh-dependent results, e.g. Rots (1988). Because of this, new higher order continuum models are developed, which will be presented by De Borst in this conference workshop. In, for example, gradient theories the softening fracture model is split into two parts, a 'computational softening diagram' and a term with dimension length, Pamin (1994). These models are generally mesh-independent. How the model parameters should be measured in the laboratory is still open to further study.

Softening fracture laws are sometimes also used in conjunction with discrete fracture models where a crack develops between the element boundaries, e.g. Reich et al.(1993). Other discrete models adopt different material parameters such as critical stress intensity factors, or J-integrals.

The measurement of all the different parameters should best be done in conjunction with the development of a given model. Perhaps the best example is the Fictitious Crack Model where a proposal for experimental validation, as described above, was presented as an un-breakable part of the model itself.

#### **4 Mesoscopic fracture models for concrete**

A distinct manner to incorporate effects due to heterogeneity of the material in a fracture model is to directly incorporate the heterogeneity in the finite element mesh. Thus, the particle structure, pore structure and interface between aggregates and matrix are incorporated in the finite element mesh, for example by using images of real concrete directly, see for example Schlangen (1995). Similar approaches were used earlier by, for example Roelfstra (1989) and Stankowsky (1990). In the lattice model adopted by Schlangen different properties are assigned to lattice beams falling in different parts of the projected material structure. In the finite element models proposed by Roelfstra and Stankowsky, continuum elements (plane stress in 2D simulations) are used for the aggregate and matrix phase, whereas special spring elements are used to model the interface between aggregate and matrix.

Another option is to use stochastic material properties in numerical simulations as for example attempted by Carmeliet & De Borst (1994). Different properties of the material can be selected as a stochastic property, for example, the initial damage threshold in a continuum damage model. As an effect a correlation distance of the random field is introduced, which has to be determined in addition to the internal length scale of the higher order continuum (non-local) model that is used in order to avoid mesh dependency in the localization phase.

Thus, as we can see, new material properties are introduced in the meso-level models. In the models where the particle structure is generated directly, and projected on top of the finite element mesh, the particle size distribution, pore size distribution and constitutive equations for the aggregate, matrix and interface zone should be determined. Most parameters are geometric in nature, and are known for a given concrete (or rock). Most difficult are the constitutive equations, which however might

turn out to be more simple than for the equivalent (macroscopic) continuum model. The interface zone properties might be most difficult to determine. On the other hand, the stochastic continuum models require as input the random distribution of the selected material property and the correlation length. In addition, when higher order continuum models are used, an internal length scale must be determined as well. In general, procedures for measurement of all the new variables is lacking, and more seriously, a true physical meaning is not always given. Therefore it is considered highly desirable to describe experimental methods that could be used to quantify the above mentioned material properties. In this respect it would be extremely useful to strive to a more close cooperation between theoreticians and experimentalists.

## **5 The role of standard testing**

During the past decades, an increasing amount of fracture experiments on plain concrete (and rock) have been carried out. The largest explosion has been generated with the introduction of the Fictitious Crack Model, in which it was proposed to use a 'simple' uniaxial tension test (where the specimen is clamped between non-rotating loading platens) to determine the tensile strength and the fracture energy of the material under consideration. In a RILEM TC50FMC draft recommendation it was proposed to adopt a bending test rather than a uniaxial tensile test for the determination of the fracture energy. The tensile strength should in that case be measured in a separate splitting tensile test. The main reason for introducing the bending test was to allow for measurement of fracture energy in laboratories where the appropriate tensile test equipment was lacking. However, the 3 point bend test introduced additional experimental difficulties as was clearly elucidated by Elices et al. (1992). An alternative experiment is the splitting tensile test introduced by Tschegg and Linsbauer (1986) and Brühwiler & Wittmann (1990). Again, it is attempted to measure the tensile fracture energy by loading a specially designed test specimen in a compressive machine. Each of the above methods, uniaxial tension, three-point-bending and tensile splitting testing introduces experimental errors on the fracture energy. Moreover, it should be debated if the real fracture parameter that we are interested in is determined in such experiments. Are the 'assumed' crack physics truly captured by the experiment that is carried out? Such questions will be addressed in the Cardiff workshop preceding the FraMCoS-2 conference. The outcome of the workshop will be presented in this conference workshop. However,

irrespective of the outcome of the Cardiff workshop, it is felt that a unified method for measuring fracture mechanics parameters should be introduced on short notice. It would help to make the various experimental investigations comparable to one another, and perhaps, the relation between different parameters used in the various models could be further elucidated.

## 6 Questions to be addressed

Before the workshop, a long list of questions was circulated among the introductory speakers. For the different type of models, i.e. stochastic models, higher order continua, Hillerborg type models, and discrete crack models (both based on  $G_f$  and  $K_c$ ), it is asked to make an exhaustive list of all parameters needed in the model. This includes not only the material parameters, but also model-related parameters. Furthermore, it should be elucidated whether the model is based on direct physical mechanisms, or whether the model is phenomenological in nature. In the latter case it should be clarified how extrapolation outside the range of experimental

Table 1. Classification of model and experimental issues to be raised

Model parameters	<ul style="list-style-type: none"> <li>- which originate from the material ?</li> <li>- which are additional from the numerical technique ?</li> <li>- which are additional from the experiment ?</li> </ul>
Model type	<ul style="list-style-type: none"> <li>- does a sound physical background exist ?</li> <li>- or is it phenomenological ?</li> </ul>
Standard test for benchmark analysis	<ul style="list-style-type: none"> <li>- which geometry is most suitable ?</li> <li>- of which size and under which boundary conditions ?</li> <li>- what would be the most suitable output parameter for critical comparison of model and experiment ?</li> </ul>

observations can be realised. Moreover, the introductory speakers are asked to reflect on the question whether a standard test would be helpful for a better understanding of the model and its parameters. And, are benchmark

tests a valuable source of information to tune numerical models ? Can it be clearly specified which output is requested from such benchmark experiments ? Which experiments (specimen size and shape, and boundary conditions, or perhaps a combination of these parameters) would then be most suitable ?

The most important issues are summarised in Table 1. It would be helpful if the Table could be completed on the basis of the various contributions.

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