APPLICATIONS OF FRACTURE MECHANICS TO ANCHORS AND BOND

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Abstract
Design methods for anchors and bond are compared to models based on fracture mechanics and test results. Size effect factors are beginning to be used in design methods but brittleness factors including the fracture energy G_F and properties of high performance concrete are still to be introduced.

1 Introduction. Brittleness and ductility

Anchoring and bond problems are fundamental in the design of reinforced concrete structures. As tensile stresses occur in the concrete when forces from reinforcement or anchors are carried into a structure, it is essential that tensile fracture is modelled in a relevant way.

How far have we progressed in this area? Some answers and lines of development will be presented in this paper.

A basic parameter in fracture mechanics is the *brittleness number B*. It can be defined in the following way. Let us study a tensile test of a concrete prism, see Fig. 2.1. Up to the maximum load (with *stress* f_t, and *deformation* $\delta_E = \varepsilon_t L$) the prism basicly behaves in an elastic way (*strain* $\varepsilon_t = f_t / E$). After maximum, a narrow fracture zone (FZ) deforms further

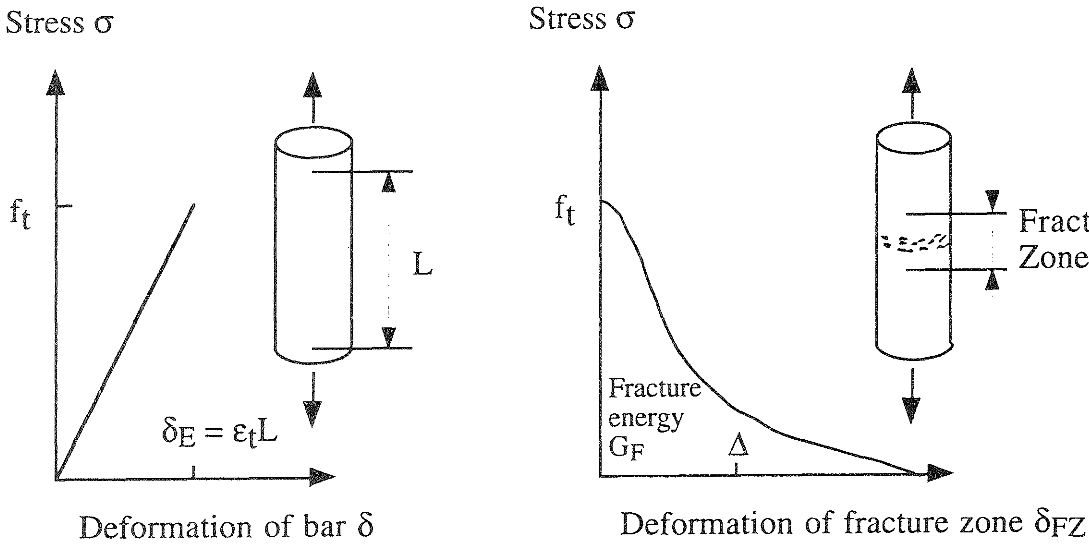


Fig 2.1 Loading of a concrete prism. Definition of basic parameters for ductility and brittleness. From Bache (1995), modified.

under falling load. At the same time the material outside the fracture zone is relieved elastically - largely following the first curve back to the origin. The area under the descending curve is defined as the *fracture energy* G_F which is needed in order to separate the prism into two parts. A characteristic *failure zone deformation* Δ can also be defined as $\Delta = G_f / f_t$.

Structures can be defined as *brittle* when the elastic deformation δ_E dominates, whereas the behaviour can be defined as *ductile* when the deformation of the fracture zone Δ dominates. The *brittleness number B* can be defined as

$$B = \delta_E / \Delta = \varepsilon_t L / \Delta = f_t^2 L / EG_F$$

The reciprocal value *1/B* can be named the *ductility number*. It can be seen that the brittleness/ductility deponds on the *length L*, the *tensile strength* f_t, the *modulus of elasticity E*, and the *fracture energy* G_F . The brittleness number is also proportional to the ratio of elastic to fracture energy:

$$\text{Elastic energy / Fracture energy} = 0.5\, f_t\, \delta_E / G_f = 0.5\, f_t^2\, L / EG_F \sim B$$

The factor EG_F / f_t^2 is a material parameter which was introduced by Hillerborg (1976, 1983) as the *characteristic length*, l_{ch} . The *brittleness number* was introduced in the 80-ies by Bache (1995), see Elfgren (1989). A basic fracture mechanics philosophy is to relate the strength of an object to its brittleness number *B* or to the components of *B* i e the length *L*, the tensile strength f_t , the modulus of elasticity *E*, and the fracture energy G_F.

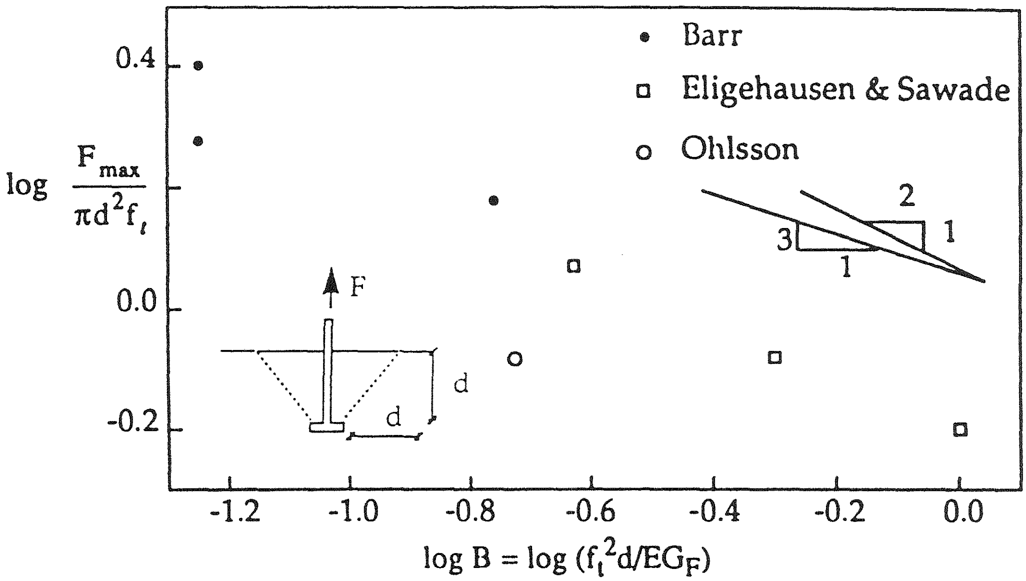


Fig. 3.1 Failure load F_{max} as function of brittleness number for anchor bolts. From Elfgren and Ohlsson (1992).

This way of describing the tensile fracture is now beginning to be introduced in modern design codes. In e g the CEB-FIP Model Code 1990 (1993) values are given for the fracture energy G_F [Nm/m^2] and a bi-linear stress - crack-opening diagram is proposed for concrete in tension. However, in most traditional codes, e g Eurocode EC-2 (1992), not much can be seen except some empirical formulae for size effect influences.

2 Anchors

Analyses and tests of anchors have been carried out by many researchers during the last fifteen years, see e.g. Rehm et al (1991), Elfgren (1992), and Eligehausen (1994).

Some test results are plotted in Fig. 3.1 as a function of the brittleness number B. A curve through the points would have a slope of $k = -1/3$ or $k = -1/2$. For varying slopes we get the following formulae for the maximum load F_{max}.

$k = 0$	very small B	$F_{max} \sim d^2 f_t$	no size effect
$k = -1/3$	medium B	$F_{max} \sim d^{5/3} f_t^{1/3} E^{1/3} G_F^{1/3}$	
$k = -1/2$	large B	$F_{max} \sim d^{3/2} E^{1/2} G_F^{1/2}$	(LEFM)

In design codes often the value of k = 0 is used which gives no size effect.

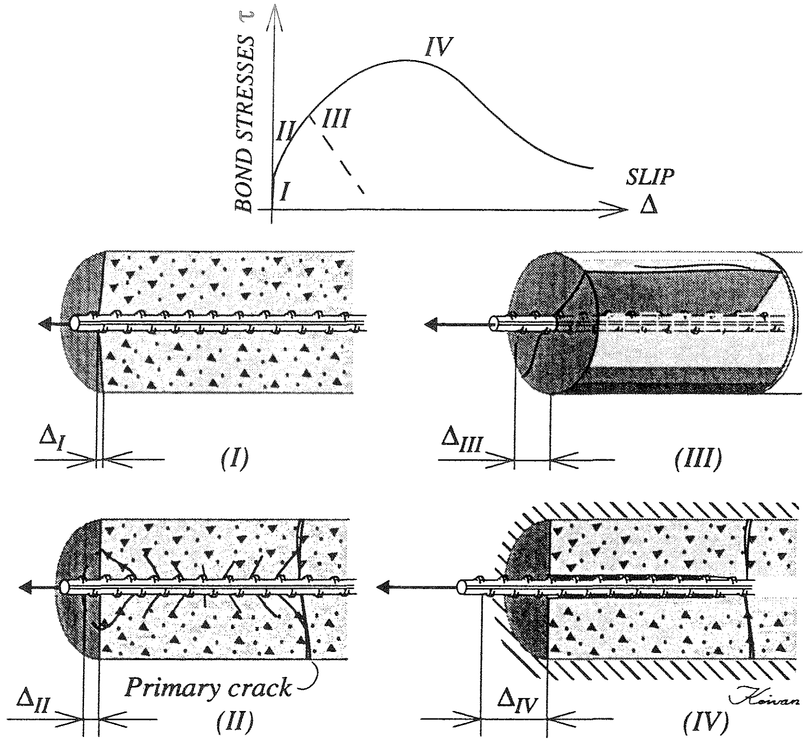
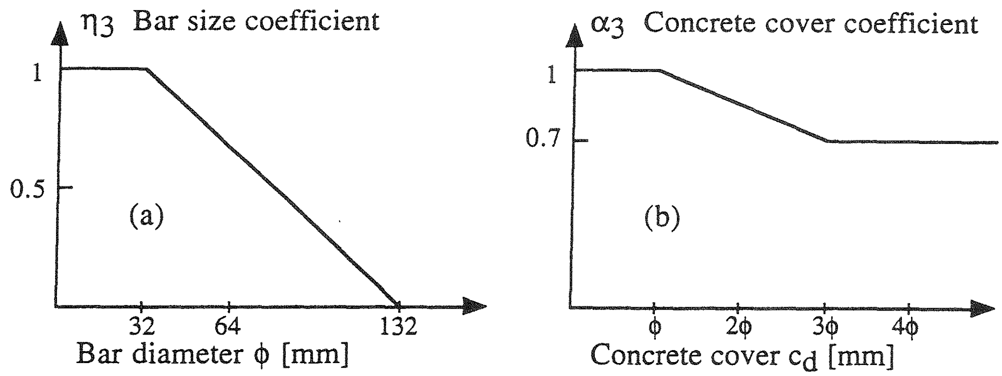


Fig. 4.1 Typical local bond-slip relationship: (I) Elastic deformation, (II) Primary (vertical) and secondary (conical) cracking, (III) splitting (longitudinal, radial cracking), and (IV) crushing in front of the ribs of the reinforcing bar. From Noghabai (1995), Gambarova et al (1989), and Rots (1989). See also Uijl (1994) and Åkesson (1993).

Here unsafe results can be obtained as the influnce of the embedment depth *d* is over estimated for large embedment depths. The value of k = -1/2 gives a line with the same slope as is obtained with linear elastic fracture mechanis (LEFM). This is realistic for large bolts and/or brittle concrete. In the future it is likely that models will appear which will be able to describe the maximum load as a function of the brittleness parameters in a more refined way than with the straight lines given above.

3 Bond

Reviews of bond models have been presented in e.g. Tepfers (1982), Rots (1989), and Skudra (1992). The general behaviour can be illustrated in Fig 4.1. In the CEB-FIP Model Code (1993) bond is treated as in Fig 4.2. Fracture mechanics influences can be observed in the way the thickness of the concrete cover c_d influences the bond length (coefficient α_3) and in the way the bar thickness ϕ influences the bond stress (coefficient η_3). Let us compare this method with some recent developments regarding the splitting failure which have been presented by Olofsson et al (1995) and Noghabai (1995). The model in Fig. 4.3 is used and results are shown in Fig. 4.4-5.



The **CEB-FIP Model Code 1990** (1993) gives the bond stress f_b by

$$\mathbf{f_b = \eta_1\, \eta_2\, \eta_3\, f_{ctd}} \quad \text{where}$$

η_1 considers the type of reinforcement: $\eta_1 = 1.0$ for plain bars, $\eta_1 = 1.4$ for intended bars and $\eta_1 = 2.25$ for ribbed bars

η_2 considers the position of the bar during concreting: $\eta_2 = 1.0$ for good bonding conditions and $\eta_2 = 0.7$ for other cases

η_3 considers the bar diameter: $\eta_3 = 1.0$ for $\phi \leq 32$ mm and $\eta_3 = (132 - \phi)/100$ for $\phi > 32$ mm, see Fig (a) above

f_{ctd} is the design value of the concrete tensile strength ($= f_{ctk,min}/1.5$)

The bond length l_b for a reinforcement bar with yield stress f_{yd} is given by the equilibrium equation

$$\mathbf{l_b\, f_b\, \phi\, \pi = f_{yd}\, \pi\, \phi^2/4} \quad \text{from which} \quad \mathbf{l_b = 0.25\, \phi\, f_{yd}/f_b}$$

The design anchoring length $l_{b,net}$ is given by

$$\mathbf{l_{b,net} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_b\, A_{s,cal}/A_{s,ef}} \quad \text{where}$$

$A_{s,cal}$ is the calculated area of the reinforcement required by design

$A_{s,ef}$ is the area of the reinforcement provided

α_1 is a coefficient taking into account the form of the bar (= 0.7 - 1.0)

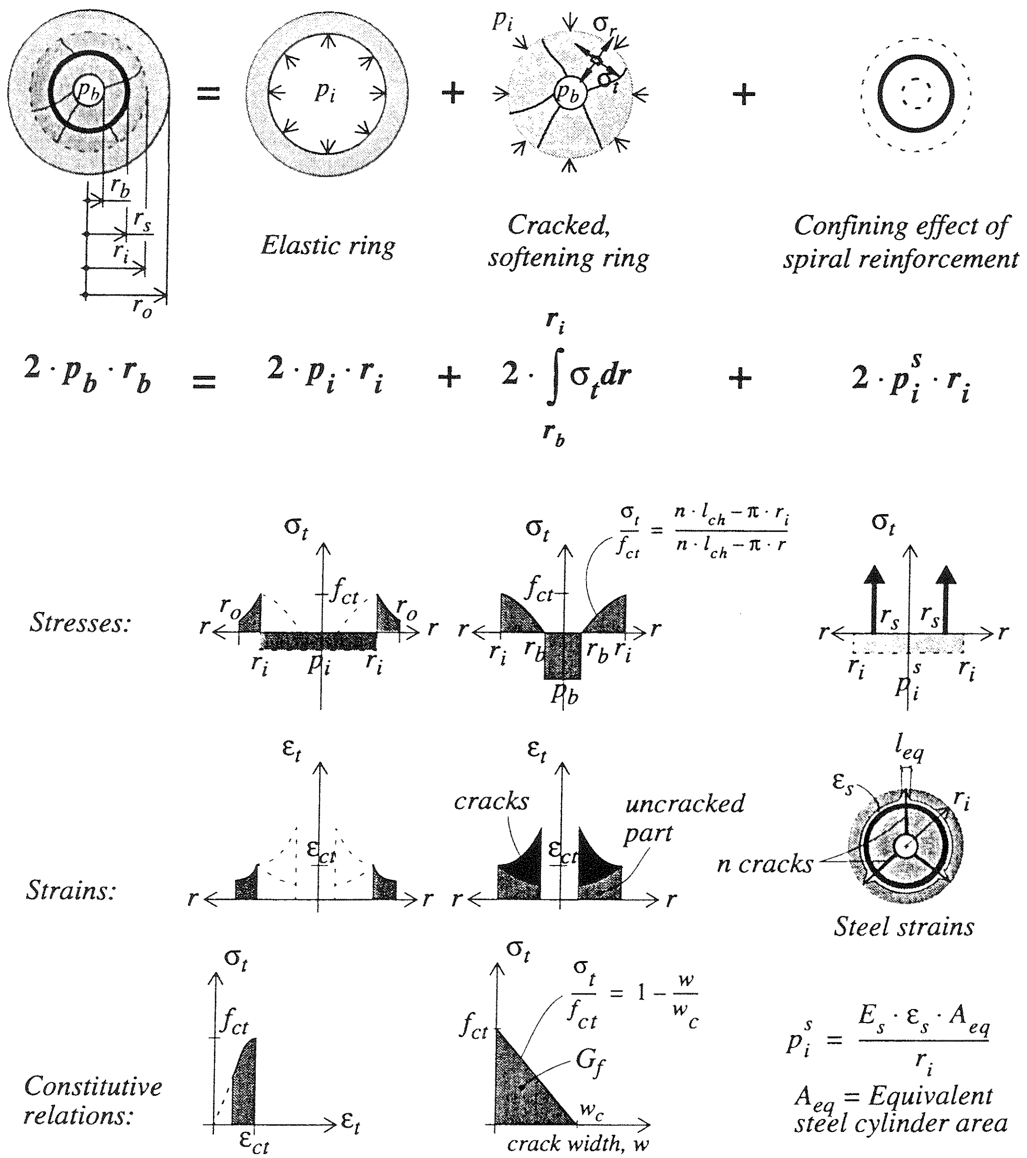
α_2 is a coefficient taking into account the influence of one or more welded transverse bars along the design anchoring length (= 0.7)

α_3 is a coefficient taking into account the effect of the confinement by the concrete cover: $\alpha_3' = 1 - 0.15\,(c_d - \phi)/\phi$ with $0.7 \leq \alpha_3 \leq 1.0$ for bars without hooks, see Fig (b) above

α_4 is a coefficient taking into account the effect of confinement by transverse reinforcement $0.7 \leq \alpha_4 \leq 1.0$

α_5 is a coefficient taking into account the effect of the pressure p transverse to the plane of splitting along the design anchoring length: $\alpha_5 = 1 - 0.04\,p$ with $0.7 \leq \alpha_5 \leq 1.0$

Fig. 4.2 Size effect coefficients for bond in the CEB-FIP Model Code 1990. (a) Influence of bar size ϕ, (b) influence of concrete cover c_d



$$\frac{p_b}{f_{ct}} = \frac{r_i}{r_b} \cdot \left(\frac{r_o^2 - r_i^2}{r_o^2 + r_i^2} \right) + \frac{n \cdot l_{ch} - \pi \cdot r_i}{\pi \cdot r_b} \cdot \ln\left(\frac{n \cdot l_{ch} - \pi \cdot r_b}{n \cdot l_{ch} - \pi \cdot r_i} \right) +$$

$$+ \frac{E_s \cdot A_{eq}}{r_s \cdot E_c} \cdot \left[1 + \frac{l_{ch} \cdot (r_s - r_i) \cdot (2 \cdot \pi \cdot r_s - n \cdot l_{eq})}{l_{eq} \cdot r_i \cdot (\pi \cdot r_s - n \cdot l_{ch})} \right]$$

Fig. 4.3. General features of fracture mechanics model for splitting due to e g bond stresses. From Noghabai (1995), modified. The first fracture mechanics models for splitting were due to Tepfers(1973) and Veen (1990).

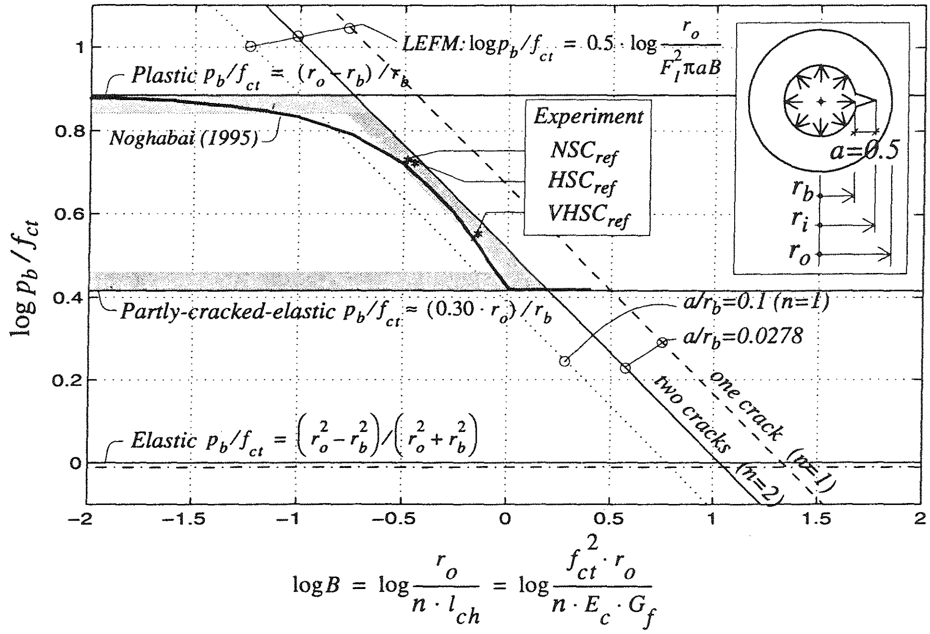


Fig. 4.4 Ultimate relative splitting pressures given by different models for a concrete ring (r_b = 18 and r_o = 156.5 mm) as a function of the brittleness number B. The coefficient F_I is depending on a, r_b and r_o and is determined with linear elastic fracture mechanics (LEFM). NSC, HSC and VHSC refer to tests with normal, high, and very high strength concrete with compressive strengths of 57, 105 and 157 MPa respectively. From Noghabai (1995), modified.

From the model the influence of *the thickness of the concrete cover* c_d can be studied, se Fig. 4.5. From the figure it can be seen that for a bar with ϕ=16 mm a concrete cover of $c_d = 3\phi$ has the maximum relative pressure of 5.2 while a concrete cover of $c_d = \phi$ has the relative pressure of 1.8, which gives a ratio of 5.2/1.8 = 2.9. This is much more than the CEB-FIP Model Code gives, where the ratio of the values of α_3 for the corresponding cases is 1/0.7 = 1.43 compare with Fig. 4.2b.

Also the influence of *the bar size* ϕ can be studied in Fig. 4.5. A comparison of the relative pressures for $c_d = \phi$ gives the following values for ϕ = 32 and 64 mm respectively: 1.8 and 1.7 with the ratio 1.7/1.8 = 0.94. This indicates a smaller influence than the change in η_3 in the code which sinks from 1 to 0.7 in Fig. 4.2a. For $c_d = 3\phi$ we get relative pressures of 4.6 and 3.5 with the ratio 3.5/4.6 = 0.76. This value should also be compared to the η_3 ratio 0.7. Thus the code gives slightly bigger reductions due to size effects than the model.

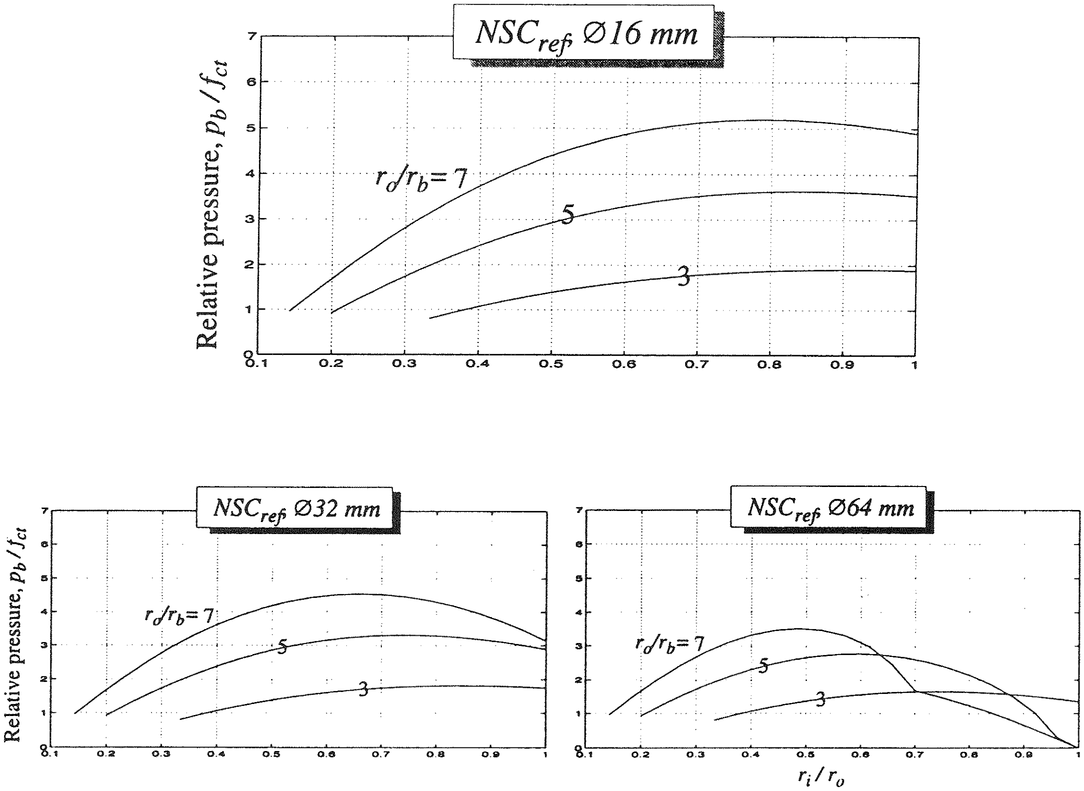


Fig. 4.5 Relative pressure p_b/f_{ct} as function of crack length r_i and concrete cover $c_d = r_o - r_b$ for bar diameters ϕ = 16, 32 and 64 mm. (The ratios r_o/r_b = 7, 5 and 3 correspond to $c_d = 3\phi$, 2ϕ and ϕ respectively). From Noghabai (1995), modified and extended.

4 Externally bonded reiforcement

Täljsten (1994) has studied the strengthening of existing concrete structures with externally bonded reinforcement plates of steel or fibre reinforced concrete. For the analysis of the bond stresses he has derived formulae based on non linear fracture mechanics, see Fig. 4.6.

5 Conclusions

Great steps forward have been taken during the last few years and models are becoming available for a correct way of designing structures with regard to brittleness and ductility. In the future also G_F and E ought to be taken into consideration in design codes and not only the size effect.

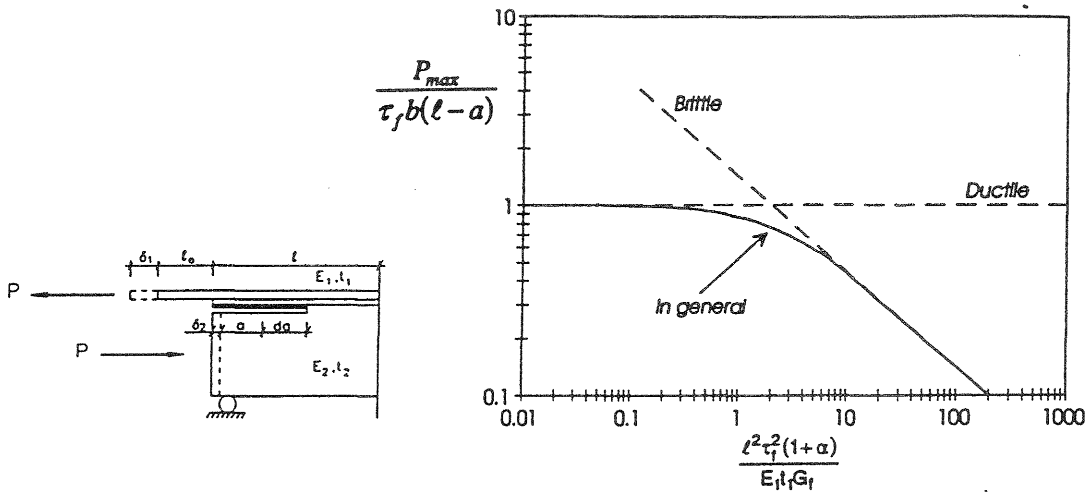


Fig. 4.6. Normalized joint strength as function of brittleness ratio for symmetric and non-symmetric lap joints. From Täljsten (1994). See also Gustafsson and Wernersson (1991).

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