FRACTURE MECHANICS IN THE DESIGN OF CONCRETE PIPES

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Abstract
The use of results of non-linear fracture mechanics in the design of plain concrete pipes is discussed. Several computational results obtained by the fictitious crack model are indicated. The results are compared to tests and to other theories: linear elastic brittle theory, plastic theory and Weibull theory.

1 Introduction

Fracture mechanics is very useful in design and applied strength analysis of pipes. Piping systems for waste water, commonly made of concrete, are an essential and costly part of the infrastructure of a developed society. As an example, in Sweden, a country with 8 million inhabitants, 84,860 km of waste water pipes are in operation (VA-VERK, 1992) and each year about 1000 km are installed. About 80% of the piping currently in operation is made of concrete.

The magnitude of the investment in concrete piping demonstrates the need for good methods for design. In this context it has been found necessary to consider the non-zero tensile fracture toughness of concrete and, accordingly, the consequences of the gradual damage and the strain localisation during fracture. Numerical results obtained by fracture mechanics (Gustafsson, 1983) have been included in practice codes in the form of tables and simplified design equations. Applied fracture mechanics analyses have also been beneficial in terms of giving rational explanations for a number of effects known from practice but contradicting the conventional linear elastic brittle theory. Results from fracture mechanics have moreover produced ideas and knowledge for new or modified concrete pipe test methods, aiming at improved validity of the results and/or savings in the continuing quality testing.

Examples of computational results are shown in this paper. The material model used in the various studies referred to, and ranging from 1983 (Gustafsson, 1983) to the present (Dahlblom and Gustafsson, 1995), is basically the well-known fictitious crack model proposed by Hillerborg and co-workers (Hillerborg, Modéer and Petersson, 1976). In connection with the computational results, comparisons are made to tests and to other methods of analysis. The discussion is basically restricted to pipes without reinforcement, but several of the general conclusions regarding load at failure or crack development should also be relevant for reinforced pipes.

2 Circular pipe: effect of mode of bending, size, shape and fracture toughness

There are essentially two modes of bending failure: One is the ring bend-

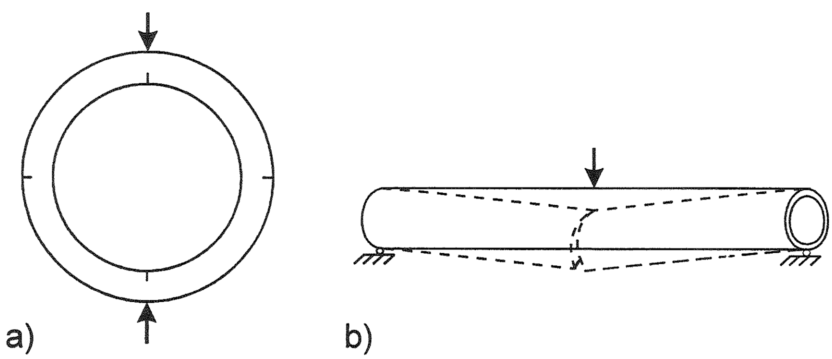

Fig. 1. a) Ring bending failure
b) Beam bending failure

ing failure, where the pipe is loaded and supported along its length. The other, see Figure 1b), is the beam bending failure, where the pipe is supported and loaded like a beam.

In Figure 2 the formal bending strength f_f is indicated for the two modes of bending. The formal bending strength f_f is the maximum stress at maximum load, calculated according to the conventional theory of linear elasticity. f_f is normalised with respect to f_t, the tensile strength of the concrete, and shown against normalised size of the pipe, d_i/l_{ch}, where l_{ch} is an intrinsic length of material and equal to EG_f/f_t^2, where E and G_f are the modulus of elasticity and fracture energy, respectively, of the concrete.

From the computational results in Figure 2 it is evident that the formal bending strength is much higher for ring bending than for beam bending. Since tests of pipes usually are made in ring bending, it is very important in design to note this difference in strength. Figure 2 also indicates a significant size effect, in particular in the case of ring bending. The effect of the fracture energy of concrete, G_f, is equal to the effect of the inverse of the size, d_i^{-1}. Since $G_f = 0$ would give $f_f/f_t = 1.0$, it is evident that the tensile fracture toughness of concrete, corresponding typically to $0.2 \leq d_i/l_{ch} \leq 5$, is of great importance for the load bearing capacity of pipes. The effect of the geometrical shape of a pipe, defined by ratio t/d_i, seems to be fairly small.

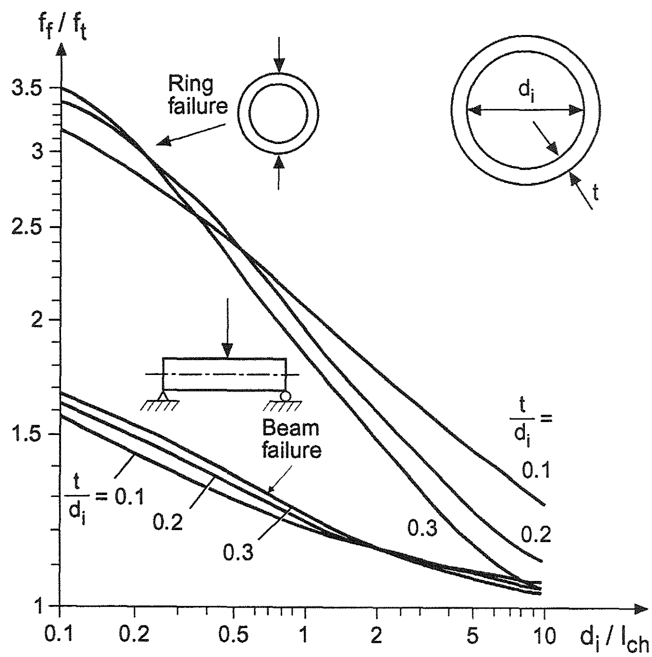

Fig. 2. Variation of the formal bending strength of a pipe

The relationships according to Figure 2 are in good agreement with various test results found in the literature (Gustafsson, 1985). A small additional experimental study, see Table 1, gave a similarly good result: the tensile strength of the actual concrete as predicted theoretically from the various bending strength test results is almost constant. The evaluation of f_t from the recorded f_f were made on the assumption $l_{ch} = 380$ mm. This assumption was based on fracture mechanics property tests (Petersson, 1981) of concretes with mixes similar to the actual pipe concrete. The shape of the $\sigma - w$ curve, i.e. cohesive stress versus crack opening, used in calculations by the fictitious crack model was assumed to be bi-linear according to the proposal of Petersson (1981).

Table 1. Tested bending strengths and the corresponding tensile strengths of the concrete as predicted by fracture mechanics

Pipe geometry		Mode of bending	f_f (MPa)	f_t (MPa)
t (mm)	d_i (mm)			
34	100	Beam	7.4	4.9
34	150	- " -	6.8	4.9
34	225	- " -	6.6	5.1
34	225	Ring	11.0	4.9
55	400	- " -	9.6	4.9

A comparison with respect to formal bending strength versus tensile strength is indicated in Table 2 (Gustafsson, 1988). The tensile strength of the concrete was tested by splitting tests of cylinders sawn from the pipes. 15 splitting tests and 15 ring bending pipe tests were made.

Table 2. Tested bending strength, the theoretically corresponding tensile strength and the tested splitting tensile strength

Pipe geometry		Tested	Theoretically corresponding	Tested
t (mm)	d_i (mm)	f_f (MPa)	f_t (MPa)	f_t (MPa)
29	150	11.6	3.9	4.2
35	225	11.1	4.2	4.1
60	400	10.0	4.3	4.0

The results indicated in Table 2 suggested that the strength of a pipe can be estimated by splitting tests of the concrete, and, vice versa, that the tensile strength of the concrete can be estimated from the results of pipe tests.

3 Flat bottom pipe: effect of load distribution

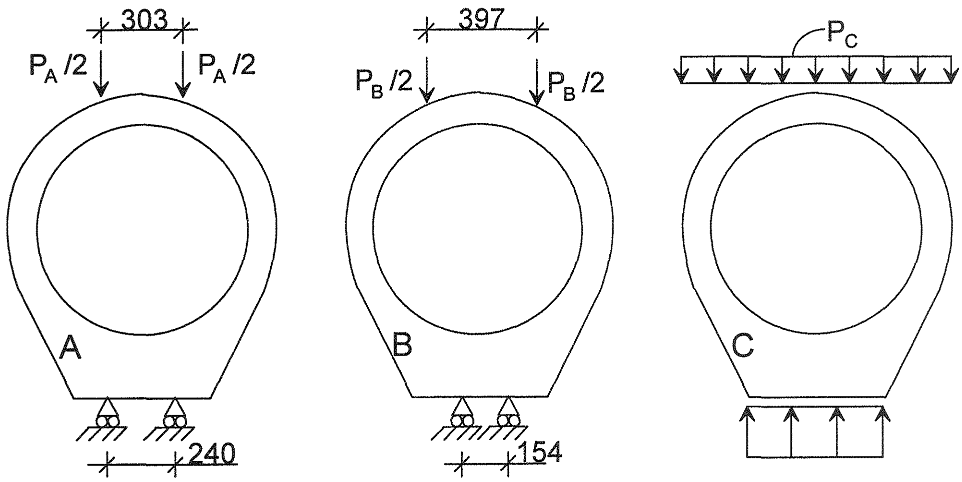

Fig. 3. Test setup *A*, test setup *B* and design load distribution (*C*) for a flat bottom pipe with $d_i = 800$ mm

Flat bottom plain concrete pipes (Ingwersen and Thygesen, 1994) have been tested for many years according to setup *A*, Figure 3. From the tested load bearing capacity, the load bearing capacity for the design load distribution, *C*, is calculated. The conversion factor from loading *A* to loading *C* used in design is according to linear elasticity. Since loading *C* is not very different from loading *A* it has until recently been assumed that the elastic conversion factors are sufficiently accurate. However, because of instances of damage, the conversion factors were questioned and it was decided to make an experimental study where nominally identical pipes with $d_i = 800$ mm were tested according to setup *A* and the alternative setup *B*. According to the elastic theory one should expect $P_{B,failure}/P_{A,failure} = 1.21$. The testing (Olesen and Pedersen, 1991) of 5+5 pipes, however, gave $P_{B,failure}/P_{A,failure} = 1.01$, see Table 3.

Table 3. Test results obtained for pipes "ig-800" (Olesen and Pedersen, 1991)

Loading	Failure loads kN/m	Mean kN/m	Standard deviation kN/m
A	174, 180, 175, 179, 173	176	3
B	171, 187, 179, 181, 171	178	7

To study whether the test results could be explained by fracture mechanics, finite element analyses were made (Dahlblom and Gustafsson, 1995), Figure 4. While the circular pipes were studied by a discrete crack implementation of the fictitious crack model, the flat bottom pipes were analysed by a smeared cracking implementation of the same model. Table 4 shows the results obtained with $f_t = 2.85$ MPa, assuming a single straight line $\sigma - w$ curve. The tensile strength value 2.85 MPa, is chosen so that an agreement with the test results is obtained for loading A when $l_{ch} = 430$ mm.

Table 4. Failure loads in kN/m calculated for $f_t = 2.85$ mm and various $l_{ch}(= EG_f/f_t^2)$

loading	l_{ch} = 0 mm	260 mm	430 mm	∞ mm
A	80	147	176	389
B	96	168	190	404
B/A	1.21	1.14	1.08	1.04

The results show that the fracture toughness of concrete affects the conversion factor $P_{B failure}/P_{A failure}$ and may explain, at least partly, the test results indicated in Table 3.

Table 5. Failure loads in kN/m calculated for $f_t = 2.85$ MPa and the loading conversion factor C/B for various l_{ch}

loading	l_{ch} = 0 mm	260 mm	∞ mm
B	96	168	404
C	121	205	493
C/B	1.251	1.223	1.221

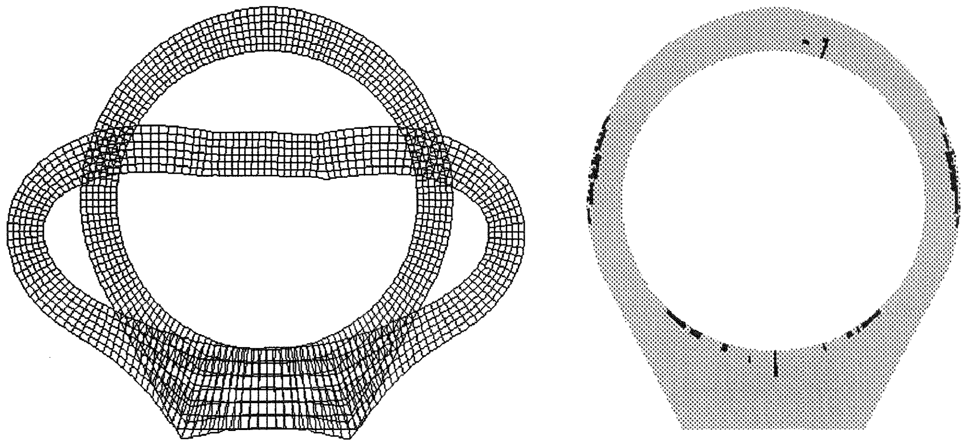

Fig. 4. Finite element mesh (8-node plane elements), deformation and cracking pattern for loading *A*.

The global responses of the pipes are indicated in Figure 5 . It can be noted that the concrete starts to fracture long before the maximum load is reached. The results in Tables 4 and 5 for $l_{ch} = \infty$ were obtained by limit load analysis on the theory of ideal plasticity.

From the results it is obvious that the effect of the fracture mechanics properties of the actual concrete should be considered in the design conversion factor. An alternative, however, is to find, if possible, a test setup such that the conversion factor to the loading *C* becomes insensitive to the value of l_{ch}. Various calculations for pipes with $d_i = 800$ mm, Table 5, as well as for larger pipes have suggested that loading *B* is in fact such a setup. It is therefore probable that future testing will be carried out according to *B*.

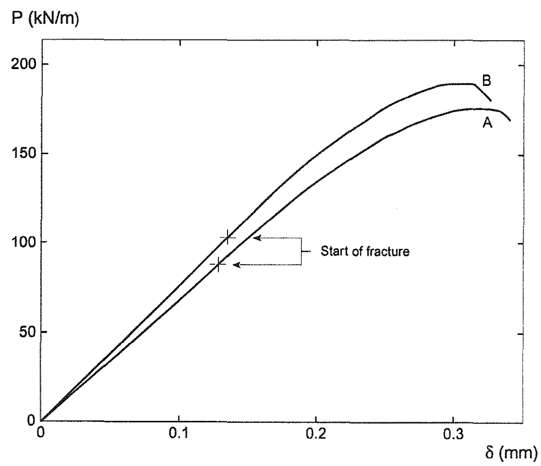

Fig. 5. Load against deflection

4 Comparison to other theories

In Table 1 test results obtained for various circular pipes made of the same concrete were evaluated by a non-linear fracture mechanics model, the fictitious crack model (FCM). In Table 6 the same test series are evaluated also by other theories: the conventional linear elastic brittle theory, a theory of ideal plasticity and the Weibull weakest link theory. Linear elastic fracture mechanics is not applicable to pipes lacking a pre-existing sharp crack of a known and considerable length.

Table 6. Tested bending strengths and the corresponding tensile strengths of the actual concrete as predicted by various theories

Pipe geometry t (mm)	d_i (mm)	Mode of bending	Tested bending strength (MPa)	Corresponding tensile strength of the concrete (MPa) according to elasticity	plasticity	Weibull	FCM
34	100	Beam	7.4	7.4	2.8	6.3	4.9
34	150	" - "	6.8	6.8	2.8	5.9	4.9
34	225	" - "	6.6	6.6	2.9	6.0	5.1
34	225	Ring	11.0	11.0	2.8	10.0	4.9
55	400	" - "	9.6	9.6	2.5	9.3	4.9
			Mean value	8.3	2.8	7.5	4.9
			Variation	24%	6%	26%	1%

In the evaluation by the plastic theory it was assumed that the concrete performs in an ideal plastic manner in compression as well as in tension, assuming the yield stress ratio f_c/f_t to be 12. In the case of yielding only in tension, almost the same results as for $f_c/f_t = 12$ are obtained.

Evaluation by the Weibull theory requires integration over the volume of the pipe with respect to probability of failure. In the present evaluation the Weibull parameter, m, was set equal to 14, corresponding to 10 per cent variation of the strength of the concrete within a pipe. In the evaluation by the fictitious crack model, l_{ch} was assumed to be 380 mm. This estimation of l_{ch} was based on the mix of the concrete. Information about the computational methods used for the various evaluations can be found in Gustafsson (1983 and 1985).

The true tensile strength of the concrete is not known. However, from the mix of the concrete, f_t was estimated to be about 4.3 MPa from tests reported in literature (Petersson, 1981). The direct tensile tests were made on prisms with a volume of 0.2 dm^3, also used as the reference volume in the Weibull analysis.

From Table 6 and a number of similar evaluations it has been concluded that non-linear fracture mechanics (FCM) is a very good theory and, at present, the best theory available for design analysis of concrete pipes. Having in mind that the pipes are made of plain concrete, the theory of ideal plasticity has consistently been found to give surprisingly good results. The size effect, however, can not be analysed, and when predicting load carrying capacity from the basic direct tensile strength, far too high values will be obtained. The Weibull theory, although predicting a size effect, has been found to give bad results. The conventional theory has been found to give the worst results and appears able to produce surprisingly bad predictions.

5 Concluding remarks

It appears from knowledge gained during the past decade that fracture mechanics must be included in any rational theory expected to predict in a realistic manner the load bearing capacity of concrete pipes from basic material parameters. Linear elastic fracture mechanics is, however, not applicable. Instead, some non-linear model, such as the fictitious crack model, must be used.

Although the non-linear fracture models available today are useful in design, they can most certainly be improved and complemented, e.g. with respect to consideration of the scatter of the strength in the concrete and of the tensile plastic hardening. The fracture mechanics methods of design calculations will, in addition to modernization of the practice codes, call for knowledge about the fracture softening properties of the concretes used in producing pipes. When extreme qualities of concrete such as fibre reinforced concrete (Thygesen, 1995) are used, knowledge about the G_f–value is not sufficient; the shape of the tensile stress-deformation response must also be known.

If results of non-linear analysis are not available in a design situation, the ideal plastic theory can be helpful. Knowing the load capacity of one pipe, the plastic theory enables the load capacity of other pipes or differently loaded pipes to be estimated. The conventional linear elastic brittle theory should be abandoned as far as design and applied strength analysis of concrete pipes are concerned.

6 References

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