INTRINSIC MATERIAL LAW FOR PREDICTING SIZE EFFECT IN CONCRETE STRUCTURES

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Abstract

This paper presents some aspects of size effect in concrete structures by means of numerical analyses. Main source of the size effect is the heterogeneity of material structure and the intrinsic material law is the tension softening property. On the basis of the numerical study, existing scaling laws are critically discussed.

1 Introduction

Concrete structures have been usually designed according to empirical formula which were deduced from test results. Recently sizes of concrete structures have become larger and at the same time the strength of concrete has increased. If the strength of concrete structures is independent of the size, prediction of the capacity of large structures can be possible from the existing formula and/or experimental results in a testing laboratory. As already observed by many experimental researchers, however, it is not quite true in many cases. Namely, there is a size effect which is usually defined as the change in the nominal strength due to a change in the size of geometrically similar specimens.

While many experimental results and theoretical studies were reported on the size effect, the questions of scaling law have become one of major issues in the field of fracture mechanics of concrete structures in the 1990s. An international workshop on "Size Effect in Concrete Structures" was organized by Japan Concrete Institute in 1993 and the proceedings was published in 1994 [Mihashi et al., (1994)]. In the proceedings, various experimental aspects of the size effect, the physical mechanisms that cause the size effect, theoretical models and analytical techniques for predicting the size effect, and the design methods for concrete structures that can take the size effect property into consideration are shown. In addition to the papers presented in the workshop, an annotated bibliography on size effect in concrete structures during 1970 and 1993 is also contained.

As concrete is a composite material with at least two phases that is mortar and aggregate, the material structure is disordered. Besides this intrinsic randomness, a boundary layer near the surface of concrete structures has a different composition and strength than the interior because of the wall effect and of diffusion process effects [for example, ACI (1992), and Mirza (1992)]. As a result, strength of concrete structures is random. While the weakest link theory or Weibull model was often used to interpret the randomness [for example, Jayatilaka (1979)], the size effect was also interpreted by the same model on the hypothesis that a larger specimen has a weaker defect. If the strength is determined only by the crack initiation, the model may work but the size effect properties of concrete structures observed in experimental studies cannot be sufficiently described by this model.

Many experimental studies have revealed that most failure phenomena of concrete structures are composed of crack initiation and its propagation. Mihashi (1983) proposed a probabilistic model for quasi-brittle failure which is different from Weibull model, though Bazant and his co-workers have not recognized the difference, yet [Bazant and Xi (1991), Bazant et al. (1994)]. As one of the results obtained from the probabilistic model, Mihashi showed that the difference of the size effect in tensile strength and in compressive one is due to the mechanism of the propagation process [Mihashi (1983)].

Bazant (1984) drove the so-called size effect law given by eq. (1) from a dimensional analysis for the geometrically similar specimens with a notch of the length proportional to the specimen size considering the energy balance at crack propagation in concrete.

$$\sigma_N = Bf_t' / \left(1 + \frac{d}{\lambda_0 d_{max}}\right)^{\frac{1}{2}} \tag{1}$$

where σ_N is nominal strength at failure, f_t' is strength parameter, B and λ_0 are two empirical constants that can be determined by least square method by fitting with test results, d is characteristic specimen size, and da is the maximum aggregate size. To derive the size effect law, Bazant assumed that the potential energy released at the fracture is proportional to the crack length and the area of fracture process zone, the width of the crack band being assumed constant and proportional to the maximum aggregate size.

The size effect law shows that the nominal strength decreases more and more steeply and finally following the size effect law driven from LEFM as the size increases. Several experimental results, however, indicate that the strength of specimens without notch decreases but gradually approaches to a constant value as the size increases [for example, Walraven (1994)]. For overcoming this inadequacy of the size effect law, Kim (1989) proposed to take into consideration the variable size ratio of the characteristic flaw to the specimen.

While concrete is a heterogeneous material as above mentioned, the influence of the disorder on the mechanical properties of the material depends on the ratio of the size of the largest defect due to the heterogeneity to the macroscopic size of the specimen. From such view point, Carpinteri and his co-workers (1994) proposed the multifractal scaling law as follows:

$$\sigma_N = \left(A + \frac{B}{d}\right)^{\frac{1}{2}} \tag{2}$$

where σ_N is nominal tensile strength, d is characteristic structural size, A is a constant with physical dimensions equal to the square of stress, and B is a constant with physical dimensions equal to the square of stress intensity factor. While the two constants need to be determined by means of a non-linear least square numerical analysis for each test series, this model needs experimental results with a much wider range of the size than the size effect law. In addition to this shortcoming, the fundamental hypothesis may be wrong because it predicts that the strength increases with decreasing size even when the specimen is small in comparison with the microstructural characteristic size.

From the practical view point, both of the size effect law and the multifractal scaling law need an experimental work with large scale specimens to determine the empirical constants, though they may give a backbone curve to one design proposal. Once geometry of the structure is changed, another test series need to be carried out. In those tests, should the maximum aggregate size be changed proportionally to the size of the specimen or not necessary ?

The purpose of this paper is to present some results of numerical studies showing that main source of the size effect is the heterogeneity of material structure and that the intrinsic material law is the tension softening property. On the basis of the numerical results, existing scaling laws are critically discussed.

2 Influence of Disordered Material Structure on Fracture Mechanics Parameter

While concrete is considered as a two-phase composite material, the disordered material structure significantly influences mechanical properties of concrete. It has been generally accepted that a microcracking zone is created in front of a notch before the load reaches the maximum. It is so-called fracture process zone. Hillerborg and his co-workers (1976) proposed a fictitious crack model which can analyze the development of the fracture process zone. By means of the fictitious crack model, Petersson (1981) showed the fracture process zone and the stress distribution in front of the notch tip at the maximum load for different beam depth. He concluded that the depth of the fracture process zone increases with increasing beam depth, though the increase in the depth of the fracture process zone is less than that proportional to the increase in the beam depth and consequently the depth ratio of the fracture process zone/beam depth decreases with increasing beam depth. Rokugo and his co-workers (1991) investigated the effects of the specimen size and the cross-section shape on the flexural strength of concrete by means of the fictitious crack model.

Mihashi and Nomura (1992) reported the microcracking properties of concrete studied by means of a three-dimensional acoustic emission technique to reveal that the length of the fracture process zone ahead of the notch tip seems to be independent of the heterogeneity but the width is obviously influenced by the aggregate size which may result in a wider critical crack width of the fictitious crack model. The finding was supported by a work of Otsuka (1994) who showed by a new x-ray inspection technique that the width of the fracture process zone tends to be wider as the maximum aggregate size increases. Otsuka (1994) also reported the influence of specimen size on the length of the fracture process zone to conclude that the length of the fracture process zone at the peak load was not proportional to the specimen size.

The recent findings in experimental studies may suggest that the influence of the disordered material structure on the mechanical behavior of concrete under tensile stress can be analyzed if the constitutive law of the fracture process zone that is tension softening diagram is determined as a function of the material structure. Roelfstra and Wittmann (1986) proposed an inverse technique to determine a tension softening diagram from the given material. Wittmann and his co-workers (1988) analyzed the influence of the specimen size on the tension softening property to obtain an almost unique diagram.

3 Numerical Test Results and Discussion

3.1 Numerical CT tests with a notch proportional to the specimen size

For simulating the size effect law for concrete of different maximum aggregate size, CT specimens with three different sizes (Fig. 1) were analyzed by finite element method using the fictitious crack model. In this case, the notch length was changed proportionally to the specimen size. Three different tension softening diagrams were introduced, which

which is determined as the area under the tension softening diagram shown in Fig. 2, and defined as the amount of energy necessary to create one unit area of a crack [RILEM (1985)]. The other is G_f which is defined as the energy required for crack growth in an infinitely large specimen [RILEM (1990)]. Because of the different definitions, G_f gives about a half value of G_F for the same aggregate size though G_f is less sensitive to the change of the aggregate size (Fig. 4).

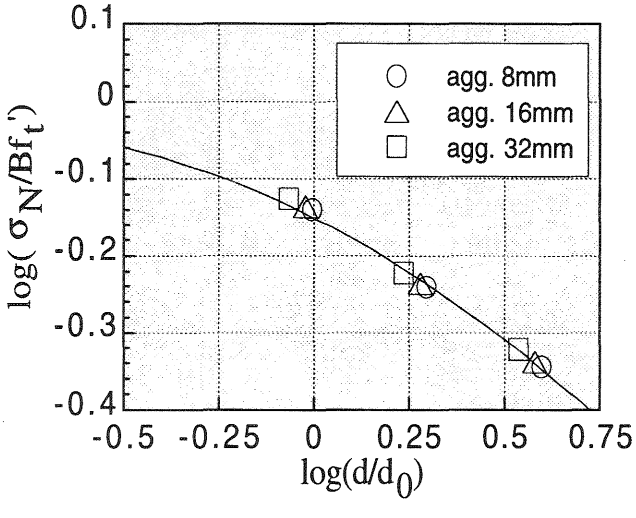

Fig.3. Results of numerical CT tests (size effect law)

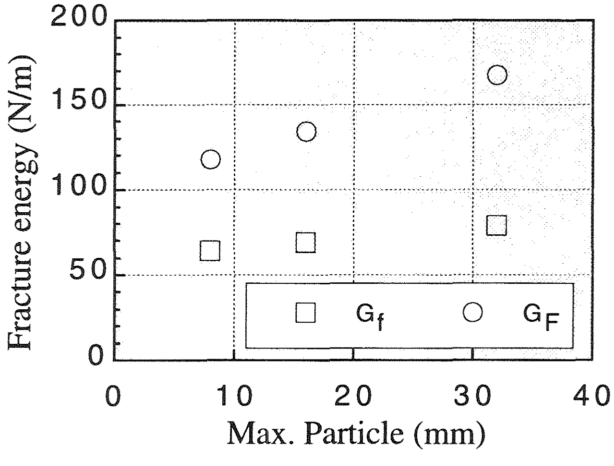

Fig.4. Fracture energy of two different definition

As Fig. 3 shows, the size effect law is supported by the present numerical study even for concrete of different size of the largest aggregate if the notch size responsible for the stress singularity is changed proportionally to the specimen size. In usual cases, however, concrete structures are unnotched and the most dominant material defect in concrete is often the interface of the largest aggregate or void which corresponds to a material structure itself but not necessarily proportional to the specimen size. Neither previous experimental studies have proved that the equivalent notch length to the fracture process zone at the peak load is proportional to the specimen size. An example of the applicable case may be the prediction of pulling out resistance of anchor bolts in a large structure if the defect to cause the stress singularity is much larger than the order of the heterogeneity of the material structure.

3.2 Numerical direct tensile tests with a notch of constant length

Assuming ordinary tensile tests of concrete, concrete plates with a notch of constant length was analyzed by the fictitious crack model (Fig. 5). In this case, the tension softening diagram for the maximum aggregate size of 16 mm was used.

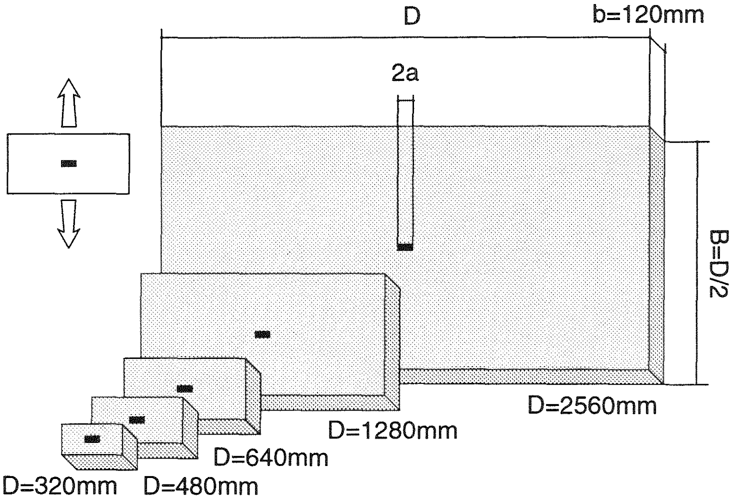

Fig.5. Direct tensile test specimens
(notch length is kept constant)

The simulated relation between the normalized nominal stress and the normalized specimen size is shown in Fig. 6 where the solid line was obtained for the comparison by the size effect law applied to the direct tensile test of a notch length varied proportionally to the specimen size. It is clearly shown that the relation gradually deviates from that of the size effect law if the initial notch length is constant as practically expected. The simulated relation is similar to experimental results of Shioya and Akiyama (1994) who obtained (-1/4)th power law by carrying out size effect tests with large reinforced concrete beams without the stirrup.

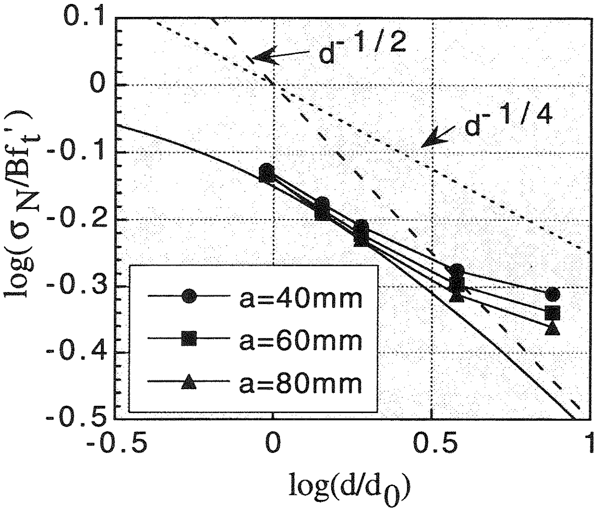

Fig.6. Results of numerical direct tensile tests

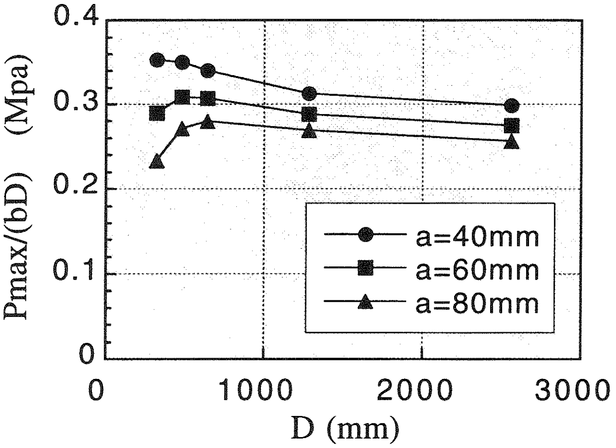

Fig.7. Relation between apparent nominal strength and specimen size

In a practical material test, the nominal strength is usually determined from the peak load divided by the apparent section area of the specimen. Fig. 7 shows the relation between the nominal strength and the specimen size on the condition corresponding to the practical test, which was recalculated from the same results as Fig. 6. While the strength increases as the specimen size decreases in case of a=40 mm, the strength decreases as the size decreases in case of a larger notch embedded. This is because the local mechanical condition is changed due to the interaction between the material defect and the boundary of the specimen when the size of the defect is large in comparison with that of the specimen. This tendency is quite different from that predicted by the multifractal scaling law, though Tanigawa (1978) showed experimentally the same results in the compressive strength as that shown in Fig.7.

4 Conclusions

Numerical studies using a fictitious crack model were carried out to clarify the influence of the material structure on the size effect of concrete structures. The following conclusive remarks were obtained:

1. If the size of the dominant material defect is proportional to the specimen size, the size effect law works well even for concrete of the different material structure.
2. If the size of the dominant material defect is independent of the specimen size, the size effect law doesn't work but the reduction rate of the strength becomes less than that predicted by the size effect law in the range of a very large size.
3. The intrinsic material law in fracture of concrete is the tension softening property. Once the tension softening diagram is determined by a laboratory test for the concrete used in the structure, a numerical simulation can predict the size effect as one example.

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represents the influence of the different maximum aggregate size (Fig. 2).

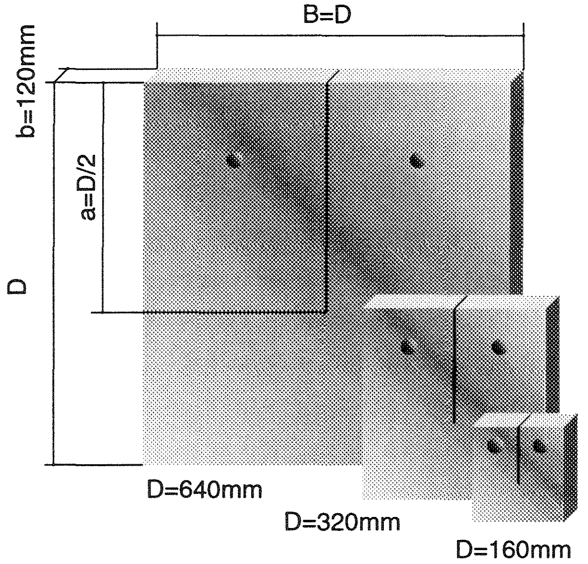

Fig.1. CT specimens of three different sizes
(notch length is changed proportionally to the specimen size)

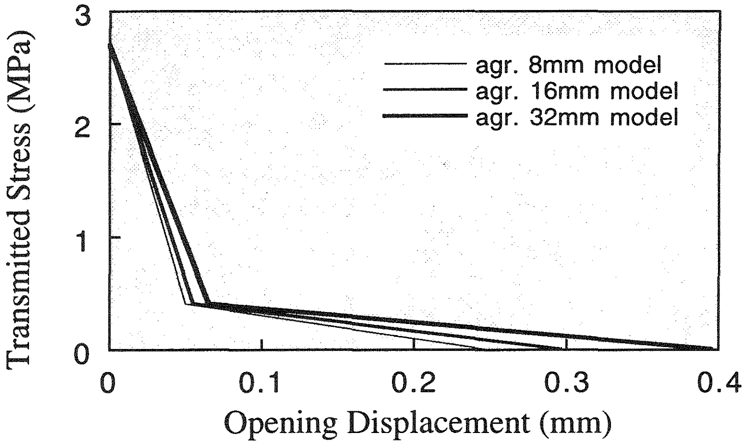

Fig.2. Tension softening diagrams used in simulation

Normalized results of the numerical CT tests are shown in Fig. 3. For each numerical concrete shown in Fig.2, two parameters were determined. When the obtained numerical results of the nominal strength and the specimen size are normalized by the parameters corresponding to each numerical concrete, a unique curve is obtained as shown in Fig.3. From the same results, two different fracture energy are evaluated. One is G_F