BRIDGES BETWEEN DAMAGE AND FRACTURE MECHANICS

Jacky Mazars and Gilles Pijaudier-Cabot,
Laboratoire de Mécanique et Technologie - Ecole Normale Supérieure
94235 Cachan-France

Abstract

Fracture mechanics and Damage mechanics are two corelated theories. In some instances, e.g. for large specimens, crack propagation may be viewed equivalently as a sudden localisation of damage. Relationship between the two theories have been shown in this paper. Two major uses of the proposed equivalence between fracture and damage are shown and applications confirm the interest of these concepts.

1 Introduction

There are two main categories of models which describe the failure processes :
1/ Fracture mechanics, well adapted to describe the separation due to the decohesion of two parts of the continuum [1] & 2/ Damage mechanics, which includes smeared or distributed crack models [2], describes the local effects of microcracking through the evolutions of the mechanical properties of the continuum (stiffness,anisotropie, permanent strain)
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The purpose of this paper is to provide a view on the possible connections between Damage and Fracture mechanics in the particular case of quasi-brittle materials (concrete, rocks, ceramics,..) for which linear approaches are realistic. The objective of this exercice is to offer the possibility to pass from one theory to the other during a same calculation or to obtain, from one theory, informations to use the other. Then, the main issue of this work is to help, at a given modelling point, at choosing the most efficient model.

2 Theoretical concepts

A unified manner to present Damage and Fracture mechanics is the thermodynamical approach [3]. From the free energy of the system considered, Ψ, the state laws give respectively for the damaged material (assumed isotropic) and for the cracked structure (A is the actual area of the crack):

$$Y = \frac{\partial \Psi}{\partial D} = -\frac{1}{2} \Lambda_{ijkl}\, \varepsilon_{ij}\, \varepsilon_{kl} \quad \text{and} \quad G = \frac{\partial \Psi}{\partial A} = \frac{1}{2} q^2 \frac{\partial K}{\partial A} \qquad (1)$$

Λ_{ijkl} and ε_{kl} are respectively the local stiffness and strain component, D is the damage variable and Y is the damage strain energy release rate. q is the displacement supported by the structure at a load Q, K is the global stiffness and G the strain fracture energy release rate.

The respect of the thermodynamical principles is insured if the Clausius-Duhem inequality is respected, which gives for the two considered cases :

$$-Y\dot{D} \geq 0 \rightarrow \frac{1}{2} \Lambda_{ijkl}\, \varepsilon_{ij}\, \varepsilon_{kl}\, \dot{D} \geq 0; -G\dot{A} \geq 0 \rightarrow \frac{1}{2} q^2 \left(-\frac{\partial K}{\partial A}\right) \dot{A} \geq 0 \quad (2)$$

these equations implie $\dot{D} \geq 0$, and $\dot{A} \geq 0$, showing that only micro or macro-cracking progression is possible.

2 Bridges between fracture and damage

2.1 Equivalent crack concept [4]

Considering the similarity of the two approaches, it seems natural to go from one concept to the other. One possible solution is to transform a given damage zone into an equivalent crack. It was shown from (2) that this equivalence is thermodynamically acceptable if the consumption of energy is the same during the 2 processes. Considering the case of LEFM, the critical condition of crack propagation is, $-G=G_C$. Then the equivalent progression dA_e of a crack to a given evolution dD of damage, at point $\underline{x}$ is (3); and if the total evolution, 0—>$D(\underline{x})$ at point $\underline{x}$, is considered, the equivalent crack for the whole process is given by (4) :

$$d A_e = \frac{\int_V -Y\, d D(\underline{x})\, d\underline{x}}{G_c} \quad (3) \; ; \quad A_e = \frac{\int_V \int_0^{D(\underline{x})} -Y\, dD\, d\underline{x}}{G_c} \qquad (4)$$

2.2 Fracture energy and non local damage, analytical way to determine Gf [5] :

Recently Planas and co-workers [6] have derived the relationship between non local models for concrete and the fictitious crack model. In these approaches the fracture energy and the softening behaviour are considered as material constants and the link between those characteristics can be derived explicitly.

Consider an infinite body subjected to uniaxial tension in direction 1, σ^0_{11} with $\sigma^0_{ij}=0$ for $i \neq 1$ and $j \neq 1$. We assume at this stage a distribution of damage, denoted as D^0 and the corresponding strain field is denoted as ε^0_{ij}. When small deviations from this equilibrium state are analysed, harmonic displacement fields are solutions of the partial differential equations : $div(\dot{\sigma}) = 0$ with $\dot{\sigma} = (1 - D^0) E \varepsilon_{11} - E \varepsilon^0_{11} \dot{D}$.

Assuming that the evolution law of damage takes the form $D=f(\bar{\varepsilon})$, where $\bar{\varepsilon}$ is the non local value of the equivalent strain $\tilde{\varepsilon}$ [4]. The wave length of these solutions is given by (5) , where $\bar{\alpha}(\omega,l_c)$ is the Fourier transform of the weight function which depends on the wave length $2\pi / \omega$ and on the characteristic length l_C. The wave length is entirely determined from the evolution law of damage and the internal length of the continuum. The calculation of the approximated fracture energy is based on the assumption that at the onset of strain localisation, i.e. at the onset of localised cracking, the distribution of strain and damage jumps suddenly from a homogeneous distribution to an harmonic solution with the smallest possible wave length. With the minimum wave length, the distribution of damage perpendicularly to the crack direction (the coordinate is denoted as y) is given by (6):

$$\frac{(1-D^0)}{\varepsilon^0_{11}\dfrac{\partial f}{\partial \bar{\varepsilon}^0}} = \bar{\alpha}(\omega,l_c) \qquad (5)$$

$$D(x_2) = \frac{\int_{-\infty}^{+\infty} \alpha(x_2 - s_2)\eta(s_2)ds_2}{\int_{-\infty}^{+\infty} \alpha(s_2)\eta(s_2)ds_2} \qquad (6)$$

with $\eta(x_2) = \cos(\omega_{\max} x_2)$ if $x_2 \in \left[\dfrac{-\pi}{2\omega_{\max}}, \dfrac{\pi}{2\omega_{\max}}\right]$ *and* $D(y) \geq 0$

$\eta(x_2) = 0$ elswhere

The energy consumption due to crack propagation is the integral of the energy dissipation at each material point of coordinate y in the fracture process zone which encountered damage up to $D(y)$, and can be deduced easily from eq.(4).

3 Applications

3.1 Behaviour of a structure with a combined approach Damage-Fracture Mechanics

The structure considered is a compact tension specimen (figure 1-a) a serie of which were tested at LMT Cachan [4]. In order to simulate the behaviour we propose 2 kinds of calculation : **1/** from O to B with a non local damage model & **2/** from B to C (softening) with Linear Elastic Fracture Mechanics.

The bridge from the first calculation to the other uses directly the equivalent crack concept previously presented, that necessitates to predetermine the evolution K=K(A) and (-dK/dA) . The following parameters have been used :

- critical fracture energy at point B : Q_B=18.9kN, q_B=0.2E-03m, K_B= 9.5E+04kN/m, $(-dK/dA)_B$=51E+05kN/m^3 which gives, $G_c = 1/2 q_B^2 (-dK/dA)_B$ = 102N/m
- LEFM calculation, from Eq. (2) one can deduce $q=\sqrt{(2G_c)/(-dK/dA)}$, which allows to determine Q=Kq for any values of A.

The part of the behaviour deduced from LEFM calculation looks in accordance with the experimental curve (fig.1-c), even if it is obvious than Gc is not constant (see fig.1-b)

3.2 The prevision of G_c for large specimen when size effect is exhibited

The structures considered for this presentation are notched beams of different sizes, the geometry of which is given figure 2. The tests have been done at Lund University and the calculations, using a non local damage model, have been performed at LMT Cachan. To emphasize the size effect, we used the classical presentation into the log-log stress-size diagram. From these results and using a regression line, the Bazant size effect law (S.E.L.[7]) has been determined (see figure 2).

The size effect method which consists in the determination of fracture parameters from asymptotic values of the S.E.L., leads to K_c=1.49 MPa$\sqrt{m}$ (or G_c=65.8 N/m).

From the non local parameters the analytical determination of G_f previouly presented leads to G_c=55.6 N/m or K_c=1.37 MPa$\sqrt{m}$ .Valide for large specimen as indicated before, this value can be reported into the log-log stress-size diagram as an asymptote, which is, *in fine,* close to the one found from the size effect law.

Remark : Conversely, it is possible to represent cracking by an equivalent damage zone. This equivalence follows from the same assumptions as those used for the derivation of the fracture energy. Given a crack observed on a structure, the approximation yields an equivalent map of damage. See [8] [9] for more details.

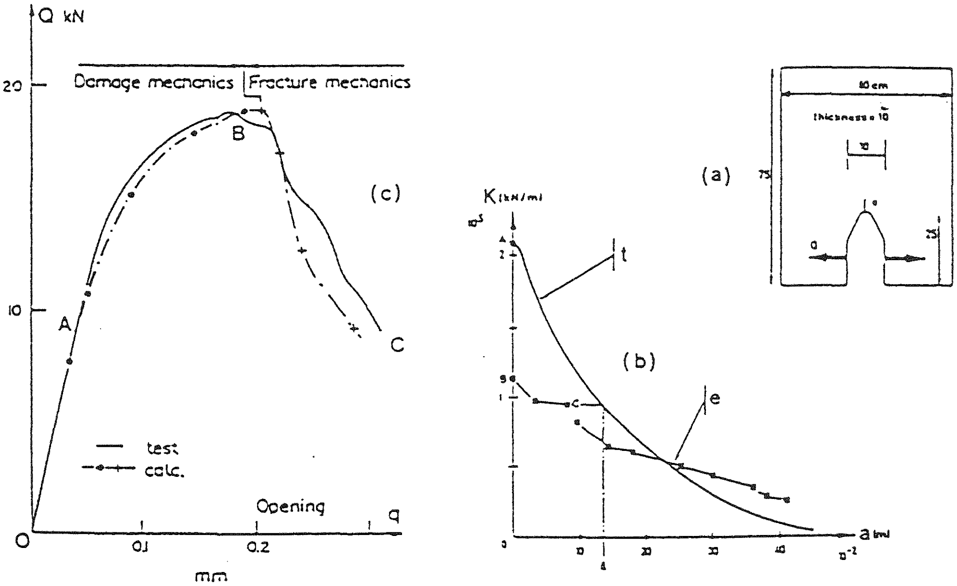

Figure 1: Compact tension specimen; a/ geometry; b/ evolution of the stiffness with the crack growth, (t) theoretical, (e) experimental; c/ global behaviour, calculation is performed using the Damage-Fracture combined approach.

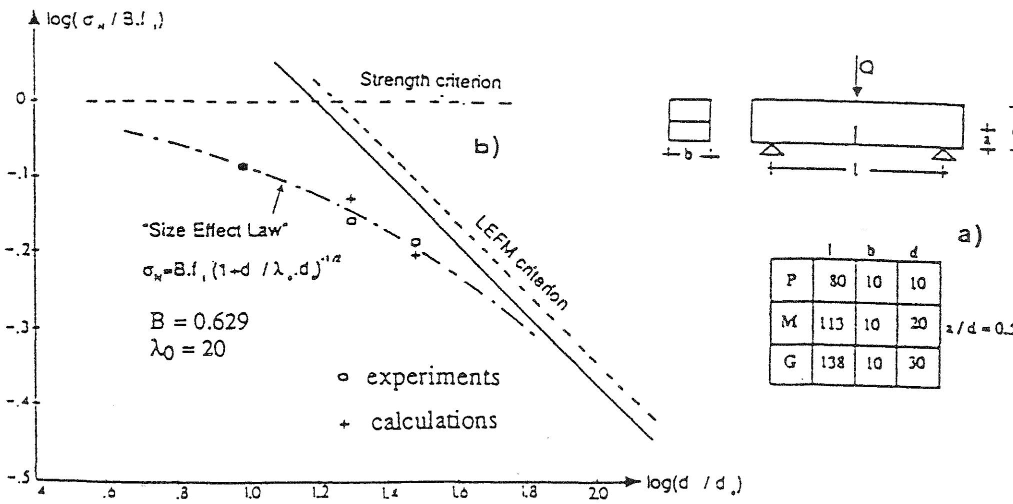

Figure 2: Notched beams; a/ the 3 different geometries P,M,G (values are given in cm); b/ comparison of the experimental and calculated beam strengths (by the non local damage model) in the "size effect log-log diagram" , (-----) asymptote issued from the "size effect law", (——) asymptote issued from the present analytical calculation of G_c.

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