Fracture Mechanics of Concrete Structures
**Proceedings FRAMCOS-3**
AEDIFICATIO Publishers, D-79104 Freiburg, Germany

# EFFECT OF SPECIMEN SIZE AND LOADING RATE ON THE TENSION-SOFTENING CURVE OBTAINED BY BACK ANALYSIS METHOD

T. Okada
Institute of Technology, Shimizu Corporation, Japan
H. Horii
Department of Civil Engineering, University of Tokyo, Japan

**Abstract**

The tension-softening curve represents material characteristics of tensile crack growth in concrete. To carry out crack growth analysis, the tension-softening curve of the material used in the large scaled structure must be determined from appropriate material tests. In the present study, wedge splitting tests are carried out and the tension-softening curves are obtained by two back analysis programs.It is shown that the tension-softening curves by two different back analysis programs more or less agree. The fracture energy calculated from the load-displacement curves agree with the area under the tension-softening curve obtained by the back analysis. While the values of the fracture energy scatter more in the small specimen, the average values of the fracture energy do not show clear dependency on the specimen size.

With increasing loading rate, the fracture energy tends to increase.

Key words : fracture energy, tension-softening curve, back analysis, specimen size, loading rate

## 1 Introduction

In simulating cracking in concrete through a numerical analysis, it becomes necessary to determine not only the fracture energy, $G_F$, but also the relationship between transfer stress and crack width in a crack growth region, i.e. the tension-softening curve of concrete. The reason is that even at the same fracture energy, the tension-softening curve of concrete may vary resulting in different load-displacement curves. With attention focused on the effects of specimen size and loading rate, in this study wedge splitting tests on concrete with 20 mm in maximum grain size are carried out.

As structures increase in size, the effect that apparent compressive, tensile and bending strengths become small, known as scale effect, becomes a problem. Essentially, it is desirable to find fracture characteristics using large-scale concrete specimens. Various methods for investigating the material characteristics of concrete are available. Lately, the scale effect is analyzed by investigating the material characteristics through tests based on fracture mechanics.

Among the tests are

(1) Notched beam bending test proposed by the RILEM

and

(2) Wedge splitting test.

These methods have their own characteristic features. To elucidate the effects of specimen size on the tension-softening curve of concrete, this study adopts the wedge splitting test. The testing method conforms to TC51-ALC, 78-MCA proposed by the RILEM AAC 13.(1992)

## 2 Experimental procedures

### 2.1 Test series and preparation of specimens

Two basic testing parameters are selected: specimen size and loading rate. The type and dimensions of test specimens used for the wedge splitting test are shown in Table 1 and Fig. 1. Specimens of three sizes, 200 x 200 x 100 mm, 800 x 800 x 400 mm, and 1200 x 1200 x 450mm are tested with four different loading (crack mouth opening displacement: CMOD) rates, 0.1 mm/min., 1.0 mm/min., 10 mm/min., and 340 mm/min. The faster rates are employed only for the small specimen. Crack velocities are measured by five clip gages attached along the groove ahead of the initial notch.

Table 2 shows the mix proportions of concrete used for the test.

Table 1. Characteristics of the different test series

| Loading rate (mm/min)* | Size of specimans (mm)** | | |
|---|---|---|---|
| | Small size 200×200×100 (Lo=100) | Medium size 800×800×400 (Lo=400) | Large size 1200×1200×450 (Lo=600) |
| Low rate 0.1 | SS [3]*** | MS [4] | LS [4] |
| Medium rate 1.0 | SM [3] | MM [4] | LM [4] |
| Fast rate 10.0 | SF [3] | MF [4] | LF [3] |
| Faster rate 340.0 | SU [2] | | |

* Velocity of crack mouth opening displacement (CMOD)
** Height, width, thickness and (ligament length Lo)
*** Number of specimens

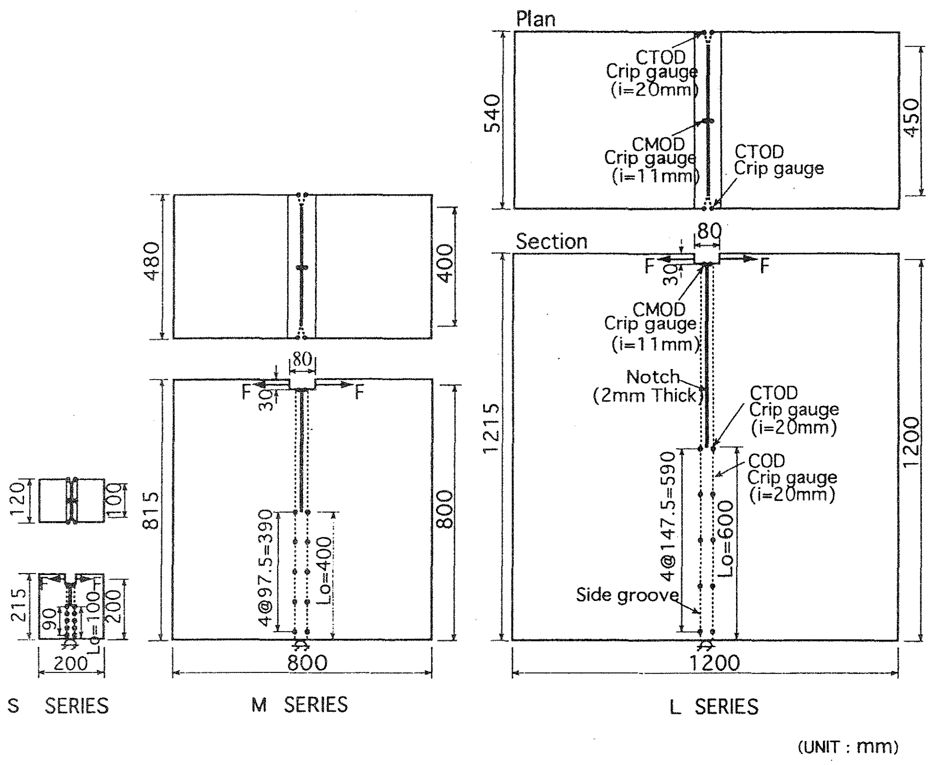


Fig.1. Geometry and dimensions of wedge splitting test specimens

Table 2. Mix proportions of concrete used

| Slump (cm) | Air (%) | Water/ cement ratio W/C (%) | Modulus of fine agg. s/a (%) | (kg/m³) Water W | Cement C | Fine agg. S | Coarse agg.* G | AE admixture |
|---|---|---|---|---|---|---|---|---|
| 3± 1.0 | 3±1.0 | 65.7 | 26.7 | 92 | 140 | 547 | 1635 | 3.92 |

* Maximum grain size, $d_{max}$ =20mm

Concrete is cast into steel forms for the specimens of three different sizes, and those for standard specimens ( JIS A 1108,1113 ) of ø150 x 300 mm and 150 x 150 x 530 mm in size for measuring standard strength. The forms are capped about 24 hours after demolding the specimens for curing at site. A notch of each test specimen is formed by embedding a 2 mm thick greased steel plate in a slit of a steel form before casting concrete, and pulling it out 24 hours after casting concrete.

At the center of the steel form, a notch-shaped steel piece is attached to form a groove having a depth 10% of the thickness of a specimen along the slit and ligament on both sides of each specimen.

### 2.2 Testing procedures

A servo controlled hydraulic jack with a maximum capacity of 500 kN is used to control CMOD at constant speeds. Four displacement rates are set: low rate of v = 0.1 mm/min. (hereinafter referred to as the quasi-static loading condition); medium rate, or a relatively low rate of v = 1 mm/min.; fast rate, or a relatively high rate of v = 10 mm/min.; and the fastest rate, or a rate of v = 340 mm/min. A total of ten clip gages are attached to the crack tip and ligament where the crack will develop on both sides of a specimen to measure crack opening displacements (hereinafter referred respectively to as CTOD and COD1 to COD4) and the distribution of the displacements.

## 3 Test results and discussions

The test is conducted on the specimens at the age of 26 to 40 days. The small difference in compressive strength obtained between the time the test is started and completed is about 10%. The averaged mechanical properties of the concrete are as follows.

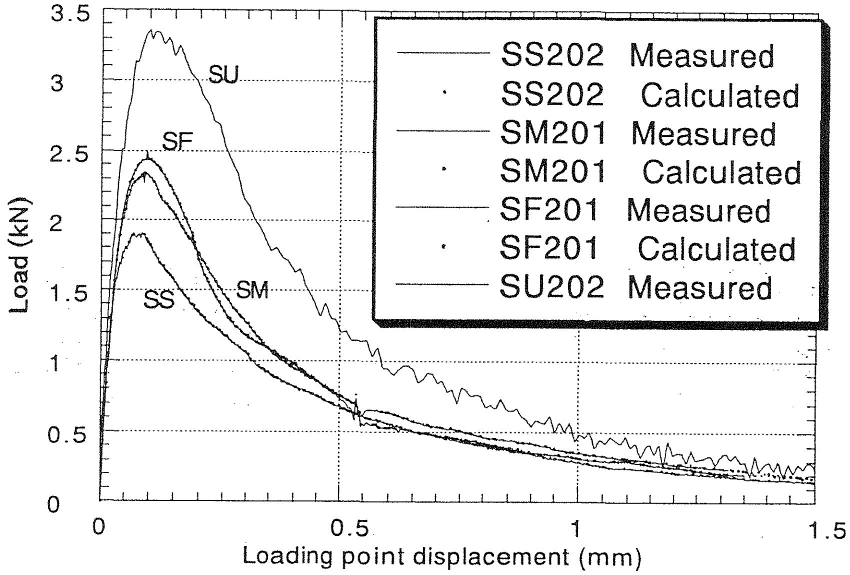


Fig.2 Measured load-displacement curve and simulated curves of specimens at different loading rates.

Compressive strength, $f_{cm}$ = 26.4 MPa
Tensile strength, $f_{ctm}$ = 2.11 MPa
Bending strength, $f_{bm}$ = 2.46 MPa
Modulus of elasticity, $E_c$ = 27.6 MPa
Poisson's ratio = 0.21

Fig. 2 shows the relationship between the load F and the loading point displacement of specimens at different loading rates. Fig. 3 shows the relationship between the load F and the loading point displacement of specimens of different sizes with different ligament lengths. The maximum value of fracture energy is calculated by dividing the area surrounded by the load F versus loading point displacement curve up to the final measuring point (a measurement is finished when the load drops to about 5-7% of the maximum load) by the area of the ligament.

With regard to the variations of the load versus loading point displacement curve of specimens of the same type, the coefficients of variation of the maximum load and fracture energy are 13% and 19%, respectively. At medium, fast and faster displacement rates the maximum load normalized by that under the quasi-static loading condition are 1.3, 1.3 and 1.8, respectively. Fig. 4 shows the relationship between the loading rate and the ratio of specific fracture energy $G_{FS}$ to fracture energy $G_{FSC}$ under the quasi-static loading condition. The normalized specific fracture energy are 1.3, 1.0 and 2.0, at the medium, fast and faster rate, respectively. Fig. 5 shows the relationship between ligament length and the ratio of specific fracture

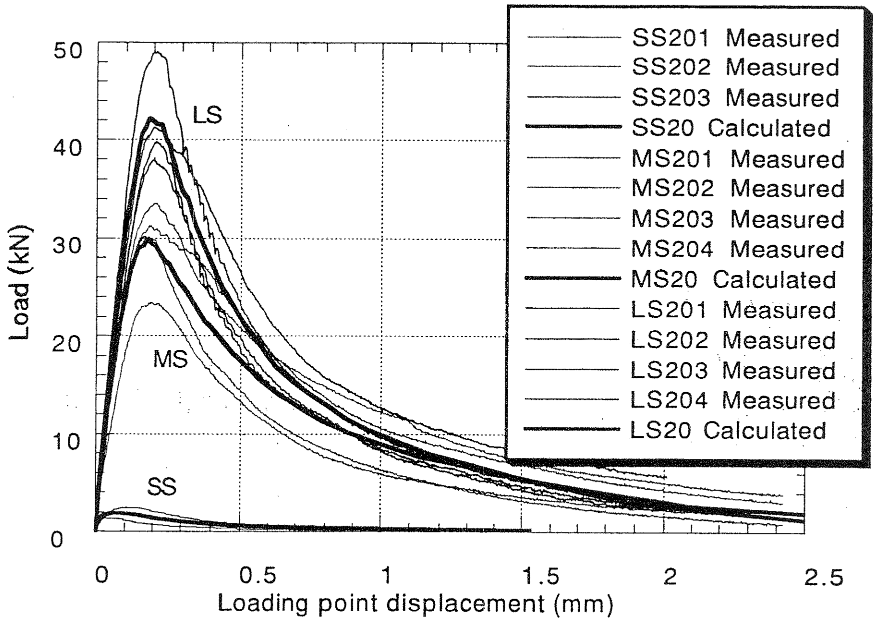


Fig.3. Measured load-displacement curve and simulated curves of specimens with different ligament lengths.

energy $G_{FS}$ to fracture energy $G_{FSS}$ for the small size specimen. The normalized fracture energy of medium- and large-sized specimens are about 1.5 and 1.0 respectively. The test data presented by Wittmann, Rokugo et al. (1998) were obtained under testing conditions similar to those for this test : that is, with a compressive strength of 43 MPa, $d_{max}$ of 16 mm , and a loading rate of 0.2 mm/min. Like in the test data presented by Wittmann, et al., the tendency of fracture energy to increase is seen as the ligament length increases from 10 cm to 30 cm, but the energy remains unchanged at the ligament length of 60 cm. It is considered that there is no noticeable tendency to increase if the large coefficient of variation of $G_{FS}$ (19 %) is noticed. This implies that the tension-softening curve can be understood as the material characteristics and the size of the specimen can be determined so that the scatter of the results is small enough with a limited number of tests. Particularly, it is necessary to increase the number of small-sized specimens to 6 to 10 because the fracture energy of small-sized specimens varies greatly.

Fig.6 shows the typical distribution of measured opening displacements at the ligament when CTOD is 0.15 mm. In this figure, the data obtained from specimens of three different sizes at two different loading rates are shown. As is obvious from the figure, crack opening displacements are distributed almost linearly from the tip of the notch.

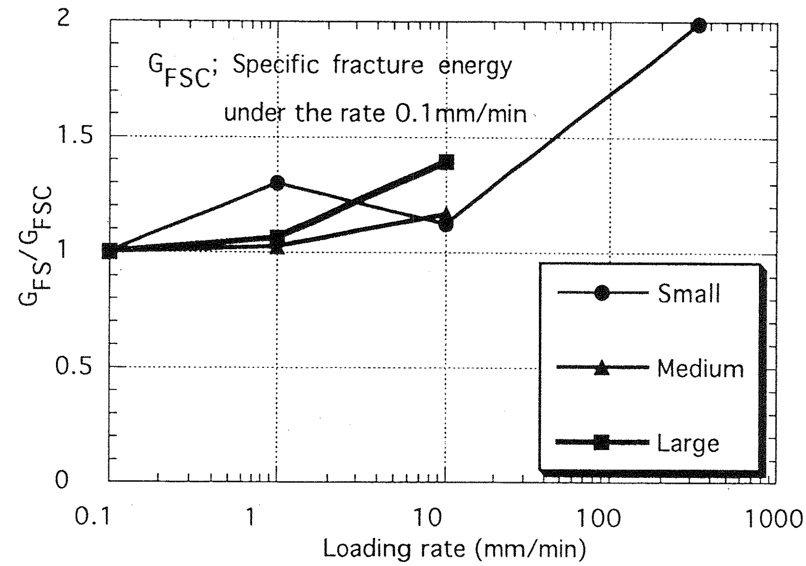


Fig.4 Effect of loading rate on $G_{FS}/G_{FSC}$

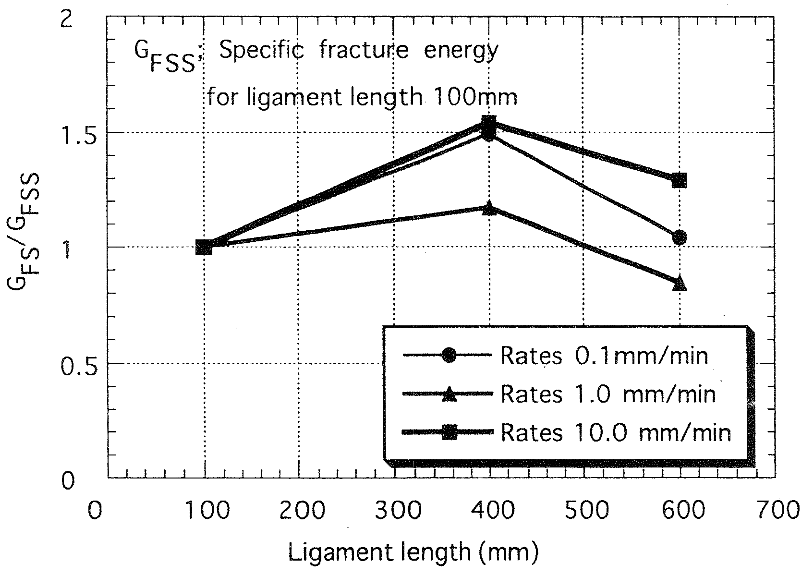


Fig.5 Effect of ligament length on $G_{FS}/G_{FSS}$

A crack is initiated when CTOD is in a range of 0.005-0.02 mm, and grows up to about 50% of the ligament by the time when the load reaches the maximum. Assuming that the distribution of crack opening displacements is linear, the location of the crack tip is estimated from the measured opening displacement at each moment. Then the crack velocity is calculated. The crack velocities in small-sized specimens

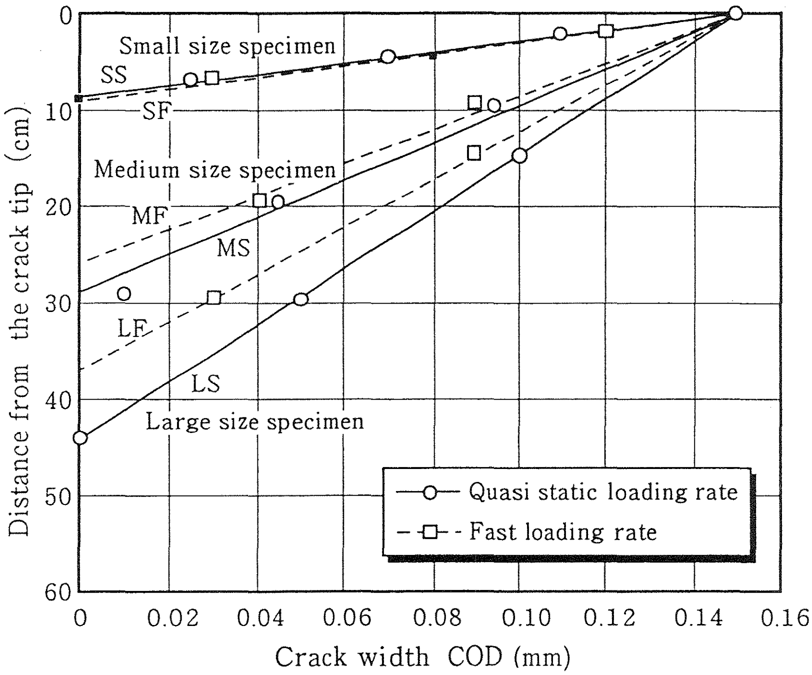


Fig.6 Distribution of crack opening displacement

estimated when the crack length is about 60-80% of the ligament are 0.3, 7, 56 and 379 mm/sec at the quasi-static loading rate, medium, fast, and faster rate, respectively. The similar velocities are also calculatedfrom medium-and large-sized specimens. It is observed that crack velocities are roughly proportional to CMOD rates.

## 4 Determination of tension-softening curve by back analysis

In order to analyze crack growth in concrete, it is necessary to obtain a tension-softening curve. However, it is very difficult to obtain the curve directly from a tensile test. Recently, a method was proposed in which the tension-softening curve is obtained by back analysis of a bending or wedge splitting test results. The back analysis is conducted by a finite element method using elements having discontinuous surfaces; see Uchida et al. (1995) and Nanakorn et al. (1996).

In the present study, the tension-softening curve is estimated from the actually measured load F versus CMOD curve by two methods: Back

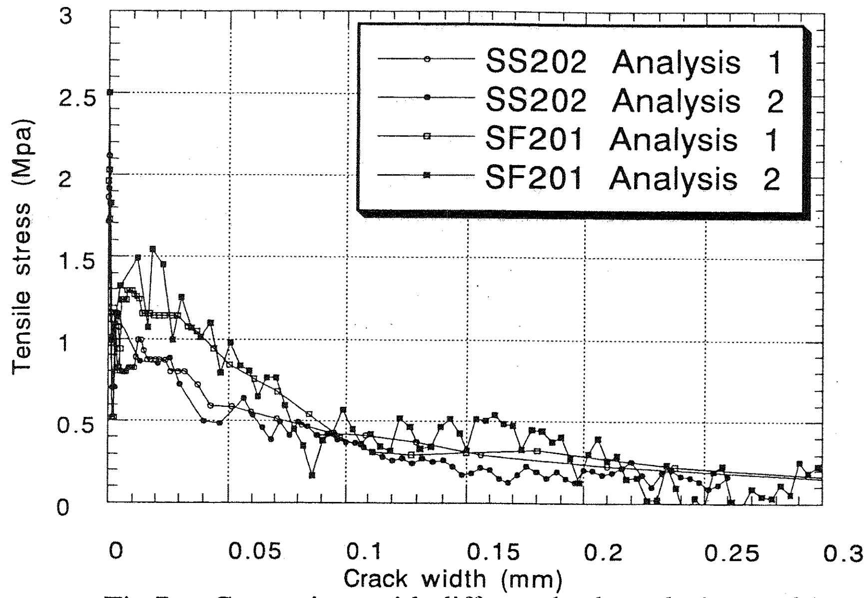


Fig.7. Comparison with different back analysis 1 and 2 on the tention softening curve

analysis program 1 by Uchida et al. (1995) and Back analysis program 2 by Nanakorn et al. (1996). These back analysis programs use a piecewise-linear function for the tension softening curve. The methods determine the slope of the tension-softening curve successively so that measured load-displacement relation agrees with analyzed one. In the present study, the number of elements along the ligament on the finite element mesh used for Back analysis program 1 and 2 is 47 and 32, respectively. Fig. 7 makes a comparison between the tension-softening curves of specimens SS202 and SF201 obtained by Back analysis program 1 and those by Back analysis program 2. The tension-softening curves of the specimens obtained by Back analysis program 1 are plotted with a open circle, and those by Back analysis program 2 with a solid circle. There is a relatively good agreement between tension-softening curves calculated by Back analysis program 1 and 2, both in the stress when softening starts and the shape of the curve.

Fig.8 and Fig.9 shows the tension-softening curves obtained by Back analysis program 1 normalized by tensile splitting strength, $f_{ctm}$, of 2.11 MPa. These curves were obtained using F vs. CMOD relation, which was an average of the data measured for all the identical specimens under the same conditions. The load F vs. loading point displacement curves, which are calculated from the obtained tension-softening curves, are shown with dots in Fig. 2 and with heavy lines in Fig. 3 together with corresponding measured data. It is natural to have agreement between the original F vs. loading point displacement curve and that

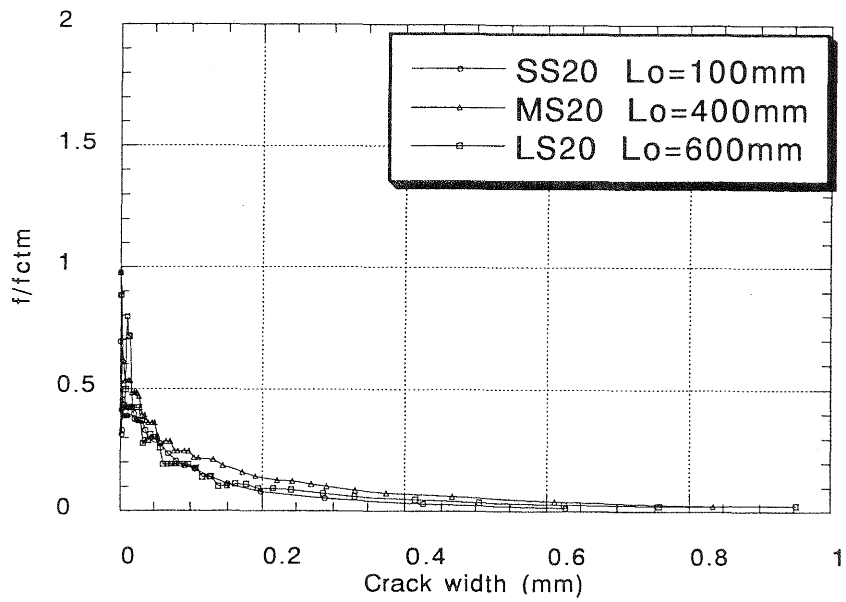


Fig.8 Effect of the different ligament length on the normalized by tensile splitting strength, $f_{ctm}$

calculated with the tension-softening curve obtained from the F vs. CMOD curve by the back analysis.

The calculated stress at which softening starts, i.e. tensile strength, is shown in Table 3. The stress at which softening starts is almost constant even when specimen size differs. Estimated tensile strength at the quasi-static loading rate is about 2.06 MPa for small- and medium-sized specimens, and 1.86 MPa for large-sized ones. The tensile splitting strength of standard specimens is 2.11 MPa. A tendency of the tensile strength obtained by the back analysis to increase with an increase in displacement rate is seen for specimens of all sizes. The tensile strength at the faster rate is about 160% of that at the quasi-static loading rate. Table 3 also shows the fracture energy, $G_F$, calculated from the area under the tension-softening curve. A clear tendency of variation of the fracture energy with changing specimen size is not seen. On the other hand, the fracture energy tends to increase with an increase in loading rate. The fracture energy at the faster rate is about 190% of that at the quasi-static loading rate. The fracture energy, $G_F$, calculated from the tension-softening curve agrees with the specific fracture energy, $G_{FS}$, from the measured load F vs. loading point displacement curve.

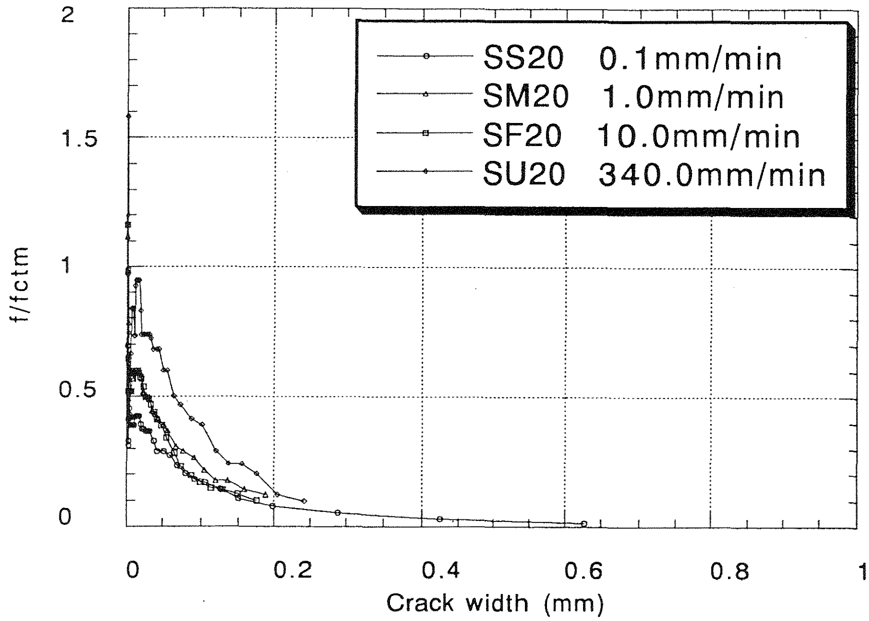


Fig.9 Effect of the different loading rate on the normalized by tensile splitting strength, $f_{ctm}$

Table.3. Numerically predicted tensile strength and $G_F$ value

| Loading rates (mm/min) | Small size (Lo=100mm) | | Medium size (Lo=400mm) | | Large size (Lo=600mm) | |
|---|---|---|---|---|---|---|
| | Tensile strength $f_t$ (MPa) | Fracture energy $G_F$ (N/m) | Tensile strength $f_t$ (MPa) | Fracture energy $G_F$ (N/m) | Tensile strength $f_t$ (MPa) | Fracture energy $G_F$ (N/m) |
| Low 0.1 | 2.06 | 122 | 2.06 | 200 | 1.86 | 158 |
| Medium 1.0 | 2.35 | 168 | 2.45 | 207 | 1.96 | 157 |
| Fast 10.0 | 2.45 | 142 | 2.74 | 230 | 2.25 | 201 |
| Faster 340.0 | 3.33 | 235 | | | | |

## 5 Conclusions

The results of this study are summarized as follows.

1. With regard to the variations of the load F versus loading point displacement curve of specimens of the same type, the coefficients of variation of the maximum load and the specific fracture energy are 13%

and 19%, respectively. The maximum load and the specific fracture energy tends to increase with the loading rates. But no noticeable effect of specimen size on the specific fracture energy is seen.

2. The average crack velocities of small-sized specimens at the quasi-static rate, medium, fast, and faster rate, which are estimated from the measured distribution of crack opening displacements along the ligament, are 0.3, 7, 56 and 379 mm/sec. respectively. Similar velocities are obtained from medium- and large-sized specimens. It is seen that crack velocities are roughly proportional to CMOD rates.

3. The tension-softening curve is calculated from the measured load F versus CMOD curve by two methods. There is a relatively good agreement between tension-softening curves and tensile strengths calculated by Back analysis program 1 and 2.

4. The estimated tensile strength at the quasi-static loading rate is close to the tensile splitting strength of the standard specimen. It is observed that the estimated tensile strength tends to increase with an increase in the loading rate.

5. With increasing loading rate, the fracture energy tends to increase. The rate of increase is small when the rate of crack mouth opening displacement is less than 10 mm/min.

6. While the values of the fracture energy scatter more in the small specimen, the average values of the fracture energy do not show clear dependency on the specimen size.

## References


Nanakorn, P., Horii, H. (1997) Back analysis of tension-softening relationship of concrete, **J. Materials, Conc. Struct., Pavements, JSCE,** 544／32, 265-275

RILEM, AAC13.1(1992) Determination of the specific fracture energy and strain softening of concrete AAC, TC51-ALC, 78-MC.

Uchida, Y., Kurihara, N., Rokugo, K. and Koyanagi, W. (1995) Determination of tension softening diagrams of various kinds of concrete by numerical analysis, **Fracture Mechanics of Concrete Structures**, Proceedings FRAMCOS-2, (ed. F.H. Wittmann), AEDIFICATIO Publishers, Germany, 17-30

Wittmann, F.H., Rokugo, K., Bruhwiler, E., Mihashi, H. and Simonin, P.(1988) Fracture energy and strain softening of concrete as determined by means of compact tension specimens, **Materials and Structures**, 21,21-32.